PUTNAM'S ARITHMETIC.

THE

AMERICAN

COMMON-SCHOOL ARITHMETIC;

IN WHICH

THE PRINCIPLES OF THE SCIENCE ARE FULLY EXPLAINED, AND APPLIED TO THE SOLUTION OF A GREAT VARIETY OF

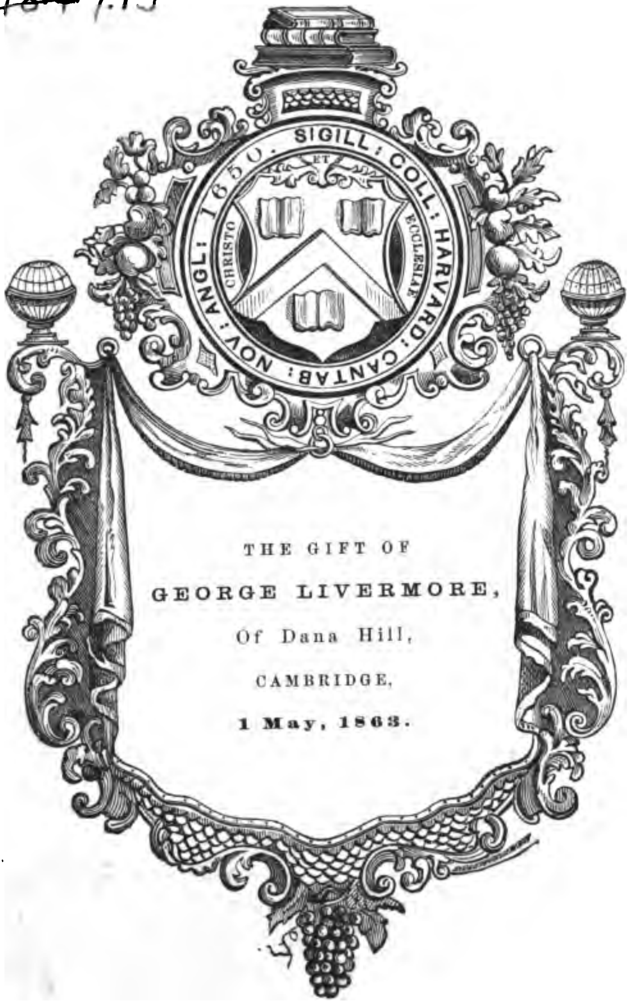
PRACTICAL EXAMPLES.

DESIGNED FOR THE USE OF

COMMON SCHOOLS AND ACADEMIES.

BY RUFUS PUTNAM,
PRINCIPAL OF THE [illegible] GRAMMAR SCHOOL, SALEM, MASS.

BOSTON:
TAPPAN, WHITTEMORE & MASON,
114 Washington Street.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PREFACE.

"WHAT are the peculiarities of the work?" "In what respects does it differ from books on this subject already in use?" are among the first questions propounded to the publisher who attempts to introduce a new school-book to a community already surfeited with text-books in the various departments of science.

To these and similar questions, it may be replied, that while the present work is, in many respects, like its predecessors, it differs from most of them in some important particulars, among which are the following.

1. In the great variety and amount of Arithmetical matter in a comparatively small compass.
2. In its practical character.
3. In treating of Integral and Fractional Decimals in connection.
4. In the number and variety of mental and oral exercises interspersed through the book, in connection with exercises for the slate.
5. In embracing the details of the science under general principles, and avoiding, as much as practicable, a multiplicity of specific rules.

A few remarks will illustrate the object and extent of these peculiarities.

1. The number of exercises for the pupil, and the amount and variety of arithmetical matter, is much greater than from the size of the book would seem possible. For example, the pupil is directed to "Multiply 56789 by 2; by 4; by 5; by 6; by 7; by 8; by 9;"—and to "Divide 30654016 by 13; by 14; by 15; by 16; by 17; by 18; by 19;"—and, in Fractions, to divide mentally $5\frac{5}{8}$ by 2; by 3; 4; 5; 6; 7; 8; 9;—and to "Compute the interest on $1418.46 from Jan. 15, 1841, to Nov. 25, 1846; from Dec. 14, 1845, to May 6, 1848; from July 4, 1776, to March 23, 1781; from Aug. 25, 1837, to Nov. 16, 1848," &c. Thus, without embarrassing the pupil, exercises are embraced in a single line, which might easily be made to occupy, and which, in most books, are made to occupy, several times as much space. By this means, a great number of mental and oral, as well as written exercises, of practical importance is introduced, without making the size and expense of the book greater than of those books from which such matter is excluded.

2. While the author has aimed to make the work strictly elementary, to lead the pupil to analyze and demonstrate, as far as possible, every important principle, and to present every topic in the manner which his experience has suggested as best adapted to the capacities of the learners, as well as best fitted to develop their mental powers, he has kept in view the maxim that "Children should be taught those things while young which they are to practise when they become men."

The modes of computing time between different dates, (Art. 69,) Discount, Accounts Current and Interest Accounts, and many other topics, though different from what the learner is taught in some text-

books in arithmetic, will be found to accord with the practice of men of business at the present time; so that the learner, when he goes from the school-room into the world, and from dealing with text-books comes to deal with men, may not be obliged to unlearn what he has learned at school, before he can put in practice the methods adopted by business men, in solving questions that occur in their intercourse with each other.

The different methods of equating accounts current and interest accounts, though discussed in very few arithmetics for common schools, are of so much practical importance to most young men of good education, that the author has introduced them into this work. These topics, when properly presented, are not so difficult to be understood as much of the less practical matter which is found in most text-books in arithmetic. If the exercises on these topics contained in this book are fully mastered, the pupil may anticipate little difficulty in understanding and adjusting such accounts when he shall meet them in actual business. The sections which treat of Mensuration, the Mechanic Powers, and Book-keeping, will be valuable to those who have not time or opportunity to study more extended treatises on these subjects.

The numerous articles on Analysis interspersed through the work will, it is believed, be peculiarly acceptable to the teacher. The author would call the attention of teachers and others to Art. 101, 122, 124, 131, 138, 146, and Sections XVIII. and XIX.

3. The authors of most arithmetics seem to have labored to divorce Integral and Fractional Decimals from each other, having almost uniformly presented Decimal Fractions to the pupil as something to be studied, if ever, after all the elementary rules, Compound Numbers, Vulgar Fractions, &c., have been learned; as though their relation to integers was so slight, and the subject so difficult, or so unimportant, that it should be wholly omitted until these other things are learned.* The consequence has been, that a large majority of those who have completed their common school education have either not studied decimals at all, or have done it with the feeling that a knowledge of them was of little practical utility; and, therefore, the knowledge they have acquired of the subject has been altogether vague and indefinite. This ignorance is the more to be lamented in our own country, from the fact that our currency is a *decimal* currency, and that operations in it can be safely trusted to those only who are familiar with the principles of Decimal Notation, fractional as well as integral. Numerous specific rules, indeed, are given to the learner for performing operations in dollars, cents, and mills; but these rules, being arbitrary, and based upon no general principle already explained and understood, are very uncertain in their application, for the reason, if for no other, that arbitrary rules are easily forgotten. It is presumed that the experience of every reader, whether a teacher or not, will fully attest to the truth of these remarks.

* The works on arithmetic by Pliny E. Chase are the only exceptions to this remark that the author has seen.

The author of this book has endeavored so to present the whole subject of decimal notation, that the pupil will become as familiar with *fractional* as with *integral* decimals, and as little liable to embarrassment or error in one case as in the other; thus rendering the whole class of specific and arbitrary rules, relating to operations in dollars, cents and mills, entirely useless. Whether the effort has been a successful one, remains for those who may use his book to decide. He has no doubt of its practicability, or of its necessity, in order to make the youth of our country expert and correct accountants in our Federal currency. This mode of presenting decimals to the learner may require some more patient labor at first, but this will be amply repaid by his subsequent progress.

4. So far as the author's experience and observation extend, mental exercises in arithmetic are very generally neglected after the pupil has commenced "ciphering" on the slate. This is doubtless owing partly to the fact that very few arithmetics have been prepared in which mental and written exercises are combined to any great extent in the same book. The author has endeavored to supply this want, by introducing a great variety of mental exercises, by which means the pupil is not only allowed to retain his familiarity with what he has learned from Colburn, but is allowed to make still greater proficiency in mental calculations.

There are in our common schools different classes of learners, whose wants the author has had in view, in the preparation of this work. The first, and probably the largest of these classes, consists of those who never progress to the more advanced rules, and whose knowledge of arithmetic must be chiefly limited to the elementary rules. In this work, a great variety of practical examples are introduced under these elementary rules, so that it is believed that the scholar who can perform intelligently and with facility the questions on the first 83 pages, can solve correctly ninety-nine in a hundred of the practical questions which occur in common business transactions.

Another class consists of those who are preparing for the counting-room and for commercial business. It is believed that this work contains more that will be practically useful to this class than most other arithmetics that have been published. Others, not expecting to be merchants, are to be farmers, mechanics, &c. To these the sections on Mensuration, the Mechanic Powers, and the elements of Book-keeping by Single Entry, will be particularly useful.

It may be remarked, in conclusion, that the introduction of mathematical puzzles has been carefully avoided in this work; and that the gradations from the easier to the more difficult topics are such that the careful pupil will generally be able to pass from one to the other without the assistance of his teacher.

The work is submitted to the public with the hope that it may be found adapted to the wants of both teachers and learners; tending to render the duties of the one less arduous, and to make the studies of the other both profitable and pleasant. RUFUS PUTNAM.

Salem, May, 1849.

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For Suggestions to the Teacher, see page 263.

ARITHMETIC.

Art. 1. **Arithmetic** is the art of computing by numbers.

Any single thing is called a *unit*. *Number* signifies a unit, or a collection of units; as, one book, two slates, five plums.

2. The ten following characters, called the Arabic figures, or digits, are used in writing numbers. 1, one; 2, two; 3, three; 4, four; 5, five; 6, six; 7, seven; 8, eight; 9, nine; and 0, naught.

They are called digits, from the Latin word *digitus*, which signifies a finger.

The first nine figures are called *significant* figures, to distinguish them from the tenth, 0, which has no value when standing alone. It is sometimes called *cipher*, or *zero*.

3. Arithmetic consists of five fundamental operations; viz., Numeration, Addition, Subtraction, Multiplication and Division.

SECTION I. — NUMERATION.

4. Numeration is the art of expressing numbers by figures, and of reading them when so expressed.

As nine is the largest number that can be expressed by any single figure, all numbers larger than nine are expressed by *combinations* of two or more of the ten Arabic figures. (**2.**)

Thus, to express ten, we combine 1 and 0, 10; to express eleven, we combine 1 and 1, thus, 11; to express twelve, 1 and 2, 12; thirteen, 13, &c. Twenty, or two tens, is written 20; twenty-one, 21; twenty-two, 22, &c., *two* figures being combined to express any number between nine and one hundred.

1. Write upon the slate all the numbers from ten to ninety-nine, inclusive.

To express numbers larger than ninety-nine, and less than one thousand, three figures are combined.

Thus, 100 is one hundred, 101 is one hundred and one, 102 is one hundred and two, 110 is one hundred and ten, 120 is one hundred and twenty, 125 is one hundred and twenty-five, 200 is two hundred, 300 is three hundred.

2. Write on the slate all the numbers from one hundred to two hundred; from three hundred to four hundred; from seven hundred and fifty to eight hundred and fifty.

5. TO TEACHERS. If the pupil is a beginner, he should write on a slate or blackboard, numbers between ten and one thousand, until he can readily write and read any of them, when they are dictated to him. He should do the same with larger numbers. *Let each succeeding exercise in the book be extended by the teacher, till the pupil is perfectly familiar with the principle and its application.*

6. The *value* of any figure, when combined with others, depends upon the place it occupies. *Its value becomes ten times as large by removing it one place to the left, and consequently, one tenth as large by removing it one place to the right.*

Thus 1, when alone, represents a *unit*, or *one*, and is called a unit of the first order. On being removed one place to the left, as in 10, it represents one *ten*, or *ten*, and is then called a unit of the second order. On being again removed one place to the left, as in 100, the 1 represents *ten* tens, or one *hundred*, and it is then called a unit of the third order. Removing the 1 again one place further to the left as in 1000, it represents *ten* hundreds, or one *thousand*. and is called a unit of the fourth order.

So also 5, when alone, or at the right hand of other figures, represents 5 *units*, or five. 50 is 5 *tens*, or fifty. 500 is 50 tens, or 5 *hundred*. 5000 is 50 hundred, or 5 *thousand*.

7. In whole numbers, we call the place of the first right hand figure the units' place; of the second figure from the right, the tens' place; of the third, the hundreds' place.

Thus, 854 is eight hundreds, five tens, and four units; or eight hundred and fifty-four. 609 is six hundreds, no tens, and nine units; or six hundred and nine. The figure 0 being always used to fill a place not occupied by a significant figure.

8. Numbers containing more than three figures, are divided into periods of three figures each, by commas, counting from the right; the first or right hand period contains units, tens, and hundreds; the second period contains units of thousands, tens of thousands, and hundreds of thousands; the third contains units of millions, tens of millions, &c., as in the following table.

NUMERATION TABLE.

Quadrillions,	Hundreds of trillions.	Tens of trillions.	Trillions.	Hundreds of billions.	Tens of billions.	Billions.	Hundreds of millions.	Tens of millions.	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.
6,	8	0	1,	0	8	0,	4	8	9,	1	6	0,	0	8	4.
6th period. Quadrillions.	5th period. Trillions.			4th period. Billions.			3d period. Millions.			2d period. Thousands.			1st period. Units.		

II. The periods above quadrillions are quintillions, sextillions, septillions, octillions, nonillions, decillions, undecillions, duodecillions; and by continuing to adopt a new name for every three figures, the number of periods may be increased indefinitely.

Dividing numbers into periods of *three* figures is the French method of Numeration. The English method has been to divide them into periods of *six* figures each; but the French method is more convenient than the other, and has been almost universally adopted.

9. TO READ NUMBERS. *Count off the figures, beginning at the units' place, into periods of three figures; and, beginning at the left, read the numbers standing in each period, adding the name of each period, except the right hand period.*

The numbers in the above table are read: Six quadrillions, eight hundred and one trillions, eighty billions, four hundred and eighty-nine millions, one hundred and sixty thousand and eighty-four.

Examples to be written in words, and read by the learner.

(1.) 10. (2.) 100. (3.) 1000. (4.) 10000. (5.) 100000. (6.) 101. (7.) 180. (8.) 1001. (9.) 1012. (10.) 2084. (11.) 7804. (12.) 10001. (13.) 30805. (14.) 38050. (15.) 30085. (16.) 500005. (17.) 3050601. (18.) 850160804.

II. (19.) 4016080900. (20.) 1851608090504. (21.) 50080090607010.

10. FOR BEGINNERS. 1. Write every third number from 1000 to 1100; thus, 1000, 1003, 1006, &c. Write every fourth number from 1300 to 1400. Write every fifth number from 2000 to 2100.

TO WRITE WHOLE NUMBERS. *Commence with the highest or left hand period, which may require one, two, or three figures, and write all the other periods in their order, allowing three places for each period, filling the vacant places with naughts.*

Numbers to be expressed in Figures.

1. Eighty-four. 2. Nine hundred and four. 3. Nine hundred and forty. 4. One thousand and one. 5. One thousand and ten. 6. One thousand one hundred and one. 7. Eighty thousand and eight. 8. Eighty thousand and eighty. 9. Eighty thousand and eighty-eight. 10. Five hundred and seventy-five thousand, six hundred and thirty-seven. 11. Eight millions and thirty-five. 12. Thirty-four millions, thirty-four thousand and thirty-four. 13. Five hundred millions and fifty. 14. Six billions, six thousand and six hundred. II. 15. Fifteen quintillions, forty millions, eight thousand and forty. 16. Sixty octillions, ninety trillions, and three thousand. 17. Fifteen decillions, eight sextillions, four hundred billions, and eight millions.

NUMERATION OF DECIMALS.

11. If a number be divided into 10 equal parts, one of the parts is called one tenth of that number. Thus, one tenth of 10 apples is one apple, because if ten apples be divided into 10 equal parts, one part will be one apple. If 20 apples be divided into 10 equal parts, what will you call one of the parts? *Ans.* One tenth of twenty apples. How many apples is one tenth of twenty apples? Why? One tenth of 30 apples? Why? One tenth of 50 apples? Why? Of 60 apples? Why? 70 apples? Why? 80 apples? Why?

If 100 apples be divided into ten equal parts, what part of 100 apples will one part be? *Ans.* One tenth of 100 apples. Why? How many apples is one tenth of 100 apples? Why? Of 200? Why? 300? Why? 400? 500? 600 apples? Why?

How many apples is one tenth of 1000 apples? Why? 2000 apples? Why? 3000? Why? 4000 apples? Why?

If a single thing be divided into 10 equal parts, one of the parts is called one tenth of that thing; two such parts are called two tenths of it; 3 such parts, three tenths, &c. If it be divided into 100 equal parts, one of the parts is called 1 hundredth of that thing; two such parts are called 2 hundredths of it; three such parts, 3 hundredths, &c.

What is meant by 1 tenth of 1 apple? 2 tenths? 3 tenths? 4 tenths? What is meant by 1 hundredth of one apple? 2 hundredths? 3 hundredths? &c.

If 2 apples be divided into 10 equal parts, what part of 1 apple will one of the parts be? How much is 1 tenth of 2 apples? Why? Of 3 apples? Why? 5? 7? Why?

How much is one tenth of 1 unit? Why? Of 2 units? Why? Of 3 units? Why?

If an apple be divided into 10 equal parts, and then each of these parts be divided into 10 equal parts, what part of 1 tenth will one of these parts be? Why?

Into how many equal parts will the whole apple be divided? What part of the whole apple will one part be? Why?

If 2 apples be divided in the same manner, what part of 1 apple will one part be? Why? What is one hundredth of 1 unit? Of 2 units? Why? Of 3 units? Why?

12. As tens are tenths of one hundred, and are written one place to the right of hundreds; and as units are tenths of one ten, and are written one place to the right of tens; so tenths of units are written one place to the right of units; and tenths of tenths, or hundredths, are written one place to the right of tens; thousandths, one place to the right of hundredths, &c.

Such parts of units as tenths, hundredths, thousandths, &c., are called *decimals*, from the Latin *decem*, ten; because their value diminishes toward the right in a tenfold ratio. A point called the *decimal point*, is placed to the left of the tenths' place to separate the decimals from whole numbers. Thus, 3.6 is 3 and 6 tenths; 3.06 is 3 and 6 hundredths; 3.25 is 3 and 25 hundredths; 3.005 is 3 and 5 thousandths; 3.465 is 3 and 465 thousandths. .3 is 3 tenths; .08 is 8 hundredths; .015 is 15 thousandths.

Write one tenth of 1; of 2; of 3; of 5. Write one hundredth of 1; of 2; of 8; of 15; of 54. Write one thousandth of 1; of 5; of 8; of 18; of 27; of 89; of 548.

DECIMAL TABLE.

Thousands, &c.	Hundreds.	Tens.	Units.	*Decimal point.*	Tenths.	Hundredths.	Thousandths.	Ten thousandths.	Hund. thousandths.	Millionths.	Ten millionths.	Hund. millionths.	Billionths.	Ten Billionths.	Hund. Billionths.	Trillionths, &c.
3	0	8	5	.	6	1	8,	6	0	0,	8	5	0,	1	6	8

TO READ DECIMALS. *Read the figures as in whole numbers, and add the name of the lowest decimal place. In reading whole numbers and decimals, read the whole numbers first, and then the decimals.*

Thus .7 is 7 *tenths;* .18 is 18 *hundredths;* .00108 is 108 *hundred thousandths:* 15.006 is 15 and 6 *thousandths;* 16.0064 is 16 and 64 *ten thousandths.* The number in the table is three thousand and

eighty-five, and 618 billion, 600 million, 850 thousand and 168 trillionths.

Examples to be written in words, and read by the pupil.

(1.) .5. (2.) .05. (3.) .037. (4.) .1008. (5.) 501.08. (6.) .7108. (7.) .008. (8.) .001001. (9.) 16.16. (10.) 180.018. (11.) 7.00016. (12.) 30175.04. (13.) 301680.45. (14.) 30168.045. (15.) 3016.8045. (16.) 301.68045. (17.) 30.168045. II. (18.) 3.0168045. (19.) .30168045. (20.) 35.000100041. (21.) 10010806.3000010806. (22.) 581006-.00940008607.

NOTE. The teacher will find it a profitable exercise to write on the blackboard a number of figures, example 21st or 22d above, for instance, or a shorter example for beginners, and then, removing the decimal point successively from one place to another, require his pupils to read the figures as pointed, and also to point out the local value of any figure or group of figures, either of the whole numbers or the decimals, which he shall underscore. He should continue this exercise till they can readily read the number, and name the local value of any figure or figures, whatever position the decimal point may occupy.

13. TO WRITE DECIMALS. *Write the decimal as if it were a whole number, and then prefix* as many naughts as may be necessary to reduce it to the proper denomination.*

Suppose it is required to write five thousand and eighteen ten millionths. Write 5,018, and as ten millionths (see table) is the seventh place from the decimal point, three naughts must be prefixed:— .0005,018.

EXAMPLES TO BE WRITTEN IN FIGURES.

(1.) Four *hundredths*. (2.) Four *thousandths*. (3.) Five *ten thousandths*. (4.) 6 *hundred thousandths*. (5.) 8 *millionths*. (6.) 14 *hundredths*. (7.) 14 *thousandths*. (8.) 104 *thousandths*. (9.) 75 *ten thousandths*. (10.) 851 *ten thousandths*. (11.) 1845 *hundred thousandths*. (12.) 16 *hundred thousandths*. (13.) 58 *millionths*. (14.) 2506 *millionths*. (15.) Three,—and eight *tenths*. (16.) 41,—and 7 *hundredths*. (17.) 400,—and 17 *thousandths*. (18.) 4,—and 107 *ten thousandths*. (19.) Eighty-four thousand and sixteen,—and fifty-four *ten thousandths*.†

* PREFIX, means *to place before;* ANNEX, *to place after.*

† For more exampels see page 247.

II. (20.) Five million and forty, — and 97 ten millionths. (21.) Fifty billion, 40 million, 75 thousand and 54, — and 5 thousand and 15 *hundred thousandths*. (22.) 5 trillion, 90 million, 5 thousand, one hundred, — and 16 thousand and 5 *hundred millionths*. (23.) 84 thousand and sixteen, — and three-thousand and 24 *ten millionths*.

14.* 1. In one unit how many tenths? How many hundredths? Thousandths?

2. In two units how many tenths, &c.? In three units, &c.?

3. In one ten how many units? How many tenths? Hundredths? Thousandths?

4. In two tens how many units, &c.? In three tens, &c.?

5. In one hundred how many tens? Units? Tenths? Hundredths? Thousandths?

6. In two hundred how many tens, &c.?

7. In one tenth how many hundredths? Thousandths? Ten thousandths?

8. In two tenths how many hundredths, &c.?

9. In one hundredth how many thousandths? Ten thousandths?

10. How many units and tenths in 14 tenths? *Ans.* 1 unit and 4 tenths. In 87 tenths? In 401 tenths? In 806 tenths?

11. How many units, tenths, and hundredths in 347 hundredths? *Ans.* 3 units, 4 tenths, and 7 hundredths. In 584 hundredths? In 307 hundredths?

12. In 8516 how many hundreds? *Ans.* 85 hundreds. How many tens? *Ans.* 851 tens. How many units? Tenths? Hundredths?

13. In 45.16 how many tens? Units? Tenths? Hundredths?

14. In 156.04 how many hundreds? Tens? Units? Tenths? Hundredths?

15. In .3 how many hundredths? How many thousandths?

NOTE. *Naughts placed to the right hand of decimals do not alter their value; for .3 is equivalent to .30; .35 is equivalent* † *to .350 or* .3500.

II. 16. In 1701.06 how many hundreds? Tens? Units? Tenths? Hundredths? Thousandths? Ten thousandths? Hundred thousandths? Millionths?

17. In 80416.05807 how many ten thousands? Thousands? Hundreds? Tens? Units? Tenths? Hundredths? Thousandths? Ten thousandths? Millionths? Ten millionths?

18. Write 850 tens. *Ans.* 8500. (19.) Write 8057 tens. (20.) 9010 hundreds. (21.) 75 tenths. (22.) 875 tenths.

* *Beginners* may use the slate in Article 14.
† EQUIVALENT means of *equal value*.

(23.) 9045 hundredths. (24.) 1645 ten millionths. (25.) 105617 thousandths.

FEDERAL MONEY. (45.)

15. Federal Money is the national currency* of the United States. Every system of national currency has its unit of measure.

In the United States, the unit is the dollar, which is marked thus, $.

The denominations of Federal money are the eagle, the dollar, the dime, the cent, and the mill.† These denominations increase in a tenfold ratio; that is, 10 mills make one cent; 10 cents make one dime; 10 dimes make one dollar; 10 dollars make one eagle. So that all operations in dollars, cents, and mills, are performed as in whole numbers and decimals.

Dollars are written as whole numbers; cents, as so many hundredths, and mills as so many thousandths.. The figures, therefore, at the left of the decimal point, express dollars; the first two at the right of the point express cents, and the third, mills. Thus, $7.08 is read 7 dollars and 8 cents; $100.943 is 100 dollars, 94 cents, and 3 mills.

Read the following Numbers. (1.) $8. (2.) $16.00. (3.) $25.14. (4.) $168.07. (5.) $1016.08. (6.) $45.001. (7.) $1000.011. (8.) $8049.108. (9.) $10010.101.

Write in Figures—(10.) Twenty dollars. (11.) Eighty-seven dollars, 25 cents. (12.) One thousand and seven dollars, and nine cents. (13.) Five dollars and eight mills. (14.) Fifteen dollars, three cents and seven mills. (15.) 1000 dollars, 1 cent and 8 mills. (16.) Thirty thousand and eighteen dollars, six cents and three mills.

QUESTIONS FOR EXAMINATION.

What is arithmetic? What is a unit? What is number? What characters are used in writing numbers? Why are they called *digits?* What are *significant* figures? Of how many fundamental operations does arithmetic consist? What is *numeration?* How are the numbers from 1 to 9 expressed? How are numbers larger than 9 expressed? Give examples. How many figures are combined to express numbers between 9 and one hundred? To express numbers larger than ninety-nine and less than one thousand? Give examples. Upon what does the value of any figure when combined with others depend? How is

* The term *currency* signifies *money*, or the *circulating medium*.
† The mill is merely nominal; there being no coin of this denomination.

its value increased? How diminished? Give examples. In whole numbers, which is the place of units? Of tens? Of hundreds? For what purpose is 0 used? How are numbers containing more than three figures to be divided? What does the first, or right hand period contain? The second? The third? The fourth? The fifth? The sixth? Name the periods above the sixth? What is the rule for reading whole numbers? What is the rule for writing whole numbers in figures? Repeat the table, beginning at units.

If a number be divided into ten equal parts, what is one of the parts called? Give an example. If a single thing be divided into ten equal parts, what is one of the parts called? Two of the parts? Four parts? If one thing be divided into one hundred equal parts, what is one of the parts called? Two of the parts? What part of hundreds are *tens?* What part of tens are *units?* Where is each written? What part of units are *tenths?* Where are *tenths* written? What part of tenths are *hundredths?* Where are *hundredths* written? Where are *thousandths* written? What are *decimals?* Why are they so called? What is the decimal point? Where placed? Give examples. Repeat the *decimal table*, beginning at units. What is the rule for *reading decimals?* For reading *whole numbers and decimals?* What is the rule for *writing decimals?* What is the meaning of *prefix?* Of *annex?* How is the value of decimals affected by placing naughts at the right of them?

What is *Federal Money?* What is the meaning of *currency?* What is the unit of value in the United States currency? How is it marked? What are the denominations of Federal money? What relations do these denominations bear to each other? How are operations in dollars, cents, and mills performed? How are dollars written? Cents? Mills? What do the figures to the left of the decimal point express? What do those at the right express? Give examples.

16. SECTION II — ADDITION.

ADDITION is the method of finding the sum or amount of two or more numbers.

Operations in arithmetic are often represented by signs. The following are used in addition.

$=$ Sign of equality; as 100 cents $=$ 1 dollar; which is read, 100 cents are equal to one dollar.

$+$ Sign of addition, or plus sign; as, $15+6=21$; which is read, 15 and 6 are 21; or 15 plus 6 is equal to 21.

17. 1. How many are $2+2$? $3+2$? $4+2$? $5+2$? $6+2$? $7+2$? $8+2$? $9+2$? $10+2$? $11+2$?

2. Add 3 to all the numbers from 2 to 50. Thus, $2+3=5$; $3+3=6$; $4+3=7$, &c.

3. Add 4 to all the numbers from 2 to 50.

4. Add 5 in the same manner; and 6; and 7; and 8; and 9.

5. Repeat every third number from 100 to 200; thus, $100+3=103$; $103+3=106$; $106+3=109$.

6. Name every fourth number in the same manner.

7. Name every fifth number from 1 to 101. Every sixth, from 100 to 202.

8. Name every seventh number from 201 to 300. Every eighth, from 505 to 600.

The pupil should be exercised in this way till he can add units with facility.

9. How many are $10+10$? $20+10$? $30+10$? $40+10$? $50+10$? $60+10$? $70+10$? $80+10$?

10. How many are $10+20$? $20+20$? $30+20$? $40+20$? &c. $10+30$? $20+30$? $30+30$? &c. $10+40$? $20+40$? $30+40$? &c. $10+50$? $20+50$? &c. $10+60$? $20+60$? &c.

11. How many are $10+15$? Say 10 and 10 are 20, and 5 are 25; therefore, 10 and 15 are 25.

12. How many are $12+14$? Say 10 and 10 are 20; 2 and 4 are 6, which added to 20 makes 26; therefore, 12 and 14 are 26. $13+12$? $14+13$? $15+12$? $17+11$?

13. How many are $23+11$? *Ans.* 20 and 10 are 30; 3 and 1 are 4, which added to 30, makes 34; therefore, 23 and 11 are 34. $25+14$? $22+17$? $34+15$? $38+21$? $42+35$? $43+24$? $48+32$? $53+17$? $65+15$? $70+28$?

14. How many are $15+18$? Say 10 and 10 are 20; 5 and 8 are 13, which is equal to 10 and 3; 20 and 10 are 30 and 3 are 33; therefore, 15 and 18 are 33. $28+36$? Say 20 and 30 are 50; 8 and 6 are 14, which is equal to 10 and 4; 50 and 10 are 60, and 4 are 64; therefore, 28 and 36 are 64.

15. In the same way add $18+13$; $17+15$; $19+18$; $23+19$; $28+17$; $34+17$; $39+18$; $44+18$; $55+27$; $48+27$; $54+37$; $22+34$; $34+28$; $39+47$.

16. James has 46 cents and Charles has 27 cents; how many have both?

17. John gave 35 cents for an arithmetic and 37 cents for a reader; how much did he give for both?

18. A man spent the first 25 years of his life in the country, and he has lived 38 years in the city; how old is he?

19. A farmer bought 2 cows; for one he gave 48 dollars, and for the other 35 dollars; how much did he give for both?

18. A wood merchant buys of one man 310 cords of wood, of another 264 cords, of another 85 cords, and of another 460 cords. How many cords does he buy in all?

These numbers are larger than those we have been adding, and it is not so easy to add them mentally. We will arrange them under each other, so that the units shall stand in one column, the tens in another, &c.

310
264
85
460
1119

First add the units' column: 5 units and 4 units are 9 units, which write under the units' column. Then add the tens: 6 tens and 8 tens are 14 tens, and 6 tens are 20 tens, and 1 ten are 21 tens, equal to 2 hundreds and 1 ten; write the 1 ten in the tens' place, and carry the 2 hundreds to be added with the hundreds. 2 hundreds and 4 hundreds are 6 hundreds, and 2 hundreds are 8 hundreds, and 3 hundreds are 11 hundreds, equal to 1 thousand and 1 hundred, which write down, making the sum 1119. From this example we may derive the following

RULE FOR ADDITION. *Write the numbers under each other, so that units shall stand under units, tens under tens, tenths under tenths, &c. Then add the figures in the right hand column. If the amount does not exceed* 9, *write it under that column. If it is* 10 *or more, write the units' figure of the amount under the column added, and add the tens with the figures of the next column. In the same manner add the figures in each succeeding column, writing down the whole amount of the last column.*

PROOF. *Add the columns both upwards and downwards. The amounts should be alike.*

EXAMPLES FOR PRACTICE.

1. Acres.	2. Pounds.	3. Dollars.	4. People.	5. Cents.	6.
184	3178	40016	7160809	4018645	$380.001
359	5184	3108	301684	31486	41.17
467	2001	179	4100169	4197	27.036
107	8154	4807	70019	8197645	684.324

7.	8.	9.	10.
$1680.40	$3001.01	174.861	5187.31459
1.25	4896.375	30.80	30.007
.425	394.1875	5.09	169.8054
1.375	1869.875	876.485	30.0075

The population of each of the United States was,

New England States.	In 1830.	In 1840.	*Middle States.*	In 1830.	In 1840.
Maine,	399,955	501,793	N. York,	1,918,608	2,428,921
N. Hamp.,	269,328	284,574	N. Jersey,	320,823	373,306
Vermont,	280,652	291,948	Penn.,	1,348,233	1,724,033
Mass.,	610,408	737,699	Delaware,	76,748	78,085
R. Island,	97,199	108,830			
Conn.,	297,665	309,978			

Southern States.	In 1830.	In 1840.	*Western States*	In 1830.	In 1840.
Dist. of C.,	39,834	43,712	Tennessee,	681,904	829,210
Maryland,	447,040	470,019	Kentucky,	687,917	779,828
Virginia,	1,211,405	1,239,797	Ohio,	937,903	1,519,467
N. Carol.,	737,987	753,419	Indiana,	343,031	685,866
S. Carol.,	581,185	594,398	Illinois,	157,445	476,183
Georgia,	516,823	691,392	Michigan,	31,639	212,267
Alabama,	309,527	590,756	Missouri,	140,455	383,702
Miss.,	136,621	375,651	Arkansas,	30,388	97,574
Louisiana,	215,739	352,411	Iowa,		43,112
Florida,	34,730	54,477	Wisconsin,		30,945

11. How many inhabitants in the New England States in 1830? In 1840?

12. How many in the Middle States in 1830? In 1840?

13. How many in the Southern States in 1830? In 1840?

14. How many in the Western States in 1830? In 1840?

15. How many in all the states in 1830? In 1840?

16. A farmer sells 5 loads of potatoes; viz., one load containing 37 bushels, one 54, one 46, one 25, and another 17 bushels. How many bushels are there in the five loads?

17. A butcher buys 4 hogs, one weighing 324 pounds, another 287 pounds, another 409 pounds, and another 310 pounds. How many pounds in all?

18. A farmer buys a yoke of oxen for $87.26, an ox wagon for $125.17, a horse for 75 dollars, 3 cows for $24.25 apiece. How much did they all come to?

19. A man owes to A $341.17, to B $4016.35, to C $3101.01, to D $184.16, and to E $907.40. How much does he owe them all?

20. Add $38.017, $401.601, $3918.48, $4197, $50375.18, and $0.375.

21. Add 39.018, 401.007, 8160.1, 501.6803. 50.16803. 5016.803 and 501680.3.

22. Add twenty thousand two hundred and two; three hundred and eighty thousand and thirty-eight; fifty-seven million, five hundred and seventy thousand, and five hundred and seventy; and four millions and one hundred.

23. Add three hundred and seven, — and eighty-four *hundredths;* five thousand and twenty-one, — and ninety-three *thousandths;* six million, — and eighty-five *ten thousandths;* one hundred and ten thousand, nine hundred and four, — and

seven hundred and one *ten thousandths*; forty-eight thousand, —and forty-eight *thousandths*.

24. Add fifteen dollars and seven cents; eighty-nine dollars and forty cents; sixty-seven dollars and eight mills; three hundred and eight dollars, nine cents and one mill.

25. A farmer carried a load of potatoes to market and peddled them as follows: 3.5 bushels for two dollars and sixty cents; 8.75 bushels for six dollars, fifty cents; 15 bushels for eleven dollars, thirty-seven cents, five mills; and 7.25 bushels for five dollars, sixty-two cents, five mills. How many bushels were there, and how much did his load come to?

26. There are three hundred and sixty-five, and twenty-five *hundredths* days in a year. How many days are there in 4 years? In 8 years? In 10 years? In 14 years? In 20 years?

27. A man bought 4 barrels of flour at $5.25 a barrel, 5 barrels at $5.375 a barrel, and 3 barrels at $6.31 a barrel. How many barrels did he buy, and how much did he give for them?

28. A gentleman owns a house worth $4567, a store worth $2584, 6 acres of land worth $175 per acre; he has notes amounting to $3594, railroad and bank stock worth $2106.75, and other personal property worth $2184.50. How much is he worth?

19. Add the following numbers:

(1.) 134 + 27. Say 130 and 20 are 150; 4 and 7 are 11, which is equal to 10 and 1; 150 and 10 are 160, and 1 are 161: therefore, 134 and 27 are 161. (2.) 138 + 89. 30 and 80 are 110, which added to 100 makes 210; 8 and 9 are 17, which is equal to 10 and 7; 210 and 10 are 220, and 7 are 227; therefore, 138 and 89 are 227.

(3.) 275 + 348. 200 + 300 are 500; 70 and 40 are 110, which added to 500 make 610; 5 and 8 are 13, equal to 10 and 3; 610 and 10 are 620, and 3 are 623; therefore, 275 and 348 are 623.

(4.) 165 + 32. 2841 + 165. (5.) 310 + 675. 844 + 148.
(6.) 697 + 249. (7.) 638 + 286. (8.) 564 + 379.

NOTE. The teacher will best judge whether such mental operations are at present too difficult for the pupil or not. They are introduced merely as hints to the teacher, and it is hoped that he will not allow the pupil to pass over such exercises without a fair trial. A few similar exercises should, if possible, form a part of every recitation in arithmetic, at least till the learner can perform them with facility.

QUESTIONS. What is addition? What is the *sign* of *equality*? The *sign* of *addition*? Give an example. What is the rule for *addition*? What is the method of *proof*?

SECTION III. — SUBTRACTION.

20. SUBTRACTION is the method of finding the difference between two numbers, by taking the smaller from the greater.

— Sign of subtraction, or *minus* sign, written between two numbers, shows that the latter is to be taken from the former; as 15 — 4, which means that 4 subtracted from 15 leaves 11. It is read 15 less 4 is equal to eleven; or 15 minus 4 is equal to 11.

The less number, or number to be subtracted, is called the *subtrahend;* that from which it is to be subtracted is called the *minuend.* The difference is called the *remainder.*

1. How much is 1 less 1? 2 less 1? 3 — 1? 4 — 1? 5 — 1? 6 — 1? 7 — 1? 8 — 1? 9 — 1? 10 — 1? 11 — 1? 12 — 1? 13 — 1? 14 — 1? 15 — 1? 16 — 1? 17 — 1? 18 — 1? 19 — 1? 20 — 1?

2. How much is 2 — 2? 3 — 2? 4 — 2? &c., to 20 — 2?

3. How much is 3 — 3? 4 — 3? 5 — 3? &c., to 20 — 3? 4 — 4? 5 — 4? &c., to 20 — 4?

Continue this exercise, using all the numbers from 4 to 20, as *subtrahends.*

21. Subtract 543 from 864, thus:

Minuend,	864
Subtrahend,	543
Remainder,	321

EXAMPLES FOR PRACTICE.

(1.) 459 — 237. (2.) 6849 — 3526. (3.) 7684 — 5322. (4.) 69184 — 20032. (5.) 410647 — 200645. (6.) 84675 — 32423. (7.) 8547016 — 3333016. 8. 7840196 — 3430096.

William had 15 cents, and Henry had 9. How many more had William than Henry? Their father gave each of them 4 cents more. How many more had William than Henry then? William has spent 7 cents for paper, and Henry has spent 7 cents for pens. How many more cents has William than Henry now?

From these examples we see that if the same quantity be either added to, or subtracted from, two numbers, their difference will remain the same as before.

From 6135 subtract 3617. In this example we cannot take 7 units from 5 units; we therefore add 10 units to the 5, making 15 units; and subtracting 7 units from 15 units, we write the remainder 8. Then, as we added 10 units to the minuend, we must add 1 ten to the subtrahend, which makes 2 tens; 2 tens from 3 tens leaves 1 ten. Again, we cannot take 6 hundreds from 1 hundred; adding 10 hundreds to the 1 hundred in the upper line, makes 11 hundreds; 6 hundreds from 11 hundreds leaves 5

6135
3617
2518
6135

hundreds. Then adding 10 hundred, or 1 thousand to the lower line, makes 4 thousands. 4 thousands from 6 thousands leaves 2 thousands. The remainder is 2518.

The difference between two numbers shows how much must be added to the less to make the greater. We may therefore prove the above remainder to be correct, by adding it to the smaller number. The sum just equals the larger.

RULE FOR SUBTRACTION. *Write the numbers as in addition, units under units, &c., placing the subtrahend under the minuend. Beginning at the right hand, subtract each lower figure from the one above it, where it can be done. Where this cannot be done, add* 10 *to the figure in the upper line, and subtract; and add one to the next figure of the lower line.*

PROOF. *Add the remainder and subtrahend together. The sum should be equal to the minuend.*

EXAMPLES FOR PRACTICE.

	9.	10.	11.	12.
From	801647381	79108654	867899	3040605
Subtract	310864532	34745735	308907	2060807

	13.	14.	15.	16.
From	3000.8007	3017.056	$300.10	$5106.089
Subtract	1000.9008	104.875	16.75	35.19

	17.	18.	19.
From	$3570.165	$4186.006	3100080.00005
Subtract	15.25	307.108	1000181.405067

NOTE. Naughts may be annexed to the minuend when necessary.

20. How much is $18.00 — $8.25? $1561.001 — $316.108? 3487 — 18.3684?

21. How much is 175 — 18.003? 3001.01 — 185.0304? 71090.007 — 3898.1049?

22. How many more inhabitants were there in the New England States in 1840 than in 1830? In the Middle States? In the Southern States? In the Western States?

23. How much more is 816841 than 610489?

24. What must be added to 35618 to make 195816?

25. What is the difference between fifty-million, three hundred and one, — and seven hundred and eighty-seven thousand and ninety?

26. Subtract five hundred thousand and seventy-five, — and one hundred and eighty-three *ten thousandths*, from four million two hundred thousand, — and sixteen *hundred thousandths*.

27. A man bought a keg of molasses for three dollars, sixty-two cents, five mills. He gave the retailer a five dollar bill; how much should he receive?

28. A farmer sold a load of hay for 15 dollars, 75 cents, and received in pay a barrel of flour worth $7.375, and the rest in money. How much money did he receive?

29. A man is worth $1875.45. How much more must he lay up before he is worth $3000.00?

22. The number which must be added to another number to make it equal to a unit of the next higher order (**6.**) is called the complement of that number. Thus 3 is the complement of 7, it being what 7 wants of being one ten. So 37 is the complement of 53, being what 53 wants of 100. 374 is the complement of 626. Why? So 90 is the complement of 10; 88, of 12. Why?

1. What is the complement of 10? 12? 15? 17? 19? 13? 18? 20? 24? 28? 23? 29? 30? 33? 35? 38? 40? 42? 47? 44? 49? 50? 58? 53? 60? 62? 68? 65? 70? 74? 76? 78? 80? 84? 87? 90? 93?

2. James buys a book for 28 cents. How much must he receive in exchange for a dollar bill

3. A man buys 15 lbs. of sugar for $1.34. How much must he receive in exchange for a three dollar bill? (**5.**)

QUESTIONS. What is *subtraction?* What is the *sign* of *subtraction?* Give an example. What is the *subtrahend?* The *minuend?* The *remainder?* What if the same quantity be either added to or subtracted from two numbers? What does the difference between two numbers show? What is the *rule* for subtraction? What is the method of *proof?* Why? What is to be done when the subtrahend has more decimal places than the minuend?

Miscellaneous Examples in Addition and Subtraction.

23. 1. A farmer sells a merchant 7 bushels of potatoes at 78 cents a bushel, and takes in return 8 gallons of molasses at $0.375 per gallon. How much money must he receive besides?

2. A shoemaker sells a farmer 3 pairs of shoes at $0.875 a pair, and receives in exchange 4 bushels of corn at $0.625 a bushel. How much is the balance, and which must pay it?

3. A man bought a coat for $8.75, a vest for $1.75, and a pair of pants for $4.375. How much did they come to? He gave the tailor 2 ten dollar bills. How much change should he receive?

4. A man owed $864.95. He has paid one debt of $75.84,

another of $108.375, another of $15.17, and another of $5.625. How much does he still owe?

5. What is the difference between 318.067 + 4000.91 and 184001.4501?

6. A man owes to A $694.81, to B $354.17, to C $1000-.075. How much does he owe in all? How much more to A than B? To C than B? To C than A?

7. The man spoken of in the last example owns a farm worth $3000. How much will he be worth after paying his debts?

8. What must be added to 1849017 + 3591 to make 3848-108?

9. A man owes $346.50, which he is to pay in monthly payments of $57.75. How much will he owe after making the first payment? After the second? After the third? How many payments in all must he make to cancel the debt?

10. A man having $3467.85, invested $1018.25 in railroad stock, $387.95 in bank stock, and bought 5 acres of land at $125.00 per acre. How much had he left?

11. A man died, leaving an estate of 60187 dollars. He gave to his wife $10000, to his 4 daughters $5200 each, and to his 3 sons $7500 each; the remainder he gave to benevolent institutions. How much did his charities amount to?

12. A man bought a quantity of flour for $6184.58, a quantity of coffee for $584.75, and a quantity of sugar for $215.18. In exchange he gave 3725 dollars worth of corn, a quantity of butter worth $385.75, and a quantity of potatoes worth $548-.78. How much did he then owe?

24. 1. How many are 54 + 16? (**17.**) 49 + 23? 83 + 17? 47 — 17? 48 + 19? 45 — 14? 68 — 32? 55 — 23? 126 — 112? 384 — 213? 684 — 243? 126 + 112?

2. How much is 42 — 18? Say 42 is equal to 30 + 12; 10 from 30 leaves 20, 8 from 12 leaves 4, which added to 20 makes 24; therefore, 42 less 18 is 24.

3. 63 — 37? *Solution.* 63 = 50 + 13; 50 — 30 = 20; 13 — 7 = 6; 20 + 6 = 26; therefore, 63 less 37 is 26.

4. How much is 48 — 29? 73 — 48? 65 — 47? 87 — 39? 58 — 34? 67 + 248? 187 — 38?

5. How much is 154 — 68? *Solution.* 154 — 100 = 54, to which add the complement of 68, which is 32. 54 and 32 are 86; therefore, 154 — 68 is 86. 436 — 78? 464 — 356? 146 — 47? 865 — 48?

6. How much is 436 — 278? *Solution.* 436 — 300 = 136, to which add the complement of 78; 136 + 22 = 158. How much is 567 — 384? 327 — 149? 564 — 237? 746 — 489? (**5** and **19**) note.

SECTION IV. — MULTIPLICATION.

25. MULTIPLICATION is a short method of adding equal numbers. By it we find the amount of any number repeated a given number of times.

What will 5 bushels of corn come to at 67 cents a bushel?

To perform this question, we may write down 67 five times, and add as in addition. But as the same number is five times repeated, we may write it down as in the margin, and say 5 times 7 units are 35 units, or 3 tens and 5 units. Write down the 5 units, and reserve the 3 tens to be added to the tens. 5 times 6 tens are 30 tens, and 3 tens are 33 tens, or 3 hundred and 3 tens.

Answer, 335.

$$\begin{array}{r} 67 \\ 5 \\ \hline 335 \end{array}$$

The terms used in multiplication are the *multiplicand*, the *multiplier*, and the *product*. The *multiplicand* is the number to be multiplied or repeated. The *multiplier* is the number to multiply by, and shows how many times the multiplicand is to be repeated. The *product* is the number resulting from the multiplication. The multiplier and multiplicand are called *factors* of the product. In the above example, 67 is the multiplicand, or the number to be repeated; 5 the multiplier, or the number of times the 67 is to be repeated; 335 is the product, or number resulting from the multiplication; and 67 and 5 are factors of 335.

The oblique cross $\times$ is the sign of multiplication; as, $8 \times 4 = 32$, which is read, 8 multiplied by 4 equals 32, or 4 times 8 are 32.

Let the learner copy upon paper the following table, supplying the product of each combination of factors.

Multiplication Table.

2 times	3 times	4 times	5 times	6 times	7 times
0 are 0	0 are 0	0 are 0	0 are 0	0 are 0	0 are 0
1 " 2	1 " 3	1 " 4	1 " 5	1 " 6	1 "
2 " 4	2 "	2 "	2 "	2 "	2 "
3 "	3 "	3 "	3 "	3 "	3 "
4 "	4 "	4 "	4 "	4 "	4 "
5 "	5 "	5 "	5 "	5 "	5 "
6 "	6 "	6 "	6 "	6 "	6 "
7 "	7 "	7 "	7 "	7 "	7 "
8 "	8 "	8 "	8 "	8 "	8 "
9 "	9 "	9 "	9 "	9 "	9 "
10 "	10 "	10 "	10 "	10 "	10 "
11 "	11 "	11 "	11 "	11 "	11 "
12 "	12 "	12 "	12 "	12 "	12 "

8 times	9 times	10 times	11 times	12 times
0 are 0	0 are 0	0 " 0	0 are 0	0 are 0
1 "	1 "	1 "	1 "	1 "
2 "	2 "	2 "	2 "	2 "
3 "	3 "	3 "	3 "	3 "
4 "	4 "	4 "	4 "	4 "
5 "	5 "	5 "	5 "	5 "
6 "	6 "	6 "	6 "	6 "
7 "	7 "	7 "	7 "	7 "
8 "	8 "	8 "	8 "	8 "
9 "	9 "	9 "	9 "	9 "
10 "	10 "	10 "	10 "	10 "
11 "	11 "	11 "	11 "	11 "
12 "	12 "	12 "	12 "	12 "

The pupil should be taught to recite the above table, not only as it is written, as 2 times 0 are 0; 2 times 1 are 2; 2 times 2 are 4; 2 times 3 are 6, &c., but also to recite it by using the figures in the several columns as multipliers, and the number at the top as a multiplicand; thus, 0 times 2 are 0; once 2 is 2; 2 times 2 are 4; 3 times 2 are 6; 4 times 2 are 8; 5 times 2 are 10, &c. So, 0 times 6 are 0; once 6 is 6; 2 times 6 are 12; 3 times 6 are 18; 4 times 6 are 24, &c.

26. 1. If 1 pound of sugar is worth 8 cents, how much are 5 pounds worth?

Ans. 5 pounds are worth 5 times as much as one pound; 5 times 8 cents are 40 cents. How many are 5 times 8? 8 times 5?

2. How much will 6 pounds of beef come to at 9 cents a pound? How many are 6 times 9? 9 times 6?

3. At 12 cents a dozen, how much will 4 dozen of eggs come to? How many are 4 times 12? 12 times 4? 5 times 9? 9 times 5?

4. How many are 3 times 9? 9 × 7? 7 × 9? 9 × 11? 11 × 9? 8 × 7? 7 × 8? 8 × 5? 5 × 8? 8 × 9? 9 × 8?

5. At 17 cents a pound, how much will 4 pounds of butter cost? *Ans.* 4 pounds will cost 4 times as much as 1 pound. 4 times 10 are 40; 4 times 7 are 28, or 2 tens and 8, which added to 40 makes 68; therefore, 4 pounds of butter at 17 cents a pound will cost 68 cents.

6. How many are 3 times 14? 4 times 13? 5 times 15? 6 times 14? 4 times 16? 5 times 19? 6 times 17? 9 times 15? 8 times 19?

7. How many are 2 times 20? 3 times 20? 4 times 20? 20 × 5? 20 × 6? 20 × 7? 20 × 8? 20 × 9? 20 × 10?

8. How many are 3 times 21? 4 times 27? *Ans.* 4 times 20 are 80; 4 times 7 are 28, or 2 tens and 8, which added to 80 makes 108; therefore, 4 times 27 are 108. 5 times 37? 6 times 34? 8 times 56? 38 × 9?

9. How many are 45 × 9? 36 × 7? 69 × 7? 85 × 6? 93 × 7? 87 × 5? 58 × 4? (**5.**)

27. If 1 acre of land is worth 243 dollars, how many dollars are 6 acres worth?

To perform this question, write the multiplier, 6, under the right hand figure of the multiplicand, and draw a line underneath. Then say, 6 times 3 units are 18 units, or 1 ten and 8 units. Write the 8 units, and reserve the 1 ten. 6 times 4 tens are 24 tens, and one ten added make 25 tens, or 2 hundreds and 5 tens. Write the 5 tens, and reserve the 2 hundreds. 6 times 2 hundreds are 12 hundreds, and 2 hundreds added make 14 hundreds, or one thousand and 4 hundreds, which write. The product or answer is 1458 dollars.

$$\begin{array}{r} 243 \\ 6 \\ \hline 1458 \end{array}$$

RULE FOR MULTIPLICATION WHEN THE MULTIPLIER HAS BUT ONE SIGNIFICANT FIGURE. *Write the significant figure of the multiplier under the right hand significant figure of the multiplicand. Multiply each figure of the multiplicand by the multiplier, beginning at the right hand, and write the result as in addition.*

EXAMPLES FOR PRACTICE.

1. Multiply 123 by 2; by 3; by 4; by 6; by 8; by 9.
2. Multiply 2345 by 3; by 4; by 5; by 6; by 7; by 8.
3. Multiply 56789 by 2; by 4; by 5; by 7; by 8; by 9.
4. How much is 617 × 4? 843 × 5? 7016 × 6? 3917 × 7? 84167 × 11? 50800905 by 12?
5. Multiply 36.5 by 5.

Say 5 times 5 tenths are 25 tenths, or 2 units and 5 tenths. Write the 5 tenths, and reserve the 2 units. 4 times 6 units are 24 units, and 2 units added make 26 units, or 2 tens and 6 units, &c. Ans. 182.5

$$\begin{array}{r} 36.5 \\ 5 \\ \hline 182.5 \end{array}$$

6. Multiply 35.6 by 4; 6.3 by 8; 17010.8 by 7. Multiply 180109.15 by 5. Say 5 times 5 hundredths are 25 hundredths, or 2 tens and 5 hundredths. 5 times 1 tenth are 5 tenths, &c. *Ans.* 900,545.75.

RULE. *If there are decimal places in the multiplicand only, point off as many figures for decimals in the product as there are decimals in the multiplicand.*

7. Multiply 301.87 by 8; 40180.901 by 6; $145.19 by 7.
8. What will 5 barrels of flour come to at $7.875 a barrel?
9. What will 8 loads of wood come to at $6.35 a load?
10. A man sold 5 barrels of apples at $1 875 a barrel, and 5 bushels of potatoes at 75 cents a bushel. What did the whole come to?

How much is 4 multiplied by 2? by .2? by .02? by .002? If 4 be multiplied by 2, the product will be 8. If it be multi-

plied by .2, the product will be 1 tenth as large, or .8. If it be multiplied by .02, the product will be 1 hundredth as large, or .08. If multiplied by .002, the product will be .008.

Again, if .1 be multiplied by 1, the product will be .1; if multiplied by .1, the product will be 1 tenth as large, or .01. If multiplied by .01 it will be 1 tenth as large as when multiplied by .1, or 001.

Again, .01 multiplied by 1 is .01; if multiplied by .1, it is 1 tenth as large, or .001.

All these multiplications may be thus expressed:

Multiplicands,	4	4	4	4	.1	.1	.1	.01	.01
Multipliers,	2	.2	.02	.002	1	.1	.01	1	.1
Products,	8	.8	.08	.008	.1	.01	.001	.01	.001

RULE FOR MULTIPLICATION OF DECIMALS. *Multiply as in whole numbers, and point off in the product as many places for decimals as there are decimal places in both the factors. If the product does not contain so many figures, prefix as many naughts as are necessary to make the required number.*

11. How many are 307 × .5? 8065 × .08? (12.) 910.8 × 6? 50.471 × .09? (13.) 3.0169 × .007?

(14.) Multiply 350.006 by .009; 860.1049 by .0004; .8401 by .007.

28. A number that is composed of two or more factors, as $14 = 2 \times 7$; or $30 = 2 \times 3 \times 5$, is called a *Composite* number.

A *Prime* number is one that has no factors except itself or unity; as, 1, 2, 3, 5, 7, 11.

1. Write all the prime numbers from 1 to 100.

2. What are the factors of 4? 6? 8? 9? 10? 12? 14? 16? 18? 20? 21? 22? 24? 25? 26? 27? 28? 30? 32? 36? 42? 45? 48? 54? 56? 64? 72? 100?

RULE. TO MULTIPLY BY A COMPOSITE NUMBER. *When the multiplier is a composite number larger than* 12, *first multiply by one of its factors, and then that product by another, and so on till all have been used.*

If 1 bushel of corn is worth 65 cents, how much are 28 bushels worth?

As 7 times 4 are 28, we may find the price of 4 bushels, and then of 28 bushels, which is 7 times the price of 4 bushels.

	$0.65
	4
Price of 4 bushels,	$2.60
	7
Price of 28 bushels,	$18.20

In the same manner multiply the following:

(3.) 19 × 21; 64.7 × 36. (4.) 6.17 × 42; 3016.8 × 45. (5.) 674 by 72; 8.041 by 96.

6. How many are 10 times 4? 100 times 4? 1000 times 4?

7. How many are 10 times .4? 100 times .04? .004 × 1000?

RULE. *When the multiplier is 10, 100, 1000, &c., the multiplication is performed by simply annexing the naughts to the right of the multiplicand; or, in decimals, by removing the decimal point in the multiplicand as many places to the right as there are naughts in the multiplier.*

The reason for this is, that by removing the decimal point one place to the right, tenths become units, units become tens, tens become hundreds, &c. So that the value of each figure is increased tenfold.

Thus, 10 times 354 are 3540; 100 times 516 are 51600; 31.85 × 10 = 318.5; 418.06 × 100 = 41806; 5180.46 × 10000 = 51804600.

(8.) How much is 840 × 10? 7916 × 100? (9.) 8451 × 10000? 31.04 × 10? (10.) 40.168 × 100? 31.6008 × 1000? (11.) 308.09 × 1000? $49.75 × 1000?

12. How many are 817 × 20?

Multiply first by 2, and then that product by 10. See rule for multiplying by composite numbers.

$$\begin{array}{r} 817 \\ 20 \\ \hline 16340 \end{array}$$

13. Multiply 84.17 by 30. *Solution.* 3 times 84.17 are 252.51, and 10 times 252.51 are 2525.1.

14. Multiply 3618 by 400. Multiply first by 4, then by 100. (15.) Multiply 381679 by 700; 37019 by 9000; 45 by 50000.

16. Multiply 801.7 by 4000. 4 times 801.7 are 3206.8, and by removing the decimal point three places to the right, 1000 times 3206.8 are 3206800.

17. How much is 875.4 × 400? 510.78 × 9000?

18. How many are 5 times 700? 8 times 37500? Multiply the 375 by 8, and annex the naughts afterwards. How many are 7 times 156000?

RULE. *If there are naughts at the right of either of the factors, they may be omitted in the multiplication, and annexed afterwards.*

	19.	20.	21.	22.	23.
Multiply	304000	516800	31400	1964000	510460000
By	70	80000	80	30000	6000
	21280000				

[redacted]

29. Multiply 1875 by 5037. In this example the multiplier is 5000 + 30 + 7. We may therefore multiply by each of these numbers and add their products together; thus.

1875	1875	1875
7	30	5000
13125	56250	9375000

13125	product of 1875 by 7
56250	product of 1875 by 30
9375000	product of 1875 by 5000
9444375	product of 1875 by 5037

But the work is more easily performed in this manner.

1875	
5037	
13125	product of 1875 by 7 units.
5625	product of 1875 by 3 tens.
9375	product of 1875 by 5 thousands.
9444375	product of 1875 by 5037

It will be seen that this is the same as the other, with the omission of the naughts at the right of the 2d and 3d products.

In multiplying by the tens the first figure of the product is written in the tens' place, and in multiplying by the thousands the first figure is written in the thousands' place; because the product of units by tens is tens, and the product of units by thousands is thousands.

RULE. *When there are more than one significant figure in the multiplier, multiply by each figure separately, writing the first figure of each product under the figure by which you are multiplying. The sum of the several products will be the product required. If there are decimals in either of the factors, point off the decimals in the product as before directed.* (**37.**)

NOTE. If naughts occur in the multiplier between the other figures, pass them over, and multiply by the next significant figure; placing the first figure of the product as the rule directs.

PROOF OF MULTIPLICATION. *Make the multiplicand the multiplier, and the multiplier the multiplicand, and repeat the operation. The product should be the same as before.*

EXAMPLES FOR PRACTICE.

	1.	2.	3.	4.	5.
Multiply	8570016	245.007	61450000	8076.15	617.0809
by	305	3029000	540	384000	507.08

6. How many are 401765 × 4060? 510.689 × 68000?
7. 950487 × 470.601? 80.16 × 5.07?

QUESTIONS. What is *multiplication?* What do we find by it? What *terms* are used in multiplication? What is the *multiplicand?* The *multiplier?* The *product?* What are called *factors* of the product? Give an example. What is the sign of multiplication? Give an example. What is the rule for multiplication when the multiplier has but one significant figure? Repeat the rule for the multiplication of decimals. What is a *composite* number? A *prime* number? How may multiplication be performed when the multiplier is a composite number? Give an example. How may you multiply by 10, 100, 1000, &c.? What is the reason for this? How may you perform the multiplication when there are naughts at the right of either factor? What is the rule for multiplication when there are more than one significant figure in the multiplier? What is to be done if naughts occur in the multiplier between other figures? How may multiplication be proved?

Miscellaneous Exercises in Addition, Subtraction, and Multiplication.

30. 1. What is the sum of 27 + 5 + 7 + 9 + 3 + 5 + 8 + 7 + 12?

2. Add 15 + 7 + 8 + 9 + 7 + 6 + 4 + 8 + 15. 125 + 7 + 8 10 + 7 + 9 + 3 + 5.

3. How much is 45 — 23? 59 — 36? 84 — 13? 168 — 87? 386 — 143?

4. How much is 35 — 17? (**24.**) 38 — 19? 44 — 27? 53 — 25? 65 — 46? 117 — 38? 257 — 38? 324 — 138? 647 — 359? 845 — 257? 705 — 187? 917 — 358?

5. How many are 36 × 4? 48 × 5? 57 × 6? 75 × 8? 95 × 9? 178 × 4?

6. Repeat the table, Art. **49.**

7. How many shillings in 1 pound? In £5? £8? £1 5s.? £3 15s.?

8. How many pence in 1 shilling? 5s.? 12s.? 1s. 3d.? 2s. 6d.? 4s. 8d.? 5s. 9d.? 7s. 8d.? 10s. 4d.? £1 1s.? £1 1s. 1d.? £1 3s.? £1 3s. 8d.? £2 3s. 6d.?

9. How many farthings in 1 penny? 8d.? 1d. 2qr.? 4d. 3qr.? 6d. 1qr.? 1s. 3d.? 1s. 3d. 2qr.? 2s. 2d. 2qr.? 3s. 5d. 2qr.? 7s. 8d. 3qr.? £1 12s. 4d. 2qr.?

10. Repeat the table, Art. **51.**

11. How many ounces in 1 lb. Troy? 5lb.? 2 lb. 3 oz.? 7 lb. 8 oz.? 8 lb. 10 oz.?

12. How many pennyweights in 1 oz.? 8 oz.? 5 oz. 3 dwt.? 9 oz. 10 dwt.?

13. How many grains in 1 dwt.? 4 dwt.? 3 dwt. 5 gr.? 8 dwt. 12 gr.?

14. Repeat the table, Art. **53.**

15. How many quarters in 3 cwt.? 3 cwt. 3 qr.? 5 cwt. 1 qr.? 15 cwt. 3 qr.?

16. How many pounds in 1 qr.? 1 qr. 15 lb.? 3 qr. 17 lb.? 2 qr. 23 lb.? 1 cwt. 3 qr. 15 lb.?

17. How many ounces avoirdupois in 1 lb.? 2 lb.? 3 lb. 8 oz.? 5 lb. 7 oz.? 3 lb. 14 oz.?

18. Repeat the table, Art. 54.

19. How many furlongs in 1 mile? 3 m.? 2 fur.? 5 m. 7 fur.? 15 m. 3 fur.?

20. How many rods in 1 furlong? 3 fur. 15 rd.? 1 mile? 1 m. 2 fur.?

21. How many feet in 1 yd.? 3 yd.? 8 yd. 2 ft.? 5yd. 3 ft.?

22. How many inches in 1 foot? 3 ft.? 2 ft. 3 in.? 5 ft. 7 in.? 8 ft. 5 in.?

23. How many inches in 1 yd.? In 1 rd.? 1 fur.? 1 m.? 1 league? 3½ miles?

24. How many feet in 1 rd.? 5 rd.? 8½ rd.? 25 rd.? In 1 fur.? In 1 mile?

25. How many yards in 8 rd.? In 9½ rd.? In 1 fur.? In 1 mile? 3½ miles?

26. How many drams in 1 lb.? In 3 lb.? In 7½ lb.? In 1 qr.? In 1 cwt.?

Exercises in Addition, Multiplication, and Division.

31. 1. What will 54 bushels of wheat come to at $1.12 a bushel?

2. In an orchard there are 27 rows of trees, and 15 trees in each row. How many trees are there in the orchard?

3. Suppose each tree to yield 3 barrels of apples, how many barrels are there in the orchard, and what will they come to at $1.625 a barrel?

4. The gathering and marketing of the above apples cost 17 cents a barrel. What was the net* value of the apples?

5. A man bought 28 acres of wood land at $27.25 per acre. How much did it come to? How many cords of wood were there on the land, supposing it to yield 37.5 cords per acre?

6. What would the wood come to at $1.875 a cord, after paying 45 cents a cord for cutting?

7. A man sold 75 bushels of wheat at 95 cents per bushel, and received in exchange 75 gallons of molasses at 28 cents a gallon; 48 pounds of sugar at 10 cents a pound; 3 bushels of grass seed at $2.125 a bushel; 150 pounds of salt fish at 4 cents a pound; 5 yards of broadcloth at $3.25 per yard, and the remainder in money. How much money did he receive?

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* *Net* value means the value after deducting all expenses.

In £15 15s. 8d. 3qr. how many farthings?

£15 15s. 8d. 3qr.
20
———
315 shillings.
12
———
3788 pence.
4
———
15155 farthings.

In this question we multiply £15 by 20 to reduce them to shillings, because 20 shillings make 1£, and to the product we add the 15 shillings. We multiply 315 shillings by 12, because 12 pence make 1 shilling, and to this product add the 8d. Multiplying the pence by 4, because 4 farthings make 1d., and adding to the product the 3qr., gives the number of farthings in £15 15s. 8d. 3qr. This process is called reduction.

NOTE. *Higher denominations are reduced to lower denominations by multiplication.*

In 2 yr. 48 da. 15 h. 35 m. 17 sec. how many seconds? (**60.**)

365.25 [17 sec.
2 yr. 48 da. 15h. 35m.
———
730.50
48
———
778.5 days.
24
———
3114.0
15570
———
18699 hours.
60
———
1121975 minutes.
60
———
67318517 seconds.

In 15 m. 7 fur. 19 rd. 4 yd. 2 ft. 5 in. how many inches?

[4 yd. 2 ft. 5 in.
In 15 m. 7 fur. 19 rd.
8
———
127
40
———
5099 rods.
5.5
———
2549.5
25495
———
28048.5 yards.
3
———
84147.5 feet.
12
———
1009775.0 inches.

8. How many shillings are there in £15 8s.? £25 17s.?

9. How many pence in 15s. 6d.? In £12 15s. 9d.?

10. How many farthings are there in £3 5s. 8d. 3qr.?

11. How many pence will 1 barrel of flour come to at £1 5s. 6d. per bl.? 5 bls.? 8 bls.? 25 bls.? 150 bls.?

Learn the table, Art. **60.**

12. How many days in 8 years? In 10 yr.? 15 yr. and 87 da.? 37 yr. and 150 da.?

13. How many hours in 1 week? In 17 w.? In 1 yr. 35 da. 19 h.?

14. How many minutes in 1 day and 6 hours? In 1 w. 3 da. 18 h.? In 17 da. 4 h. 31 m.?

15. How many seconds in 35 m. 18 s.? In 16 h. 35 m. 24 s.? In 1 yr. 15 da. 8 h. 3 m. 4 s.?

16. In 15 lb. Troy, how many oz.? How many dwt.? How many gr.?

17. In 10 lb. 8 oz. Troy, how many oz.? dwt.? gr.?

18. In 10 lb. 5 oz. 3 dwt. 15 gr. how many grains?

19. How much are 3 lb. 5 oz. 6 dwt. of silver worth at $0.06 a dwt.?

Learn the table, Art. **59.**

20. In 189 gallons, how many quarts? pints? gills?

21. In 267 gallons, how many quarts? gills?

22. In 318 gal. 6 qt. 1 pt. how many pints?

23. What will 204 gal. 2 qt. of molasses come to, at 6 cts. per quart?

24. How much will 17 gal. 1 qt. 1 pt. of milk come to at 2 cts. a pint?

Learn the table, Art. **58.**

25. How many pints in 3 bu. 3 pk. 3 qt.? In 15 bu. 7 qt.?

26. What will 2 bu. 3 pk. 5 qt. of beans come to at 8 cts. a quart?

27. How many pints in 4 bu. 3 pk. 3 qt.? In 9 bu. 3 pt.?

28. What will 3 bu. 2 pk. 3 qt. of salt come to, at 2 cts. a qt.?

29. How many rods in 1 mile? 10 m. 3 fur.? 15 m. 2 fur. 15 rd.?

30. How many yards in 1 mile? In 2 m. 3 fur. 7 rd.?

31. How many inches in a mile? In 2 m. 3 fur. 12 ft. 4 in.?

32. What will the grading of 25 miles of railroad cost at 15875 dollars a mile?

33. The rails weigh 18 pounds per foot. How much will the rails weigh for a single track 1 mile long? 5 miles? 25 miles?

34. What will the rails for 1 mile cost at 3½ cents a pound? For 5 miles? For 25 miles? For 60 miles?

Learn the table, Art. **56.**

35. How many rods in 25 acres? In 10 acres 2 roods? 7 acres 8 sq. rods?

36. In 35 sq. rods, 15 sq. yards how many sq. yards? How many sq. feet?

37. In 29 sq. yd. 6 sq. ft. 100 sq. in. how many sq. in.?

38. What will 1 sq. acre come to at \$0.25 a sq. ft.?

39. How many oz. in 1 qr. 15 lb. 7 oz.? In 2 cwt. 3 qr. 5 lb. 7 oz.?

40. What will 1764 lb. of sugar cost at \$0.085 a pound?

41. A farmer carried to market 5 bushels of potatoes, 75 pounds of squashes, 4 bushels of apples, 3 bushels of turnips, 10 gallons of milk, 3 dozen of eggs, and 25 bunches of onions. He sold his potatoes at 18 cents a peck, squashes at 1 cent 5 mills a pound, apples at 17 cents a bushel, turnips at \$0.125 a peck, milk at 4 cents 5 mills a quart, eggs at \$0.155 a dozen, and onions at 2 cents a bunch. How much did his load come to?

42. The farmer named in the last question bought for his family 3 gallons of molasses at 24 cents a gallon, 15 pounds of sugar at 7 cents a pound, 20 pounds of coffee at 11 cents a pound, 5 pounds of fresh fish at 4 cents a pound, 25 pounds of rice at \$0.055 a pound, and a pair of boots for \$3.25. How much of his market money did he carry home?

43. A sold B 584 gallons of molasses at 23 cents per gallon, and B sold A 365 pounds of butter at 18 cents a pound, and 115 pounds of cheese at \$0.075 a pound. How much was the balance and to whom was it due?

44. What will 7606 pounds of lead come to at \$0.082 a pound?

45. A farmer sold 175 bushels of potatoes at \$0.625 a bushel, 25 barrels of apples at \$2.125 a barrel, and 200 pounds of squashes at \$1.75 per 100 pounds. He received in pay a horse, a cow worth \$40.75, and 3 barrels of flour worth \$6.75 a barrel. How much was the horse valued at?

46. In \$1057 how many cents? How many mills?

47. How many cents and mills in \$1041? In \$1475.01?

48. Reduce to cents and mills \$5.01; \$3000.001; \$81000; \$547.91; \$5761.408.

49. Multiply 4104.75 by 310.6407; by 210.0475.

50. Multiply 65041000 by 4070000; add 2504000 to the product, and from the sum subtract 58175 $\times$ 601.08.

51. Multiply 581.09106 by .00045; add to the product 15.0084, and from the sum subtract 3.6810084.

II. 52. Add 510.8075 + 3516.087 + 410.08009; multiply the sum by 510875000, and subtract 857601.007 from the product.

53. Multiply 7160.0804 by 850600000; subtract 5040800.7051 from the product, and to the remainder add 5418.008 $\times$ 3.10804.

I. 54. What is the amount of the following bills of parcels?

Mr. WILLIAM SMITH, Salem, April 12, 1849.

Bought of THOMAS MORTON,

15 lb.	Tea,	at	45 cts.,		$6.75
7 "	Chocolate,	"	35 "		
8 "	Coffee,	"	12 "		
13 "	Sugar,	"	11 "		
1 bl.	Flour,	"	$7.12 "		

Received Payment, THOMAS MORTON,
By Horace Nelson.

55. Boston, March 25, 1849.

Mr. BENJAMIN WILLIAMS,

Bought of JONAS STEPHENS,

4 yd.	Black Broadcloth,	at	$3.50,	$
6 "	Cambric,	"	.25,	
18 "	Cotton Cloth,	"	.08,	
7 "	Cassimere,	"	1.125,	
4 "	Doeskin,	"	0.875,	

Received Payment, JONAS STEVENS.

56. Danvers, Nov. 15, 1848.

Mr. HENRY SULLIVAN,

Bought of ISRAEL PUTNAM,

75 bush.	Potatoes,	at	$0.75,	$
10 "	Turnips,	"	.25,	
3 "	Beets,	"	.30,	
3 "	Onions,	"	.60,	
6 doz.	Cabbages,	"	.25,	
150 lb.	Squashes,	"	.02,	

Received Payment, ISRAEL PUTNAM.

57. Multiply fifty-four thousand and seventy-five by nine thousand and twenty-four.

58. Multiply six hundred and seventy-five thousand and twenty-five by fifty-four,—and eighty-seven *ten thousandths.*

59. Multiply fifteen million, five hundred and ninety-six thousand, eight hundred and fifty-four, by five thousand and sixteen.

II. 60. Multiply five billion, forty-one million, three thousand and sixty-five, by ninety-five thousand and fifty-four.

61. Multiply fifty-seven million, five thousand and twenty-four, — and fifty-four thousand and thirty-five *millionths*, by five hundred and four *thousandths*.

62. Multiply sixteen thousand and one, — ninety-five ten millionths, by one hundred and seven thousand and fourteen, — and seventy-five thousandths.

II. **32.** 1. How much are 4 times 146? Say 4 times 100 are 400, 4 times 40 are 160, which added to 400 is 560, 4 times 6 are 24, 560 and 24 are 584: therefore, 4 times 146 are 584.

2. Multiply 254 by 4; by 5; by 6; by 7; by 8; by 9.

3. How much is 375×2? 416×3? 512×4? 215×6? 719×2? 914×3? 567×8?

4. How much are 6 times 140? 395? 486? 1004? 1064?

5. How much are 8 times 37? 196? 287? 1087? 3284?

6. How much are 9 times 25? 115? 614? 1016? 3165? 8467?

SECTION V. — DIVISION.

33. DIVISION is the method of finding how many times one given number is contained in another.

The terms used in division are the *dividend*, or the number to be divided; the *divisor*, or the number to divide by; and the *quotient*, which shows how many times the divisor is contained in the dividend. That part of the dividend which remains after division is called the *remainder*.

A man divided 28 cents among some children, giving 7 cents to each. How many children were there? *Ans.* There were as many children as there are times 7 cents in 28 cents. 4 times 7 are 28; therefore 7 cents is contained in 28 cents 4 times. In this example 28 is the dividend, 7 is the divisor, and 4 is the quotient, or number of times the divisor is contained in the dividend.

We see, also, that the divisor and quotient are *factors* of the dividend, just as in multiplication the multiplicand and multiplier are factors of the product.

The sign of division is a short horizontal line with a dot above and below it; thus, $\div$, and shows that the number before it is to be divided by the one after it; as $28 \div 7 = 4$; which is read 28 divided by 7 is equal to 4. Or the dividend may be placed above a short horizontal line and the divisor below it, as $\frac{28}{7} = 4$, which is also read, 28 divided by 7 is equal to 4. *Numbers placed above one another in this way always show that the number above the line is to be divided by the one below it.*

The learner should copy this table, supplying the quotient of each division, and then commit it to memory.

DIVISION TABLE.

2 in	3 in	4 in	5 in	6 in	7 in
2, once.	3, once.	4, once.	5, once.	6, once.	7, once.
4, 2 times.	6, 2 times.	8, 2 times.	10,	12,	14,
6, 3 times.	9, 3 times.	12, times.	15,	18,	21,
8, 4 times.	12, 4 times.	16, times.	20,	24,	28,
10, 5 times.	15, 5 times.	20, times.	25,	30,	35,
12, 6 times.	18, times.	24, times.	30,	36,	42,
14, times.	21, times.	28, times.	35,	42,	49,
16, times.	24, times.	32, times.	40,	48,	56,
18, times.	27, times.	36, times.	45,	54,	63,
20, times.	30, times.	40, times.	50,	60,	70,
22, times.	33, times.	44, times.	55,	66,	77,
24, times.	36, times.	48, times.	60,	72,	84,

8 in	9 in	10 in	11 in	12 in
8, once.	9,	10,	11,	12,
16, times.	18,	20,	22,	24,
24,	27,	30,	33,	36,
32,	36,	40,	44,	48,
40,	45,	50,	55,	60,
48,	54,	60,	66,	72,
56,	63,	70,	77,	84,
64,	72,	80,	88,	96,
72,	81,	90,	99,	108,
80,	90,	100,	110,	120,
88,	99,	110,	121,	132,
96,	108,	120,	132,	144,

1. How much is 8 divided by 2? $12 \div 3$? $20 \div 4$? $16 \div 8$? $21 \div 3$? $21 \div 7$? $32 \div 4$?

2. How much is 14 divided by 7? $\frac{16}{8}$? $\frac{24}{8}$? $\frac{35}{7}$? $\frac{45}{5}$? $\frac{48}{6}$? $\frac{54}{9}$? $\frac{56}{8}$? $\frac{63}{9}$?

3. How much is $\frac{55}{11}$? $\frac{72}{12}$? $\frac{80}{10}$? $84 \div 12$? $84 \div 7$? $77 \div 11$? $96 \div 12$?

4. How many cakes at 3 cents apiece can be bought for 9 cents? *Ans.* As many cakes as there are times 3 cents in 9 cents. How many for 15 cents? 27 cents? 30 cents? 36 cents?

5. How many oranges at 4 cents apiece can be bought for 12 cents? 20 cents? 32 cents?

6. How much flour at 5 cents a pound can be bought for 20 cents? 25 cents? 35 cents? 40 cents? 45 cents? 60 cents? *Ans.* As many pounds as there are times 5 cents in 20 cents, &c.

7. How many pounds of sugar at 8 cents a pound can be bought for 24 cents? 40 cents? 56 cents? 48 cents? 64 cents? 80 cents? 96 cents?

NOTE. When a number or quantity is divided into two equal parts, one of the parts is called one half of the number or quantity. If it is divided into three equal parts, one of the parts is called one third of the number or quantity; and two of the parts are called two thirds of the quantity. If divided into 4 equal parts, one of the parts is called one fourth, two parts are called two fourths, three parts three fourths, &c.

8. If an orange be divided into five equal parts, what is one of the parts called? 2 parts? 3 parts? 4 parts?

9. If a pound of sugar be divided into eight equal parts, what is one of the parts called? 2 parts? 3 parts? 5 parts? 7 parts?

NOTE. Such expressions as one half, one third, two thirds, three fourths, are written $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$, which may be read, one divided by two, one divided by three, two divided by three, three divided by four, or one half, one third, &c.

10. If 12 be divided into 2 equal parts, how much will 1 of the parts be? How much will 1 part be if it is divided into 3 equal parts? Into 4 equal parts? Into 6 equal parts?

11. How much is $\frac{1}{2}$ of 12? $\frac{1}{3}$ of 12? $\frac{1}{4}$ of 12? $\frac{1}{6}$ of 12?

12. How much is $\frac{1}{2}$ of 16? $\frac{1}{3}$ of 15? $\frac{1}{4}$ of 20? $\frac{1}{5}$ of 20? $\frac{1}{6}$ of 18? $\frac{1}{7}$ of 28? $\frac{1}{6}$ of 30? $\frac{1}{5}$ of 30?

13. What is $\frac{1}{8}$ of 24? $\frac{1}{6}$ of 30? $\frac{1}{5}$ of 35? $\frac{1}{7}$ of 28? $\frac{1}{8}$ of 40? $\frac{1}{6}$ of 54? $\frac{1}{8}$ of 48? $\frac{1}{9}$ of 27?

14. What is $\frac{1}{5}$ of 40? $\frac{1}{8}$ of 56? $\frac{1}{9}$ of 54? $\frac{1}{7}$ of 56? $\frac{1}{6}$ of 48? $\frac{1}{9}$ of 63? $\frac{1}{7}$ of 63? $\frac{1}{8}$ of 72?

15. What is $\frac{1}{12}$ of 24? $\frac{1}{9}$ of 27? $\frac{1}{12}$ of 60? $\frac{1}{11}$ of 55? $\frac{1}{9}$ of 72? $\frac{1}{8}$ of 96? $\frac{1}{10}$ of 50? $\frac{1}{11}$ of 55?

16. What is $\frac{1}{3}$ of 18? $\frac{1}{4}$ of 28? $\frac{1}{5}$ of 45? $\frac{1}{9}$ of 27? $\frac{1}{8}$ of 48? $\frac{1}{10}$ of 70?

34. 1. What part of 4 apples is 1 apple? 2 apples? 3 apples? *Ans.* 1 apple is $\frac{1}{4}$ of 4 apples, 2 apples are 2 times one fourth, or two fourths of 4 apples.

2. What part of 5 cakes is 1 cake? 2 cakes? 3 cakes? 7 cakes?

3. What part of \$8 is \$1? What part are \$6? \$10? \$3? \$15? \$25?

4. What part of 10 pence is 1 penny? 3 pence? 9 pence? 13 pence?

5. What part of 12 is 1? 3? 5? 7? 15? 20?

6. 4 is what part of 9? Of 12? Of 13? Of 25? 37? 45? 150?

7. If 8 pounds of sugar cost 48 cents, how much is it per pound? Say, 1 pound will cost $\frac{1}{8}$ as much as 8 pounds; $\frac{1}{8}$ of 48 cents is 6 cents. How much will 3 pounds cost?

Ans. 3 pounds will cost 3 times as much as 1 pound, or 18 cents. 5 pounds? 7 pounds?

8. **How much is $\frac{1}{8}$ of 48? $\frac{3}{8}$ of 48? $\frac{5}{8}$ of 48? $\frac{7}{8}$ of 48?** *Say, since $\frac{1}{8}$ of* 48 *is* 6, *$\frac{3}{8}$ will be three times as much, or* 18; *therefore, $\frac{3}{8}$ of* 48 *are* 18.

9. How much is $\frac{1}{4}$ of 8? Of 12? $\frac{3}{4}$ of 8? Of 12? Of 16?

10. How much is $\frac{1}{6}$ of 18? 30? 42? $\frac{2}{6}$ of 18? 30? 42? $\frac{5}{6}$ of 18? 30? 42?

11. What is $\frac{1}{7}$ of 35? $\frac{3}{7}$ of 35? $\frac{4}{7}$? $\frac{6}{7}$? $\frac{1}{7}$ of 56? $\frac{3}{7}$? $\frac{5}{7}$? $\frac{4}{7}$ of 84?

12. If 7 yards of cloth cost 63 cents, what part of 63 cents will 1 yard cost? 3 yds.? 4 yds.? 6 yds.?

How much is $\frac{1}{7}$ of 63 cents? $\frac{3}{7}$? $\frac{4}{7}$? $\frac{6}{7}$?

13. If 8 bls. of flour can be bought for 56 dollars, what part of 56 dollars will 1 barrel cost? 2 bls.? 5 bls.? 7 bls.? 12 bls.? 10 bls?

14. How much is $\frac{1}{8}$ of \$56? $\frac{2}{8}$? $\frac{5}{8}$? $\frac{7}{8}$? $\frac{12}{8}$? $\frac{10}{8}$ of \$56?

15. If 12 dollars' worth of provisions last 6 men 1 week, what part of 1 week will 1 dollar's worth last them? 3 dollars' worth? 8 dollars' worth? 15 dollars' worth?

16. If 36 dollars pay the board of 9 men 1 week, what will pay the board of 1 man? 3 men? 8 men? 10 men? 12 men?

II. 17. What is $\frac{1}{2}$ of 20? Of 40? 60? 80? 100? 120? 140? 160? 180?

18. What is $\frac{1}{2}$ of 26? *Say one half of* 20 *is* 10, *and one half of* 6 *is* 3, *which added to* 10 *makes* 13. $\frac{1}{2}$ *of* 26 *is* 13.

19. What is $\frac{1}{2}$ of 28? 44? 48? 64? 68? 84? 86? 102? 106? 124? 128?

20. What is $\frac{1}{2}$ of 34? 34 *is equal to* $20+14$; $\frac{1}{2}$ *of* 20 *is* 10; $\frac{1}{2}$ *of* 14 *is* 7, *which added to* 10 *makes* 17. $\frac{1}{2}$ *of* 34 *is* 17. $\frac{1}{2}$ of 36? 38? 52? 56? 74? 78? 96? 92? 114? 118? 134? 158?

21. What is $\frac{1}{3}$ of 30? 60? 90? 120? 150? Say, $\frac{1}{3}$ of 15 is 5; $\frac{1}{3}$ of 150 will be 10 times as much, or 50; therefore $\frac{1}{3}$ of 150 is 50. $\frac{1}{3}$ of 180? 210? 240? 270? 300? $\frac{2}{3}$ of 30? 60? 90? 120? 150? 180? 210? &c.

22. What is $\frac{1}{3}$ of 36? 39? $\frac{1}{3}$ *of* 30 *is* 10, *and* $\frac{1}{3}$ *of* 9 *is* 3, *which added to* 10 *makes* 13; $\frac{1}{3}$ *of* 39 *is* 13. $\frac{1}{3}$ of 63? 66? 96? 99? 123? 156? $\frac{2}{3}$ of 36? 39?

23. What is $\frac{1}{3}$ of 42? *Solution.* $42=30+12$. $\frac{1}{3}$ of 30 is 10; $\frac{1}{3}$ of 12 is 4; 10 and 4 are 14; $\frac{1}{3}$ of 42 is 14. What is $\frac{1}{3}$ of 45? $\frac{1}{3}$ of 48? $\frac{1}{3}$ of 54?

NOTE. $54=30+24$. $\frac{1}{3}$ of 57? 72? 75?

24. What is $\frac{2}{3}$ of 42? Of 45? 48? 54? 57? 72? 75?

25. What is $\frac{1}{4}$ of 40? 48? $56=40+16$? $60=40+20$? 68? 80? 96?

26. What is 1 fifth of 50? $75=50+25$? 80? 95? 100? $110=100+10$? 125? 130?

27. What is 3 fifths of 50? 75? 80? 95? 100? 120? 140? 500? 540?

28. What is 1 seventh of 70? 84? 98 = 70 + 28? 126 = 70 + 56? 168 = 140 + 28?

29. What is $\frac{1}{7}$ of 70? 84? 98? 126? 168? 210? 490? 504 = 490 + 14?

30. What is $\frac{1}{8}$ of 80? 96? 128? 152 = 80 + 72? 160? 176?

31. What are $\frac{3}{8}$ of 80? $\frac{5}{8}$ of 96? $\frac{7}{8}$ of 152? 160? 2 ninths of 270?

35. 1. If an apple be equally divided between two boys, what part of an apple will each have?

2. If 3 apples be equally divided between 2 boys, how much will each have?

NOTE. First divide 2 apples, and then divide the remaining one.

3. How much is $\frac{1}{2}$ of 1? Of 2? Of 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? 15? 16? 17? 20? 21?

4. In 2 halves how many units? In 3 halves? *Ans.* One unit and one half.

5. How many units in 4 halves? 5 halves? 6 halves? 7? 8? 9?

6. If 1 orange be divided equally among 3 boys, what part of an orange will each have? *Ans.* $\frac{1}{3}$ of an orange.

7. If 2 oranges be divided equally among 3 boys, what part of an orange will each have? *Ans.* One third of 2 oranges, or $\frac{2}{3}$ of one orange.

8. If 3 oranges be thus divided, what will each have? 4 oranges? Divide 3 oranges, then the remaining one. Thus, $\frac{1}{3}$ of 3 oranges is 1 orange; to which adding $\frac{1}{3}$ of the remaining orange makes 1 orange and $\frac{1}{3}$ of an orange. 5 oranges may be divided thus: $\frac{1}{3}$ of 3 oranges is 1 orange, — and $\frac{1}{3}$ of the remaining 2 oranges is $\frac{2}{3}$ of an orange; which added to 1 orange makes 1 orange and $\frac{2}{3}$ of an orange.

9. How much is $\frac{1}{3}$ of 1? Of 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

10. In 3 thirds how many units? In 4 thirds? In 5 thirds? In 6 thirds? In 7 thirds?

11. If 1 pound of raisins be divided into 4 equal parts, what part of 1 lb. will one part contain? 2 lb.? 3 lb.? 4 lb.? 5 lb.? 6 lb.? 9 lb.?

12. How much is $\frac{1}{4}$ of 1 unit? Of 2? 3? 5? 9? 11? 13? 16? 18? 23? 31?

13. In 4 fourths how many units? In 5 fourths? In 9 fourths? In 11 fourths? 15 fourths? 18 fourths?

14. If 5 dollars buy one cord of wood, what part of a cord can be bought for one dollar? For 2 dollars? For 3 dollars?

For $6? *Ans.* 6 fifths of 1 cord, or 1 cord and $\frac{1}{5}$ of a cord.

How many for $8? For $11? $15? $18?

15. How much is 1 fifth of 1? Of 3? 7? *Ans.* $\frac{7}{5}=1\frac{2}{5}$.

How much is 1 fifth of 9? 15? 18? 24? 27? 34? 35? 43? 54?

16. How much is 1 sixth of 1? 4? 7? 9? 14? 25? 32? 37? 43? 55? 64? 72?

17. When beef is 7 cents a pound, how much can be bought for 1 cent? 2 cts.? 5 cts.? 9 cts.? 15 cts.? 25 cts.? 38 cts.? 45 cts.? 54 cts.? 68 cts.? 75 cts.? 84 cts.?

18. How much is $\frac{1}{8}$ of 1? 3? 15? 23? 27? 31? 39? 47? 58? 68? 78? 88? 96?

19. How much is 1 ninth of 1? 4? 7? 26? 36? 46? 56? 66? 76? 86? 96? 106?

20. How much is 1 tenth of 1? 8? 15? 24? 37? 45? 58? 64? 75? 86? 97?

21. What is 1 eleventh of 1? 3? 8? 14? 24? 34? 44? 54? 64? 74? 84? 94? 104? 114?

22. What is 1 twelfth of 1? 5? 11? 17? 27? 37? 47? 57? 67? 77? 87? 97? 107? 117? 127?

II. **36.** 1. How much is $\frac{1}{2}$ of 135? Say $135=120+15$. *Ans.* $67\frac{1}{2}$.

2. What is $\frac{1}{2}$ of 145? $157=140+17$? 163? 173? 185? $196=180+16$?

3. What is $\frac{1}{3}$ of 30? 33? $44=30+14$? 60? 65? 78? $83=60+23$? 90? 105? 115?

4. What is $\frac{1}{4}$ of $60=40+20$? 67? 74? 80? $93=80+13$? 115? 125? $147=120+27$?

5. What is 1 fifth of 50? 60? 75? 96? 107? 125? 143? 157? 169? 184?

6. What is 1 sixth of 60? 75? 84? 97? 118? 120? 135? 144? $387=360+27$?

7. What is 1 seventh of 70? $95=70+15$? $114=70+44$? $127=70+57$? 129? 135? 140? 148?

8. What is $\frac{1}{8}$ of 80? 111? 120? 140? 160? 175? $194=160+34$?

37. 1. How many cords of wood, at 3 dollars a cord, can be bought for 396 dollars?

Since 3 dollars will buy one cord, 396 dollars will buy as many cords as there are times 3 in 396. 396 is equal to $300+90+6$. 3 is contained in 300, 100 times; in 90, 30 times; in 6, 2 times. The answer is 132 cords.

To do this on the slate, write the numbers as in the margin, and say, 3 in 3, once: and as the 3 is hundreds, the 1 is also hundreds, and is written in the hundreds' place. 3 in 9, 3 times, to be written in the tens' place. 3 in 6, 2 times, to be written in the units' place.

$$\begin{array}{r} 3)396 \\ \hline 132 \end{array}$$

2. How many yards of cloth, at 2 dollars a yard, can be bought for $4628?

3. How many bushels are there in 4808 pecks?

Since there are 4 pecks in 1 bushel, there will be as many bushels in 4808 pecks as there are times 4 in 4808.

4. How many yards are there in 3693 feet?

5. How much is $\frac{24682}{2}$? $488448 \div 4$? $24884 \div 4$? $\frac{12660606}{6}$? $\frac{164804804}{4}$? $\frac{50055}{5}$? $\frac{12639}{3}$?

Note. If the divisor is not contained in the *first* figure of the dividend, find how many times it is contained in the *first two* figures.

At 2 cents a pound, how many pounds of squashes can be bought for 3153 cents? *Ans.* As many pounds as there are times 2 in 3153. $3153 = 2000 + 1000 + 140 + 13$. 2 is contained in 2000, 1000 times; in 1000, 500 times; in 140, 70 times; in 13, $6\frac{1}{2}$ times; which, being added, make $1576\frac{1}{2}$ pounds.

6. How many pounds at 3 cents? $3153 = 3000 + 150 + 3$. How many at 4 cents? $3153 = 2800 + 320 + 33$. How many at 5 cents? At 6 cents? At 7 cents?

To perform such divisions, we may write the numbers as in the margin, and,

$$\begin{array}{r} 2)3153 \\ \hline 1576\frac{1}{2} \end{array}$$

If after dividing any figure there is a remainder, prefix it mentally to the next figure of the dividend, and divide as before.

In this example, after dividing 3 by 2, 1 remains. Prefixing it to the next figure, makes 11. 2 in 11, 5 times, and 1 remains. Prefixing the 1 to 5, makes 15. 2 in 15, 7 times, and 1 remains. Prefixing this to the 3, makes 13. 2 in 13, $6\frac{1}{2}$ times.

7. How many times is 5 contained in 16278? *Ans.* $3255\frac{3}{5}$ times.

8. Divide 5649 by 3. 85732 by 4. 514087 by 5. 761834 by 6. 5519875 by 8.

9. Divide 30807510 by 3; by 4; by 5; by 6; by 7; by 8; by 9; by 10; 11; 12.

10. How much is 38459 ÷ 9? 10010009 ÷ 9?

11. In 184765 inches how many feet, and how many inches over? In these feet how many yards?

Since there is 1 foot in 12 inches, there will be as many feet as there are times 12 in 184765; and since there is 1 yard in 3 feet, there will be as many yards as there are times 3 in the number of feet.

```
12)184765
   ------
 3)15397 ft. and 1 inch over.
   -----
   5132 yd. and 1 ft. over.
```

12. In 48967 furlongs how many miles, and how many furlongs over? In these miles how many leagues?

Learn the table, Art. **55.**

13. In 860419 nails how many quarters? How many yards?

14. In 33167 pints how many quarts? pecks? bushels?

15. In 115418 gills how many pints? quarts? gallons?

16. In 35145 farthings how many pence? shillings? (**49.**)

17. At 7 dollars a barrel, how many barrels of flour can be bought for 8640 dollars?

38. *If there are decimals in the dividend only, divide as in whole numbers, and point off as many decimal places in the quotient as there are in the dividend.*

EXAMPLES FOR PRACTICE.

1. How many times is 4 contained in 3416.8?

```
4)3416.8
  ------
   854.2
```

2. How many times is 4 contained in 5028.16? In 30449.28?

3. Divide 2084.25 by 5. Divide 4080.012 by 6. Divide 619.0048 by 8.

4. Divide $362.168 equally among 8 men.

5. How much is 80416.008 ÷ 9? 304186.4084 ÷ 7? 4180.7616 ÷ 6?

6. Divide 4061.709 by 7.

NOTE. If there is a remainder after dividing, and more decimals are desired in the quotient, naughts may be annexed as decimals to the dividend, as in the margin. The sign + is to be annexed to the quotient if there is a remainder after performing the division as far as it is desired.

```
7)4061.7090000
  ------------
   580.2441428+
```

In the remaining examples of this Art. carry the quotient to at least 5 places of decimals, if there are remainders.

7. Divide 1843.07 by 2; by 3; by 4; by 5; by 6.
8. Divide 5106.847 by 7; by 8; by 9; by 10.
9. Divide 800700.5001 by 2; by 6; by 7; by 9.
10. Divide 501.080701 by 2; by 4; by 6.

It was shown (27) that a product must have as many decimal places as there are in its factors; therefore, the divisor and quotient, being factors of the dividend, (23,) must together have as many decimal places as the dividend.

GENERAL RULE FOR DIVISION OF DECIMALS.

Divide as in whole numbers, and point off in the quotient as many places for decimals as the decimal places in the dividend exceed those in the divisor.

The dividend must contain at least as many decimal places as the divisor. If it has not so many, annex as many decimal naughts as are needed.

11. How much is 3104.64 ÷ 4? .30415 ÷ .7?

12. How much is .000805 ÷ .005? 13.041 ÷ .006? 14 ÷ .0005?

13. Divide 3104.57 by 6; by .6; by .06; by .006; by .0006.

14. Divide 510.867 by 5; by .5; by .05; by .005.

15. Divide 87165.0008 by 7; by .08; by .009.

30. It was shown (28) that removing the decimal point one place to the right *multiplies* the number by 10; removing it two places, multiplies it by 100, &c. For a like reason, removing the decimal point one place to the left, *divides* the number by 10, since it makes units tenths, tens units, &c. Hence the following

RULE. TO DIVIDE BY 10, 100, 1000, &c. *Remove the decimal point in the dividend as many places to the left as there naughts at the right of the divisor.*

EXAMPLES.

1. Divide 304617 by 10. *Ans.* 30461.7. By 100; by 1000.
(2.) By 10000; by 100000.
3. Divide 30.4671 by 10; by 100; by 1000.

Prefix naughts to the figures of the dividend if necessary; thus, 30.4671 ÷ 1000 = .0304671.

4. Divide 85.1865 by 10000; by 100000.

5. Divide 81564 by 100; by 1000; by 10000.

6. Divide 304.06 by 20. First remove the decimal point in the dividend one place to the left, which will divide it by 10; then cancel the 0, and divide by 2. See margin.

$$2\cancel{0})30.406$$
$$15.203$$

RULE. *When the divisor has naughts on the right, remove the decimal point in the dividend as many places to the left as there are naughts on the right of the divisor; cancel the naughts, and divide by the remaining figure or figures.*

Carry the quotient in the 7th, 9th, and 10th examples to at least 6 decimal places if there are remainders.

7. Divide 51761.7 by 300; by 4000; by 60000.

8. Divide 104.57604 by 600000.

Removing the decimal point 5 places to the left; thus, .0010457604 divides the number by 100000; then dividing by 6, gives the quotient required.

9. Divide 510756 by 50; by 700; by 80000; by 90000.

10. Divide 30.1457 by 30; by 900; by 50000.

NOTE. If decimals are not desired in the quotient, the figures pointed off by the rule may be annexed to the remainder, after dividing the other figures.

11. Divide 351285 by 500. See margin.

$$5\cancel{00})3512.85$$
$$702\tfrac{285}{500}$$

12. Divide 4160841 by 8000; by 60000; by 800.

13. How many minutes are there in 6148578 seconds, and how many seconds over? In these minutes how many hours?

Since there is 1 minute in 60 seconds, there will be as many minutes as there are times 60 in 6148578; and since there is 1 hour in 60 minutes, there will be as many hours as there are times 60 in the minutes.

Ans. 1707 h. 56 min. 18 sec.

60)6148578

60)102476 min. and 18 sec.

1707 h. and 56 min. over.

NOTE. *Lower denominations are reduced to higher by division.*

14. How many minutes in 784206 seconds? How many hours?

15. How many barrels of pork will 75740 lb. make, allowing 200 lb. to a barrel?

Ans. 378 barrels and 140 lb. over.

$$2\cancel{00})\underline{757.40}$$
$$378\tfrac{140}{200}$$

16. Divide 18457 by 20. *Ans.* $922\frac{17}{20}$.

$$2\cancel{0})\underline{1845.7}$$
$$922\tfrac{17}{20}$$

17. Divide 351743 by 300; by 4000; by 60.

18. Divide 4160075 by 1200; by 90000; by 1000.

19. Divide 580165 by 500; by 100; by 1000.

20. Divide 341608 by 7000; by 80000; by 600.

NOTE. The answers to the 20th, 21st, and 22d are to be carried to six decimals, if there are remainders.

21. Divide 310.45 by 1000; by 300; by 90000.

22. Divide 54716 by 8000; by 12000.

40. All the preceding examples are performed by what is called *Short Division;* but where the divisor consists of significant figures higher than 12, it is more convenient to perform the work by *Long Division.*

How many times is 24 contained in 3373689?

$24 \times 1 = 24$
$24 \times 2 = 48$
$24 \times 3 = 72$
$24 \times 4 = 96$
$24 \times 5 = 120$
$24 \times 6 = 144$
$24 \times 7 = 168$
$24 \times 8 = 192$
$24 \times 9 = 216$

```
24)3373689(140570 9/24
   24
   --
    97
    96
    ---
     136
     120
     ---
      168
      168
      ---
        9
```

To perform this, first write in a column the products of the divisor by each of the 9 digits. Then, having written the divisor and dividend as in the margin, take as many figures at the left of the dividend as will contain the divisor once or more, and divide them by it; 24 in 33, once. Place the 1, as the first figure of the quotient, to the right of the dividend, and subtract once 24 from 33. Write the remainder, 9, and to it annex the next figure of the dividend for another partial dividend. By examining the column of products, see how many times 24 is contained in 97, which is found to be 4. Write the 4 in the quotient, and subtract the product of 4 times 24 from 97. Write the remainder, 1, and annex to it the next figure in the dividend. As 24 is not contained in 13, place a naught in the quotient, and bring down and annex the next figure of the dividend. Proceed

thus till all the figures of the dividend have been brought down and annexed.

If, after dividing all the figures of the dividend, there is a remainder, it is to be written over the divisor and annexed to the quotient, as in short division.

PROOF. *Multiply the quotient by the divisor, and to the product add the remainder; the sum should be equal to the dividend.* Or,

Subtract the remainder, if there is any, from the dividend, and divide the difference by the quotient; the result should be equal to the original divisor.

NOTE 1. Commence the division by taking the fewest figures on the left of the dividend that will contain the divisor.

2. Place either a naught or some other figure in the quotient for every figure brought down from the dividend.

3. If in any case the product exceeds the partial dividend from which it is to be subtracted, the figure put in the quotient is too large, and must be diminished.

4. The remainder must always be less than the divisor.

5. Be careful to observe the rules (Art. 38) for pointing off decimals in the quotient.

EXAMPLES FOR PRACTICE.

1. Divide 306540106 by 13; by 14. (2.) By 15; by 16. (3.) By 17; by 18; by 19.

4. Divide 417870060 by 23; by 38; by 85. (5.) By 105; by 347.

6. Divide 1074083 by 37000. 37$\cancel{000}$) 1074.083 ($29\frac{1083}{37000}$

First remove the decimal point 3 places to the left; this divides it by 1000. Then divide the whole number, 1074, by 37; the figures pointed off are a part of the remainder. See note, Art. 39.

```
 74
 ---
 334
 333
 -------
  1083 rem.
```

7. Divide 1074083 by 47500; by 7180.

8. By 94300; by 871000.

9. If 18 bags of coffee cost 486 dollars, what is 1 bag worth?

10. A man bought a field, containing 19 acres, for 11108 dollars; how much did he pay for each acre?

11. There are 320 rods in a mile; how many miles are there in 5104678 rods, and how many rods over?

12. If a rail-car can run 450 miles in 1 day, how far is that per hour?

13. What is 1 barrel of flour worth, if 350 barrels cost $2154.48?

14. Divide 3 by .028.

```
.028 ) 3.000 ( 107.1428+.
       28
       ---
        200
        196
        ---
          40
          28
          --
          120
          112
          ---
            80
            56
            --
           240
           224
           ---
            16
```

$28 \times 1 = 28$
$28 \times 2 = 56$
$28 \times 3 = 84$
$28 \times 4 = 112$
$28 \times 5 = 140$
$28 \times 6 = 168$
$28 \times 7 = 196$
$28 \times 8 = 224$
$28 \times 9 = 252$

15. Divide .00716 by 314.

```
314 ) .00716 ( .00002280255—.
        628
        ---
        880
        628
        ---
        2520
        2512
        ----
          800
          628
          ---
          1720
          1570
          ----
           1500
           1570
```

In the 14th example there are 7 decimals used in the dividend, (counting all the decimal naughts which were annexed to the remainders,) and 3 in the divisor; or 4 more decimals in the dividend than in the divisor. The sign + to the right of the quotient shows that the division can be continued further.

In the 15th example, there are 11 decimal places in the dividend, and none in the divisor; consequently 11 decimals are required in the quotient. As there are not so many, as many naughts must be prefixed as are wanted to make the number. The sign — to the right of the quotient figure indicates that the last quotient figure is a little too large.

16. How many times is 30.015 contained in 80.41607?

Carry the quotients in the 17th, 18th, 19th, and 20th examples to at least 7 decimal places, if there are remainders.

17. Divide 15 by .007; by 8.16. (18.) By .0104; by 7; by 900.

19. Divide .00005 by 270; by 6.85. (20.) By 4019; by 31.07; by 1000.

21. Divide $1000.00 equally among 187 men, getting the answer to the nearest mill.

22. Divide thirty dollars one cent equally among 13 men, as in the last example.

QUESTIONS. What is *division?* What *terms* are used in division? What is the *dividend?* The *divisor?* The *quotient?* The *remainder?* Give an example. What are *factors* of the dividend? What is the *sign* of division? Give examples showing how division may be expressed. If a number or quantity be divided into 2 equal parts, what is one of the parts called? If divided into 3 equal parts, what is one part called? 2 parts? Give examples, and show how such expressions as one half, one third, three fourths, &c., are written. Show by an example how the division of larger numbers is performed on the slate. What is to be done if the divisor is not contained in the *first figure* of the dividend? If after dividing any figure there is a remainder, what is to be done? Give an example. How is division to be performed when there are decimals in the dividend only? What may be done if, after dividing, there is a remainder, and more decimals are desired in the quotient? What is the general rule for decimals in division? What is to be done if the dividend has not so many decimal places as the divisor? How do you divide by 10, 100, 1000, &c.? Why? By 3000, 80000, &c.? Why? What is to be done with the figures pointed off at the right of the dividend, if decimals are not desired in the quotient? How are *higher* denominations reduced to *lower?* (See Art. 31, p. 34.) How are *lower* denominations reduced to *higher?* When should you divide by *short division?* When by long division? Perform an example by long division, and explain each step in the process. What is the method of *proof?* What remark is made in note 1? In note 2? Note 3? Note 4? Note 5? What does the sign + at the right of a quotient indicate? What does the sign — at the right of a quotient indicate?

41. Review Art. **34**, question 17 to 31, and Art. **36.**

1. How many times is 4 contained in 135? *Solution.* 135 = 120 + 15; 4 is contained in 120, 30 times; in 15, 3¾ times, which added to 30 makes 33¾.

2. How many times is 4 contained in 151? In 178? 369? 570?

3. How many times is 5 contained in 257? In 341? 684 = 600 + 50 + 34?

4. In 174 farthings how many pence? *Solution.* Since there are 4 farthings in 1 penny, there will be as many pence in 174 farthings as there are times 4 in 174. *Ans.* 43d. 2qr.

5. How many pence in 48 farthings? In 169 farthings? In 275 farthings?

6. How many shillings in 72 pence? In 124d.? In 367d.? *Ans.* 30s. 7d.

7. How many pounds in 25 shillings? In 47s.? In 59s.?

8. How many pounds in 26 oz. Troy? In 84 oz.? In 367 oz.?

9. How many dwt. in 24 gr.? In 32 gr.? In 48 gr.? In 56 gr.?

10. How many oz. in 20 dwt.? In 25 dwt.? In 80 dwt.? In 85 dwt.? In 109 dwt.?

11. How many oz. in 16 dr. Avoidupois? In 24 dr.? In 30 dr.? In 48 dr.? In 54 dr.?

12. How many lb. in 32 oz.? In 40 oz.? In 56 oz.? In 64 oz.?

13. How many quarters in 8 nails? In 19 na.? In 54 na.? In 160 na.? In 163 na.?

14. How many feet in 36 inches? In 48 in.? In 54 in.? In 72 in.? In 720 in.? In 725 in.?

15. How many yards in 24 feet? In 240 feet? In 2400 feet? In 25 feet? In 242 feet?

16. How many miles in 32 furlongs? In 320 fur.? In 37 fur.? In 72 fur.? In 720 fur.?

17. How many square yards in 18 sq. feet? In 180 sq. feet? In 1800 sq. feet? In 23 sq. feet? In 72 sq. feet? In 76 sq. feet?

18. How many cord feet in 16 cubic feet? In 20 cu. ft.? In 32 cu. ft.? In 37 cu. ft.? In 45 cu. ft.? In 57 cu. ft.?

19. How many cords in 8 cord feet? In 12 cord feet? In 24 C. ft.? In 48 C. ft.? In 480 C. ft.? In 72 C. ft.? In 720 C. ft.? In 784 C. ft.?

20. In 15 pints how many quarts? How many gallons?

21. How many quarts and gallons in 20 pints? In 24 pints? In 45 pints?

22. How many pecks in 48 quarts? How many bushels?

23. How many pecks and bushels in 64 quarts? In 75 quarts? In 96 quarts? In 120 quarts? In 125 quarts?

Questions to be performed by Division.

42. 1. How many bushels of wheat, at $0.875 per bushel, can be bought for $85.17?

2. How many tons of coal, at $7.75 per ton, can be bought for $186.00?

3. If 350 men are to share equally $10051, how much shall each have? Get the answer to the nearest cent.

4. How many cords of wood, at $6.50 per cord, can be bought for $135.85?

5. Bought 358 barrels of flour for $2159.25; what was the price per barrel?

6. In 1841983 farthings how many pence? In these pence how many shillings? In these shillings how many pounds? Do it by short division.

7. In 180451 mills how many cents? How many dollars?

8. In 3517047 mills how many cents? How many dollars?

9. In 61048 grains Troy how many dwt.? oz.? lb.?

24) 610487 gr.

20) 25436 dwt. 23 gr.

12) 1271 oz. 16 dwt.

Ans. 105 lb. 11 oz. 16 dwt. 23 gr.

10. In 130433717 seconds how many min.? h.? da.? yr.?

60) 130433717 sec.

60) 2173895 min. 17 sec.

24) 36231 h. 35 min.

365.25) 1509 d. 15 h.

Ans. 4 yr. 48 d. 15 h. 35 m. 17 sec.

11. In 4460175 minutes how many hours? days? years?

12. In 916000 grains apothecaries' weight how many pounds, ounces, &c.?

Reduce the grains to scruples, then these scruples to drams, &c.

13. In 8164096 drams Avoirdupois how many oz.? lb.? qr.? cwt.? tons?

14. In 2289600 inches how many ft.? yd.? rd.? fur.? m.?

15. In 876510 feet how many miles, furlongs, &c.?

Reduce the feet to yards, then these yards to rods, &c.?

16. In 681045 inches how many nails? quarters? yards?
17. In 31363200 square inches how many acres?
18. In 4817690 square yards how many square miles?
19. In 810650 cubic inches how many cubic yards?
20. In 6541 pints how many quarts? pecks? bushels?
21. In 68145 pints how many quarts? gallons?
22. In 4108000 seconds how many minutes? hours? days?
23. In 86417 cubic inches how many wine gallons? (59.) How many dry gallons, or half pecks? (58.)

Perform the following divisions, annexing decimal naughts to the dividend, if there are remainders, to 6 places of decimals.

(24.) $\frac{4187}{15}$. (25.) $\frac{1}{3}$; $\frac{2}{5}$. (26.) $\frac{3}{4}$; $\frac{8}{15}$. (27.) $\frac{5}{9}$; $\frac{457}{8}$.
(28.) $\frac{945}{162}$; $\frac{5168}{349}$; $\frac{65}{18}$.

Perform the following divisions; if there is a remainder write it over the divisor and annex it to the quotient.

(29.) $\frac{167}{15}$. *Ans.* $11\frac{2}{15}$. $\frac{1586}{14}$. (30.) $\frac{10001}{107}$; $\frac{90008}{647}$.
(31.) $\frac{8190653}{80016}$; $\frac{91368408}{314008}$. (32.) $\frac{5070901}{100701}$; $\frac{30167}{851}$.

33. What number multiplied by 684 will make 7387884?

34. A man sold 78 bushels of corn for $48.75. How much was it a bushel?

35. A farmer sold a load of potatoes, at $0.375 a bushel, for $9.1875. How many bushels were there?

36. How many bushels of beans, at $1.75 a bushel, will come to $43.75?

37. If the product of two factors is 4375, and one of the factors is 175, what is the other factor?

38. How many miles per hour must a steam-ship sail to cross the Atlantic in 12 days, the distance being 3000 miles?

39. How many miles per hour to cross it in $13\frac{1}{2}$ days $=$ 13.5 days?

40. In how many hours will she cross it if she sails 12 miles per hour? In how many days?

41. 9 times 987 is how many times 25?

42. 15 times 1040 is how many times 75?

43. How much is $(171.84 \times 10.17) \div .608$? $(9.105 \times 87100) \div .00075$?

44. The circumference of the earth is 360 degrees, and it revolves on its axis in 24 hours. How many degrees does it turn in 1 hour?

43. MISCELLANEOUS EXAMPLES IN REDUCTION.

After the exercises which the pupil has had in Reduction in the preceding articles, he will be able to understand the following definitions and rules, and to perform the miscellaneous exercises which follow them.*

Reduction is changing numbers from one denomination to another, without altering their value.

Reducing numbers to *lower* denominations, — that is, pounds to pence, miles to yards, &c., — is sometimes called Reduction descending; reducing them to *higher* denominations is called Reduction ascending.

RULE FOR REDUCING A NUMBER FROM A HIGHER DENOMINATION TO A LOWER.

Multiply the highest denomination by that number which it takes of the next lower denomination to make a unit of the higher, and to the product add the number, if any, expressed in this lower denomination in the given example. Proceed in this manner through all the denominations to the lowest.

* The exercises in this article, although in the smaller type, are intended for the slate.

RULE FOR REDUCING A NUMBER FROM A LOWER DENOMINATION TO A HIGHER.

Divide the given number by so many as make one of the next higher; set aside the remainder, if any, and proceed in the same manner through all the denominations to the highest.

EXAMPLES.

1. In 24 tons 16 cwt. 2 qr. 15 lb. 9 oz. 14 dr. how many drams?
2. In 4865392 drams how many oz.? lb.? qr.? cwt.? tons?
3. In 2578 lb. 11 oz. 15 gr. Troy how many grains?
4. In 3801765 grains Troy how many pounds, &c.?
5. In 58 yd. 3 qr. 1 na. 2 in. how many inches?
6. In 384165 nails how many English ells?
7. In £2516 8s. 3¼d. how many farthings?
8. In 81846758 farthings how many pounds?
9. In 8516 English quarters 3 bu. 2 pk. 5 qt. how many pints?
10. In 4579 gal. 3 qt. 1 gill how many gills?
11. In 817568419 gills how many gallons?
12. In 516 acres 2 R. 29 sq. rods how many sq. rods?
13. In 3 A. 35 sq. rods 25 sq. yd. 7 sq. feet how many sq. feet?
14. In 875164 sq. rods how many acres?
15. In 15 cu. yards 20 cu. ft. 1084 cu. in. how many cu. inches?
16. In 18476189 cu. inches how many cu. yards?
17. In 8416 cu. ft. 561 cu. in. how many cu. inches? How many wine gallons? How many beer gallons? How many dry gallons or half pecks?
18. In 857 bushels of wheat how many pounds? (Art. **62.**)
19. How many pounds in 859 bushels of wheat? In 859 bushels of oats? Of Indian corn? Of rye? (**62.**)
20. How many pounds in 876 barrels of beef? In 849 barrels of pork?
21. How many barrels of beef in 8176 pounds of beef?
22. How many bushels in 3018 pounds of wheat? Of Indian corn? Of oats?
23. How many pounds in 875 barrels of flour?
24. How many barrels in 841675 pounds of flour?

MISCELLANEOUS EXAMPLES.

44. Review Art. **19.** 1. Add 35 + 56; 48 + 59; 317 + 125; 358 + 75; 94 + 187.

2. Add 4.4 + 8.7. Say, 4 + 8 are 12, .4 + .7 are 11 tenths, or 1 and 1 tenth, which added to 12 makes 13.1; therefore, &c.

3. Add 3.4 + 5.2; 4.18 + 5.26; 6.007 + 8.145; 5.3 + 8.9; 25.8 + 38.9.

Review Art. **22.** 4. How much is 317 — 208? Say, 200 from

300 = 100; 8 from 17 = 9, which added to 100 makes 109; therefore, 317 less 208 is 109.

5. How much is 578 — 347? 564 — 217? 634 — 124? 718 — 305?

6. What is the complement of 35? 47? 53? 32? 87? 46? 38? 58?

7. What is the complement of 127? Take 200 from 1000, and to the remainder add the complement of 27.

What is the complement of 573? Take 600 from 1000, and add to the remainder the complement of 73.

8. What is the complement of 643? 856? 359? 436? 729? 814? 538? 476?

9. How much is 549 — 387? Say, 400 from 549 = 149, and then add the complement of 87. How much is 644 — 173? 754 — 369? 835 — 275? 987 — 174? 694 — 397?

Review Art. **32, 35,** and **36.**

10. In 2 halves how many whole ones? In 3 halves? In 4 halves? 7 halves?

11. How much is 5 times $4\frac{1}{2}$? Say, 5 times 4 are 20, 5 times $\frac{1}{2}$ are 5 halves, or 2 and $\frac{1}{2}$, which added to 20 makes $22\frac{1}{2}$; therefore, 5 times $4\frac{1}{2}$ are $22\frac{1}{2}$.

12. How much are 3 times $5\frac{1}{2}$? $8\frac{1}{2}$? $12\frac{1}{2}$? $35\frac{1}{2}$?

13. In 3 thirds how many whole ones? In $\frac{4}{3}$? $\frac{8}{3}$? $\frac{11}{3}$? $\frac{15}{3}$? $\frac{19}{3}$?

14. How much are 4 times $5\frac{1}{3}$? $6\frac{2}{3}$? $7\frac{1}{3}$? $15\frac{2}{3}$? $75\frac{1}{3}$? $116\frac{1}{3}$?

15. In 4 fourths how many whole ones? In $\frac{5}{4}$? $\frac{8}{4}$? $\frac{15}{4}$? $\frac{26}{4}$?

16. How much are 5 times $\frac{3}{4}$? 5 times $1\frac{3}{4}$? 5 times $3\frac{1}{4}$? 5 times $11\frac{3}{4}$? 5 times $15\frac{3}{4}$?

17. In 5 fifths how many whole ones? In $\frac{8}{5}$? $\frac{11}{5}$? $\frac{20}{5}$? $\frac{28}{5}$?

18. How much are 6 times $\frac{3}{5}$? 6 times $1\frac{4}{5}$? 6 times $3\frac{2}{5}$? 6 times $8\frac{3}{5}$? 6 times $14\frac{1}{5}$? 6 times $35\frac{4}{5}$?

19. In 6 sixths how many whole ones? In $\frac{11}{6}$? $\frac{15}{6}$? $\frac{18}{6}$? $\frac{40}{6}$? $\frac{65}{6}$?

20. How much are 7 times $\frac{1}{6}$? 7 times $\frac{3}{6}$? 7 times $\frac{5}{6}$? 7 times $3\frac{2}{6}$? 7 times $4\frac{5}{6}$? 7 times $8\frac{4}{6}$?

21. How much are $3\frac{1}{2}$ times 8? Say, 3 times 8 are 24, $\frac{1}{2}$ of 8 is 4, which added to 24, makes 28; therefore, $3\frac{1}{2}$ times 8 are 28.

22. How much are $4\frac{1}{2}$ times 6? $5\frac{1}{2}$ times 8? $6\frac{1}{2}$ times 9?

23. How much are $3\frac{1}{4}$ times 5? $4\frac{1}{4}$ times 10? $6\frac{1}{4}$ times 12?

How much are $4\frac{3}{4}$ times 5?

Say, 4 times 5 are 20; $\frac{1}{4}$ of 5 is $\frac{5}{4}$; $\frac{3}{4}$ of 5 is 3 times as much, = $\frac{15}{4}$, or $3\frac{3}{4}$, which added to 20 makes $23\frac{3}{4}$; therefore, $4\frac{3}{4}$ times 5 are $23\frac{3}{4}$.

24. How much are $6\frac{1}{5}$ times 5? $4\frac{1}{5}$ times 9? $8\frac{2}{5}$ times 9? $6\frac{3}{5}$ times 15? $9\frac{4}{5}$ times 15? $8\frac{3}{5}$ times 36?

25. A man earns $\frac{3}{4}$ of a dollar per day. How much can he earn in 3 days? In 6 days? In 3 weeks of 6 days each? In 8 weeks?

26. What will 6 lb. of beef cost, at $10\frac{1}{2}$ cents a lb.? 12 lb.? 9 lb.?

27. If 1 lb. of beef cost 8 cents, what will $\frac{1}{4}$ of lb. cost? $\frac{3}{4}$ lb.? $1\frac{1}{4}$ lb.?

What will $3\frac{3}{4}$ lb. cost?

If 1 lb. costs 8 cents, $\frac{1}{4}$ of a lb. will cost $\frac{1}{4}$ of 8 cents, or 2 cents; $\frac{3}{4}$ will cost 3 times as much. Therefore, &c.

28. If 1 yd. of cloth cost 12 cents, how much will $\frac{1}{4}$ of a yd. cost? $\frac{3}{4}$ yd.? $3\frac{1}{4}$ yd.? $5\frac{3}{4}$ yd.?

How much will $\frac{1}{8}$ yd. cost? $\frac{3}{8}$ yd.? $\frac{7}{8}$ yd.?

Say, $\frac{1}{8}$ of a yd. will cost $\frac{1}{8}$ of 12 cents, which is $1\frac{4}{8}$ ct.; $\frac{3}{8}$ will cost 3 times $1\frac{4}{8}$ ct.; 3 times 1 are 3, and 3 times $\frac{4}{8}$ are $\frac{12}{8}$, or $1\frac{4}{8}$, which added to 3 makes $4\frac{4}{8}$. *Ans.* $4\frac{4}{8}$ cents.

29. What will $3\frac{1}{4}$ lb. of cheese cost, at 10 cts. a lb.? $5\frac{1}{2}$ lb.?

30. What will 5 yards of ribbon cost, at $8\frac{1}{2}$ cts. per yd.?

Review Art. **34.**

31. What part of 4 lb. is 1 lb.? What part is 2 lb.? 3 lb.? 4 lb.? 5 lb.? 7 lb.? 15 lb.?

Ans. 1 lb. is $\frac{1}{4}$ of 4 lb. 7 lb. are $\frac{7}{4}$ of 4 lb., &c.

32. What part of \$8 is \$1? \$5? \$9? \$3? \$11? \$15? \$7? \$23?

33. What part of 12 is 1? 5? 7? 13? 17? 9? 25? 48? 51?

34. What part of 30 is 1? 12? 5? 7? 11? 23? 45? 57? 19?

35. What part of 7 is 1? 6? 15? 27? 48? 21? 35? 57?

36. If 5 lb. of beef cost 30 cts., what will 3 lb. cost? 7 lb.? 9 lb.? 12 lb.? 16 lb.?

Say, as 3 is $\frac{3}{5}$ of 5, 3 lb. will cost $\frac{3}{5}$ as much as 5 lb. $\frac{1}{5}$ of 30 cts. is 6 cts., and $\frac{3}{5}$ are 3 times 6 cts., or 18 cts.

37. If 3 barrels of flour cost \$15, what is the price of 2 barrels? 5 barrels? 8 barrels? 12 barrels?

38. If 8 yards of cloth cost \$24, what will be the price of 3 yards? 9 yards? 12 yards?

39. What is $\frac{1}{8}$ of 24? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{3}{4}$? $\frac{5}{6}$? $\frac{7}{8}$? $\frac{5}{12}$? $\frac{7}{12}$? $\frac{17}{12}$? $\frac{9}{4}$?

40. What is $\frac{1}{9}$ of 36? $\frac{4}{9}$? $\frac{11}{9}$? $\frac{5}{6}$? $\frac{7}{6}$? $\frac{12}{6}$? $\frac{5}{12}$? $\frac{11}{12}$? $\frac{13}{18}$?

41. What is $\frac{1}{5}$ of 38? $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$? $\frac{7}{5}$?

42. What is $\frac{1}{7}$ of 12? $\frac{2}{7}$? $\frac{3}{7}$? $\frac{4}{7}$? $\frac{5}{7}$? $\frac{6}{7}$? $\frac{10}{7}$?

43. If $\frac{1}{3}$ of a bushel of corn is worth $8\frac{1}{2}$ cents, what are $\frac{2}{3}$ worth? $\frac{4}{3}$? $\frac{5}{3}$? 1 bushel? 5 bushels? $7\frac{1}{2}$ bushels?

44. If $\frac{3}{8}$ of a barrel of beef cost \$10, what is $\frac{1}{8}$ worth? 1 barrel? $5\frac{3}{8}$ barrels?

45. Bought $2\frac{1}{8}$ cords of wood for \$17; what was the price per cord?

NOTE. $2\frac{2}{3} = \frac{8}{3}$. If $\frac{8}{3}$ of a cord cost \$17, $\frac{1}{3}$ will cost $\frac{1}{8}$ of \$17, $= 2\frac{1}{8}$; and $\frac{3}{3}$ or 1 cord will cost 3 times \$$2\frac{1}{8}$, $=$ \$$6\frac{3}{8}$. What cost 3 cords? 5 cords? 8 cords?

46. If $3\frac{3}{4}$ pounds of cheese cost 30 cents, what is 1 lb. worth? $3\frac{1}{2}$ lb.? $5\frac{1}{4}$ lb.?

MISCELLANEOUS EXAMPLES IN ALL THE PRECEDING RULES.

45. 1. Multiply one thousand seven hundred and one,— and five *hundredths*, by thirty thousand and four, and add to the product six thousand and one.

2. A borrowed \$85.00. He has paid \$30.12 at one time, \$15.125 at another, and \$18.875 at another. How much remains unpaid? How many pounds of butter, at 18 cents a pound, will it take to pay the balance?

3. A grocer buys 15 firkins of butter, each weighing 45 pounds, at 15 cents a pound. How much molasses will it take to pay for it at 24 cents a gallon?

4. A farmer sold 575 pounds of pork, at $6\frac{1}{2}$ cents a pound; 3 tons of hay, at \$.75 a cwt.; 1 ton of carrots, for \$12.00. He received in pay 2 barrels of flour, at \$6.50; 50 pounds of salt fish, at $4\frac{1}{2}$ cents a pound; 25 pounds of sugar, at $5\frac{1}{2}$ cents a pound; 15 pounds of rice, at 4 cents a pound, and 24 yards of cotton cloth, at $7\frac{1}{4}$ cents a yard. How much was the balance?

5. What will 5 bu. 3 pk. 6 qt. of meal come to, at $2\frac{1}{2}$ cents per qt.?

6. A farmer sold a fat ox. The fore quarters weighed 341 pounds each; the hind quarters, 312 pounds each; the hide weighed 75 pounds; the tallow, 63 pounds. He had $5\frac{1}{2}$ cents a pound for the fore quarters, $6\frac{1}{4}$ cents a pound for the hind quarters, 8 cents a pound for the tallow, and $6\frac{1}{2}$ cents a pound for the hide. What did the ox come to?

7. A publisher is about printing a new edition of a book. It will take 12 reams of paper, each ream weighing $29\frac{1}{2}$ lb., at 11 cts. per lb.; he is to pay \$1.50 per ream for printing, and $12\frac{1}{2}$ cts. a book for the binding. What will an edition of 1000 copies cost? How much for each book?

In this and other questions in which the answer is in Federal Money, the pupil may get the answer to the nearest mill.

8. A merchant buys 50 barrels of flour for \$257.00. At what price must he sell it per barrel to gain \$55.50 on the whole?

9. A bankrupt settles with his creditors by paying .75 of his debt. How much will he pay a creditor to whom he owes $450.00? How much will the creditor lose?

10. A man labors 307 days in the year, earning each day $1.45. What will his year's wages amount to? He pays $65.00 a year for house rent; he burns 6 cords of wood, at $4.75 a cord, and pays $250.77 for other family expenses. How much does he save per annum?

11. A young man, when 17 years old, contracts a habit of smoking tobacco. If it costs him one cent a day, how much will it have cost him when he is 30 years old? If after that time it costs him 2½ cents a day, how much will it have cost him for this article alone at the age of 50?

12. How much cloth, at 5s. per yard, can be bought for £3 15s.?

13. How much pork, at $0.075 a pound, can be bought for $75? How many barrels will it fill, if each barrel holds 200 lb.?

14. How many dwt. of silver are there in 3 lb. 5 oz. 17 dwt.? How many spoons can be made of it, each spoon weighing 13 dwt.?

15. How many coats may be made of 53 yd. 3 qr. of cloth, allowing 2 yd. 1 qr. for each coat?

16. A man sold 110 gallons of molasses, at 28½ cents a gallon, and received his pay in potatoes at 57 cents a bushel. How many bushels did he receive?

17. If 1 barrel, = 196 lb., of flour cost $5.88, what is it per lb.? How much are 5 lb. worth? 16 lb.? 25 lb.? 35½ lb.?

18. If 15 lb. of beef are worth $1.05, how much are 12 lb. worth?

19. What will 3 cwt. 3 qr. 15 lb. of tea come to, at $0.375 a pound?

20. If 354 bushels of corn are worth $223.02, what are 17 bushels worth?

21. A farmer sold a grocer, to whom he owed $87.50, 25 bushels of barley, at 53 cents a bushel, 65 bushels of corn, at 65 cents a bushel, and paid the remainder in hay at $16 a ton. How much hay did it take?

22. A merchant imported 2 bales of broadcloth, each bale containing 195 yards. The cloth cost in London $3.75 a yard; he paid $6.00 for freight; the duties were $1.35 per

yard; other expenses were \$5.50. How much did the cloth cost him? How much per yard?

23. For how much must he sell the whole to gain \$100.00? At how much per yard?

24. Divide \$350.54 among 5 men. Give the first \$20.15; the second, three times as much as the first; the third, as much as the first and second; the fourth, as much as the first three, wanting 75 dollars. How much shall the fifth have?

25. If 25 barrels of flour, at \$6.20 per barrel, are given for 28 cords of wood, what is the wood valued at a cord?

26. A grocer bought 150 lb. of butter for \$25, and sold it for \$27.25. How much did he gain on a pound?

27. If 457 lb. of cheese cost \$36.00, at what price per lb. must it be sold to gain \$5.25?

28. If 3 yards of cloth cost \$13.50, how many yards can be bought for \$67.50?

29. What will 7 shares of railroad stock cost, at \$103.50 per share?

30. How many acres of land will pay for it, at \$62.50 per acre?

31. If the interest of one dollar is \$0.06 a year, how much is the interest of \$40.00 for 1 year? For 2 years? For 3½ years?

32. What would be the interest of \$154.18 for 3 years?

33. A man owing \$300, has paid .25 of his debt at one time, .35 at another, and .33 at another. How much does he still owe?

34. If 3 horses will eat 9 bu. 3 pks. of oats in 2 weeks, how many pecks will one horse consume? 6 horses? 19 horses? How many bushels?

35. If 1 oz. of silver is worth \$1.10, what is that a dwt.? How much are 8 oz. 17 dwt. worth?

36. In £1024 15s. 7¾d. how many farthings?

37. In 119892 farthings how many pounds?

38. How many pounds Troy in a million of grains?

39. In 484 lb. 11 oz. 19 dwt. 23 gr. how many grains?

40. In 24 tons 16 cwt. 2 qr. 15 lb. 9 oz. 14 dr. how many drams?

41. In 647895 drams how many oz.? pounds?

42. In 547968 inches how many yards?

43. In 287 sq. yards 8 sq. ft. 132 sq. in. how many sq. inches?

44. In 12345678 sq. inches how many sq. yards?
45. In one cubic yard how many cubic inches? (**57.**)
46. In 43765947 cubic inches how many cubic yards?
47. In one solar year, consisting of 365 days 5 hours 48 minutes and 57 seconds, how many seconds?
48. How many days from the 24th of May to the 24th of November?
49. How many days from the 25th of March to the 24th of June?
50. How many days from the 25th of December to the 25th of March in leap year?
51. The pressure of air upon a person of moderate size is said to be about 324000 pounds; how many tons of 2000 pounds each?

BILLS.

46. A BILL is a paper given by the seller to the buyer, containing a statement of the article or articles sold, with their prices.

Find the cost of each article in the following bills, and the amount of each bill:

1.

Charlestown, Dec. 2, 1848.

Messrs. BALL & POOR,

Bought of GRANT, DANIEL & Co.,

$10\frac{1}{2}$	reams	French Letter Paper,	at $2,50,	$
4	"	Color'd Medium Print'g Paper,	" 3.00,	
6	"	Foolscap Paper, Ruled,	" 2.75,	
12	"	Demy Writing Paper,	" 8.25,	

Received Payment,
GRANT, DANIEL & Co.,
By Joseph Kimball.

2.

Lowell, Jan. 15, 1849.

Mr. ABRAM WHITTEMORE,

Bought of WILLIAM PRESCOTT,

$19\frac{1}{2}$	yds. Black Silk,	at $0.75,	$
2	Canton Crape Shawls,	" 9.50,	
2	doz. Pearl Buttons,	" $0.12\frac{1}{2}$ per doz.,	

Received Payment,
WILLIAM PRESCOTT.

When bills are paid, they are to be receipted, either by the person of whom the articles are bought, as in the second bill, or by a clerk or some other person authorized to receipt them, as in No. 1.

3. *Boston, Sept.* 5, 1848.

Mr. JAMES WILSON,

Bought of D. PROUTY & Co.,

3½	doz. Files,	at	$6.50 per doz..	$
10	lb. Nails,	"	6 cts. per lb.,	
3	Chisels,	"	$1.50,	
			———	———

4. *Salem, Dec.* 15, 1848.

Mr. THEODORE TAYLOR,

Bought of SIMON DRAPER,

15½	yards	Broadcloth,	at	$4,50,	$
84	"	Cottons,	"	8½ cts.,	
38	"	Cambric,	"	12½ "	
28	"	Chintz,	"	10½ "	
1½	lb.	Sewing Silk,	"	62½ " per oz.,	
				———	———

5. *Charlestown, Nov.* 12, 1848.

Mr. STEPHEN CARPENTER,

Bought of WILLIAM MERCHANT,

118	lb.	8d. Nails,	at	6½ cts.,	$
48	"	Sheet Lead,	"	6¼ "	
12	pair	Butt Hinges,	"	18¾ "	
6	"	" "	"	23 "	
				———	———

6. *Beverly, Dec.* 8, 1848.

Mr. SIMEON BOARDMAN,

Bought of JONATHAN FARMER,

475	lb.	Pork,	at	6½ cts.,	$
187	"	Beef,	"	7¼ "	
75	"	Butter,	"	19 "	
38	"	Cheese,	"	9½ "	
15	bush.	Potatoes,	"	58 "	
75	lb.	Squashes,	"	2¼ "	
				———	———

7. *Boston. March* 5, 1849.

MR. E. MANSFIELD,

Bought of TAPPAN, WHITTEMORE & MASON,

6 Emerson's Arithmetic, 2d part,	at $22\frac{1}{2}$ cts.,	$
$2\frac{1}{2}$ doz. Russell's Sequel,	" 25 ".	

The forms of the following bills are to be given in writing by the pupil.

8. Hartford, Jan. 17, 1849. Mr. Philip R. Stetson buys of Henry Osgood, 75 gal. of molasses, at 28 cts.; 48 lb. of sugar, at $10\frac{1}{2}$ cts.; 3 bush. of grass seed, at $2.125; 150 lb. of salt fish, at $5\frac{1}{2}$ cts.; and 5 yards of broadcloth, at $3.25; for which he pays cash. Write the bill, and receipt it for Osgood.

NOTE. It is customary in writing bills to begin the name of each article with a capital letter.

9. Danvers, Nov. 15, 1848. Mr. William Daniels buys of Timothy Preston, 5 bush. of potatoes, at 75 cts; 75 lb. of squashes, at $2\frac{1}{2}$ cts.; 3 bl. of apples, at $1.75; 2 bush. of turnips, at $37\frac{1}{2}$ cts.; 10 gal. of milk, at 18 cts.; and 25 bunches of onions, at $2\frac{1}{2}$ cts. Write a bill of parcels, and state the amount.

10. Boston, Feb. 15, 1849. Charles Tappan buys of Charles Stoddard, a black horse, warranted sound and kind, and only five years old. Write the bill, and receipt it.

11. New York, March 17, 1849. Messrs. Washington, Marshall & Co. buy of Brett & Price, 10 pairs of gaiter shoes, at $4.00; 15 pair of calf boots, at $3.75; 12 pair of French shoes, at $2.25; and 9 pair kip boots, at $1.25. Write the bill, and state the amount.

12. Boston, April 7, 1849. Messrs. B. B. Mussey & Co. buy of Tappan, Whittemore & Mason, 50 American School Readers, at 50 cts.; 25 Introduction to do., at 30 cts.; 50 Russell's Sequel, at 20 cts.; 75 Russell's Primary Reader, at 13 cts.; 25 Introduction to do., at 10 cts.; and 100 Primers, at $6\frac{1}{4}$ cts. Write a bill of the articles, and state the amount.

For other forms of bills, accounts, accounts current, &c., see Art. 124, 125.

ANALYSIS.

47. Perform the questions in this article by Analysis; that is, first find the value of a unit, or single thing, of the unknown quantity, and then find the value of the whole number required.

The pupil has already had some exercises in analysis in preceding articles. For instance, questions 7 to 15 in Art. **34**, 31 to 46 in Art. **44**, &c., are examples of analysis. But as this mode of solving questions is so commonly practised by business men, and is so interesting to the pupil, and, at the same time, so well fitted to discipline his mind, and to prepare him better to understand the more difficult parts of arithmetic, the subject of analysis should have a prominent place in a text-book on Arithmetic.

1. If 4 lb. of veal cost 28 cts., what will 8 lb. cost?

Solution. Since 4 lb. cost 28 cts., 1 lb. will cost $\frac{1}{4}$ of 28 cts., or 7 cts. If 1 lb. costs 7 cts., 8 lb. will cost 8 times as much; 8 times 7 cts. = 56 cts.; therefore, if 4 lb. of veal cost 28 cts., 8 lb. will cost 56 cts.

2. If 5 lb. of lamb cost 20 cts., what will 8 lb. cost? $10\frac{1}{2}$ lb.? $12\frac{1}{4}$ lb.?

3. If 7 yd. of cloth cost $35, what will 8 yd. cost? $4\frac{1}{2}$ yd.? $3\frac{1}{4}$ yd.? $10\frac{1}{2}$ yd.?

4. If 6 lb. of coffee cost 66 cts., what will 3 lb. cost? $3\frac{1}{2}$ lb.? $5\frac{1}{2}$ lb.? $8\frac{1}{4}$ lb.?

If 1 lb. cost 11 cts., $3\frac{1}{2}$ lb. will cost $3\frac{1}{2}$ times as much. 3 times 11 are 33; $\frac{1}{2}$ of 11 is $5\frac{1}{2}$, which added to 33 makes $38\frac{1}{2}$; therefore, if 1 lb. costs 11 cts., $3\frac{1}{2}$ lb. will cost $38\frac{1}{2}$ cts.

5. If 7 horses eat 36 bushels of oats in a week, how many bushels will 3 horses eat? 5 horses? 6 horses? 10 horses?

6. If a man travels 26 miles in 6 hours, how far will he travel in 2 hours? 5 hours? 8 hours?

7. Bought 5 yards of ribbon for 42 cents; what are 3 yards worth? 8 yards? 10 yards? 12 yards?

8. If a man earns 50 shillings in 6 days, how much will he earn in 5 days? In 8 days? In 7 days? In 12 days?

9. If 3 men eat a bushel of potatoes in 20 days, how long will they last 5 men? 8 men? 10 men? Since the bushel will last 3 men 20 days, it will last 1 man 3 times as long, or 60 days. If it last 1 man 60 days, it will last 5 men $\frac{1}{5}$ as long, or 12 days. Therefore, &c.

10. If 8 men build a wall in 7 days, how long will it take 3 men to build it? 5 men? 7 men? 10 men? 12 men?

11. If 7 men do a piece of work in 45 days, in what time will 15 men do it?

12. If \$100 worth of provisions will last 5 men 75 days, how long will they last 12 men? 18 men? 37 men?

13. If a railroad car runs 140 miles in 7 hours, how far will it run in $8\frac{1}{3}$ hours? In $7\frac{1}{2}$ hours? In $25\frac{1}{3}$ hours?

14. If 35 barrels of apples cost \$48, how much will 3 barrels cost? 8 barrels? 27 barrels?

15. A drover bought 250 sheep for \$375; how much would 75 sheep cost?

16. A merchant bought 25 barrels of flour for \$162.50; what are 8 barrels worth?

17. If $15\frac{5}{6}$ tons of coal cost \$95, what is the cost of 1 ton? $6\frac{1}{2}$ tons?

$15\frac{5}{6} = \frac{95}{6}$. If $\frac{95}{6}$ of a ton cost \$95, $\frac{1}{6}$ of a ton will cost $\frac{1}{95}$ of \$95, or \$1.

18. Bought $3\frac{3}{5}$ cords of wood for \$27; what was it per cord?

19. Gave \$43 for $5\frac{3}{8}$ yards of broadcloth; what was it per yard?

20. How many times will 95 contain $15\frac{5}{6}$?

21. How many times will 27 contain $3\frac{3}{5}$?

22. How many times will 43 contain $5\frac{3}{8}$?

23. A farmer sold $6\frac{3}{4}$ bushels of potatoes for \$3.24; how much did he get per bushel? What would $25\frac{3}{4}$ bushels come to at that price?

24. How many times will 324 contain $6\frac{3}{4}$? $7\frac{1}{7}$?

25. The farmer in the last example paid \$1.77 for a keg of molasses, containing $7\frac{3}{8}$ gallons; what was the price per gallon?

26. How many times will 354 contain $7\frac{3}{8}$? $13\frac{1}{8}$?

27. If 25 gallons 2 quarts of wine cost \$33.15, what was the price per gallon? What would $8\frac{1}{2}$ gallons come to at that price?

28. Paid \$2.70 for $3\frac{3}{4}$ bushels of Indian meal; what must I pay for $5\frac{3}{8}$ bushels at that rate?

29. How many cows at \$25.50 apiece can be bought for \$433.50?

TO THE TEACHER. If the pupil is a beginner on the slate, it may be well for him to review all the preceding articles from the beginning, performing all the examples, whether marked I. or II. He will in this way acquire a facility in performing the elementary processes which he cannot so readily acquire by going forward.

SECTION VI.—COMPOUND NUMBERS.

TABLES OF MONEY, WEIGHTS AND MEASURES.

48. *Federal Money.* (Art. 15.)

Federal Money is the national currency of the United States. Its unit is the dollar. The coins are of gold, silver, and copper. The Gold coins are the double eagle,* eagle, half-eagle, quarter-eagle, and the dollar.* The Silver coins are the dollar, half-dollar, quarter-dollar, dime, half-dime. The Copper coins are the cent and half cent.

NOTE 1. If any quantity of gold be divided into 24 equal parts, each part is called a carat. The purity of gold is indicated in this manner. If the metal is pure, it is said to be 24 carats fine. If there are 22 parts of the pure metal mixed with 2 parts of silver or other baser metal, it is said to be 22 carats fine. Gold 18 carats fine has 18 parts pure gold and 6 parts *alloy*, that is, 6 parts of a baser metal.

2. The standard of the gold coin in the United States is 22 carats of gold, 1 of silver, and 1 of copper. The standard for silver coins is 1489 parts of pure silver to 179 of pure copper. The copper coins are pure copper.

3. The eagle weighs 270 grains; the dollar, 416 grains; the cent, 11 pennyweights.

1 eagle (ea.)	= 10 dollars,	($)	1 m. = $\frac{1}{10}$ of a ct.
1 dollar	= 10 dimes,	(di.)	1 ct. = $\frac{1}{10}$ of a di.
1 dime	= 10 cents,	(ct.)	1 di. = $\frac{1}{10}$ of a $.
1 cent	= 10 mills,	(m.)	1 $ = $\frac{1}{10}$ of an ea.

ea.		*$*		*di.*		*ct.*		*mi.*
1	=	10	=	100	=	1000	=	10000
		1	=	10	=	100	=	1000
				1	=	10	=	100
						1	=	10

49. *English Money.*

English Money is the national currency of Great Britain. It was the currency of the United States till the establishment of Federal Money in 1786, and is partially used here at present. Its unit is the pound, (£.)

* The double eagle and the gold dollar were introduced by act of Congress, February, 1849.

1 pound (£)	= 20 shillings,	(s.)	1qr. = $\frac{1}{4}$ of a d.
1 shilling	= 12 pence,	(d.)	1d. = $\frac{1}{12}$ of a s.
1 penny	= 4 farthings,	(qr.*)	1s. = $\frac{1}{20}$ of a £.

$$\begin{array}{cccc} \pounds & s. & d. & qr. \\ 1 = & 20 = & 240 = & 960 \\ & 1 = & 12 = & 48 \\ & & 1 = & 4 \end{array}$$

NOTE 1. The gold coins of Great Britain are the sovereign = £1, and the half-sovereign = 10s. The silver coins are the crown = 5s., the half-crown = 2s. 6d., the shilling, and the sixpence.

2. 1 guinea = 21 shillings; its value is \$5.07$\frac{1}{2}$. The exchange value of the pound is \$4.44$\frac{4}{9}$. Its legal value at the mint is \$4.866. At the custom-house it is received in payment of duties at \$4.84. See Art. **122**.

3. The standard gold coin of Great Britain is 22 parts of pure gold and 2 parts of pure copper. The silver coin is 224 pure silver and 18 of copper.

50. *French Money.*

100 centimes = 1 franc = \$0.18$\frac{3}{5}$.

51. *Troy Weight.*

Troy Weight is used in weighing gold, silver, platina, jewels, and some liquids. Its unit is the pound, (lb.;) the standard of which in the U. S. is the weight of 22.794377 cubic inches of distilled water weighed in air. *The pound Troy is the standard of weight in the United States.*

1 pound	= 12 ounces,	(oz.)	1 gr. = $\frac{1}{24}$ of a dwt.
1 ounce	= 20 pennyweights,	(dwt.)	1 dwt. = $\frac{1}{20}$ of an oz.
1 pennyweight	= 24 grains,	(gr.)	1 oz. = $\frac{1}{12}$ of a lb.

$$\begin{array}{cccc} lb. & oz. & dwt. & gr. \\ 1 = & 12 = & 240 = & 5760 \\ & 1 = & 20 = & 480 \\ & & 1 = & 24 \end{array}$$

52. *Apothecaries' Weight.*

Apothecaries' Weight is used in mixing medicines. Drugs are, however, bought and sold by Avoirdupois Weight. The pound, ounce, and grain, are the same as in Troy Weight.

* 1qr. is often written $\frac{1}{4}$d.; 2qr. is written $\frac{1}{2}$d.; 3qr. is $\frac{3}{4}$d.; 8d. 3qr. is 8$\frac{3}{4}$d., &c.

1 pound = 12 ounces, (℥.)	1 gr. = $\frac{1}{20}$ of a ℈.
1 ounce = 8 drams, (ʒ.)	1 ℈. = $\frac{1}{3}$ of a ʒ.
1 dram = 3 scruples, (℈.)	1 ʒ. = $\frac{1}{8}$ of an ℥.
1 scruple = 20 grains, (gr.)	1 ℥. = $\frac{1}{12}$ of a lb.

53. *Avoirdupois Weight.*

Avoirdupoïs Weight is used in weighing most kinds of merchandise, and all the metals, except those mentioned in Troy Weight.

1 ton (T.) = 20 hund. wt., (cwt.)	1 dr. = $\frac{1}{16}$ of an oz.
1 hund. wt. = 4 quarters, (qr.)	1 oz. = $\frac{1}{16}$ of a lb.
1 quarter = 28 pounds, (lb.)	1 lb. = $\frac{1}{28}$ of a qr.
1 pound = 16 ounces, (oz.)	1 qr. = $\frac{1}{4}$ of a cwt.
1 ounce = 16 drams, (dr.)	1 cwt. = $\frac{1}{20}$ of a T.

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
1 =	20 =	80 =	2240 =	35840 =	573440
	1 =	4 =	112 =	1792 =	28672
		1 =	28 =	448 =	7168
			1 =	16 =	256
				1 =	16

NOTE 1. The Avoirdupois pound of the United States is determined from the Troy pound. 1 lb. Troy = 5760 grains. 1 lb. Avoirdupois = 7000 grains Troy. 1 oz. Avoirdupois contains $437\frac{1}{2}$ grains. 1 dr. = $27\frac{11}{32}$ grains. The lb. Avoirdupois is $\frac{175}{144}$ of the lb. Troy.

2. By a law of the United States the hundred weight is fixed at 100 lb. The ton of 2240 lb., the cwt. of 112 lb., and the qr. of 28 lb., are seldom used, either among merchants or at the custom-house, except in weighing hemp and dye-woods on first delivery, and coal at the mines and on first delivery; the ton of 2000 lb. and the hundred weight of 100 lb. being almost universally used. In invoices of goods from England which are sold by weight, the cwt. is estimated at 112 lb. Fish is bought by the quintal of 112 lb.

3. *Gross weight* is the weight of goods with the bags, casks, or boxes, which contain them. *Net weight* is the weight of the goods only.

54. *Long Measure.*

Long Measure is frequently called *linear* or *lineal* measure. It is used in measuring distances, as lengths, breadths, depths and heights.

1 degree (deg. or °)	$=60$	Geographical miles,	(G. m.)
1 degree	$=69\frac{1}{2}$	Statute miles,	(S. m.)
1 league	$=3$	miles,	(m.)
1 mile	$=8$	furlongs,	(fur.)
1 furlong	$=40$	rods,	(rd.)
1 rod	$=5.5=5\frac{1}{2}$	yards, (or $16\frac{1}{2}$ feet,)	(yd.)
1 yard	$=3$	feet,	(ft.)
1 foot	$=12$	inches,	(in.)

1 in. $=\frac{1}{12}$ of a ft.	1 fur. $=\frac{1}{8}$ of a m.
1 ft. $=\frac{1}{3}$ of a yd.	1 m. $=\frac{1}{3}$ of a league.
1 yd. $=\frac{2}{11}$ of a rd.	1 S. m. $=\frac{2}{139}$ of a deg.
1 rd. $=\frac{1}{40}$ of a fur.	1 G. m. $=\frac{1}{60}$ of a deg.

m.	*fur.*	*rd.*	*yd.*	*ft.*	*in.*
$1=$	$8=$	$320=$	$1760=$	$5280=$	63360
	$1=$	$40=$	$220=$	$660=$	7920
		$1=$	$5\frac{1}{2}=$	$16\frac{1}{2}=$	198
			$1=$	$3=$	36
				$1=$	12

55. *Cloth Measure.*

Cloth Measure is used in measuring cloth and other goods sold by the yard. The yard is of the same length as the yard in Long Measure.

1 English ell (E. ell.)	$=5$	quarters,	(qr.)
1 French ell (Fr. ell.)	$=6$	quarters,	—
1 Flemish ell (Fl. ell.)	$=3$	quarters,	—
1 yard (yd.)	$=4$	quarters,	—
1 quarter	$=4$	nails,	(na.)
1 nail	$=2.25=2\frac{1}{4}$	inches,	(in.)

1 in. $=\frac{4}{9}$ of a na.	1 qr. $=\frac{1}{3}$ of a Fl. ell.
1 na. $=\frac{1}{4}$ of a qr.	1 qr. $=\frac{1}{6}$ of a Fr. ell.
1 qr. $=\frac{1}{4}$ of a yd.	1 qr. $=\frac{1}{5}$ of an E. ell.

E. Fr.	*E. E.*	*yd.*	*E. Fl.*	*qr.*	*na.*	*in.*
$1=$	$1\frac{1}{5}=$	$1\frac{1}{2}=$	$2=$	$6=$	$24=$	54
	$1=$	$1\frac{1}{4}=$	$1\frac{2}{3}=$	$5=$	$20=$	45
		$1=$	$1\frac{1}{3}=$	$4=$	$16=$	36
			$1=$	$3=$	$12=$	27
				$1=$	$4=$	9
					$1=$	$2\frac{1}{4}$

56. *Square Measure.*

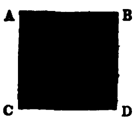
Square Measure is used in measuring surfaces, such as land, flooring, roofing, painting, and everything that has length and breadth only.

1 square mile (sq. m.)	=	640 acres,	(A.)
1 acre	=	4 roods,	(R.)
1 rood	=	40 square rods,	(sq. rd.)
1 square rod	=	30.25 square yards,	(sq. yd.)
1 square rod	=	272.25 square feet,	(sq. ft.)
1 square yard	=	9 square feet,	(sq. ft.)
1 square foot	=	144 square inches,	(sq. in.)

1 sq. in. = $\frac{1}{144}$ of a sq. ft.	1 sq. rd. = $\frac{1}{40}$ of a R.
1 sq. ft. = $\frac{1}{9}$ of a sq. yd.	1 R. = $\frac{1}{4}$ of an A.
1 sq. ft. = $\frac{4}{1089}$ of a sq. rd.	1 A. = $\frac{1}{640}$ of a sq. m.
1 sq. yd. = $\frac{4}{121}$ of a sq. rd.	

sq. m.	*A.*	*R.*	*sq. rd.*	*sq. yd.*	*sq. ft.*	*sq. in.*
1 =	640 =	2560 =	102400 =	3097600 =	27878400 =	4014489600
	1 =	4 =	160 =	4840 =	43560 =	6272640
		1 =	40 =	1210 =	10890 =	1568160
			1 =	$30\frac{1}{4}$ =	$272\frac{1}{4}$ =	39204
				1 =	9 =	1296
					1 =	144

In measuring land, surveyors use a chain which is 4 rods long, and which is divided into 100 links. 25 links make 1 rod, and $7\frac{92}{100}$ inches make 1 link. The term *pole* is often used for *rod*.



A *square* is a surface which has its four sides equal, and its angles right angles. (See page 000, def. 8.)

A *rectangle* is a surface which has its opposite sides equal, and its angles right angles.

A *square inch* is a square, each side of which is one inch long.

A *square foot* is a square, each side of which is one foot long.

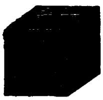
To find the number of square inches, feet, &c., in a square or rectangular surface, *Multiply the length by the breadth*, (**70**, **153**, &c.)

57. *Cubic Measure.*

Cubic Measure is used in measuring solid bodies, and capacities, or anything that has length, breadth, and thickness; as timber, stone, boxes of goods, &c.

1 cord of wood (C.)	= 128 cubic feet,	(cu. ft.)
1 cord foot (C. ft.)	= 16 cubic feet,	——
1 ton of timber (t.)	= 40 cubic feet,	——
1 cubic yard (cu. yd.)	= 27 cubic feet,	——
1 cubic foot	= 1728 cubic inches,	(cu. in.)

1 cu. in. = $\frac{1}{1728}$ of a cu. ft.	1 cu. ft. = $\frac{1}{16}$ of a C. ft.
1 cu. ft. = $\frac{1}{27}$ of a cu. yd.	1 cu. ft. = $\frac{1}{128}$ of a C.
1 cu. ft. = $\frac{1}{40}$ of a t. of timber.	

A 

A *cube* is a solid having 6 equal sides which are squares.

A *cubic inch* is a cube, each of whose sides is a square inch.

A *cubic foot* is a cube, each of whose sides is a square foot.

To find the number of cubic inches, feet, &c., in a solid, whose surfaces are all squares or rectangles, *Multiply the length by the breadth, and this product by the thickness.* (**70**, **168** and **170**.)

58. *Dry Measure.*

Dry Measure is used in measuring grain, fruit, salt, coal, and similar goods.

1 chaldron (ch.)	= 36 bushels,	(bu.)	1 pt. = $\frac{1}{2}$ of a qt.
1 Eng. quarter (qr.)	= 8 bushels,	—	1 qt. = $\frac{1}{8}$ of a pk.
1 bushel	= 4 pecks,	(pk.)	1 pk. = $\frac{1}{4}$ of a bu.
1 peck	= 8 quarts,	(qt.)	1 bu. = $\frac{1}{8}$ of a qr.
1 quart	= 2 pints,	(pt.)	1 bu. = $\frac{1}{36}$ of a ch.

bu.	*pk.*	*qt.*	*pt.*
1 =	4 =	32 =	64
	1 =	8 =	16
		1 =	2

The bushel contains 2150.4 cubic inches. The half-peck contains 268.8 cubic inches.

59. *Liquid Measure.*

Liquid Measure is used in measuring all kinds of liquids.

1 gallon	= 4 quarts,	(qt.)	1 gi. = $\frac{1}{4}$ of a pt.
1 quart	= 2 pints,	(pt.)	1 pt. = $\frac{1}{2}$ of a qt.
1 pint	= 4 gills,	(gi.)	1 qt. = $\frac{1}{4}$ of a gal.

gal.	*qt.*	*pt.*	*gi.*
1 =	4 =	8 =	32
	1 =	2 =	8
		1 =	4

NOTE 1. The *common* gallon is 231 cubic inches. The "Imperial gallon" of Great Britain contains 277.274 cubic inches. A gallon of milk, and malt liquors, is 282 cubic inches. In many places, milk is measured by the *common* gallon.

2. The hogshead is not a measure of a definite capacity. Casks of different capacities, from 60 or 70 gallons and upwards, are indiscriminately called hogsheads. They usually contain from 120 to 150 gallons. Many are larger. *The unit is the gallon.*

3. A barrel of cider, or of fish oil, such as curriers use, is understood to contain 31½ gallons. A barrel of molasses, from 28 to 31 gallons.

60. *Measure of Time.*

1 year (yr.)	= 12	months,	(mo.)
1 solar yr. = 365.25	= 365¼	days,	(da.)
1 month	= 4	weeks,	(wk.)
1 week	= 7	days,	(da.)
1 day	= 24	hours,	(h.)
1 hour	= 60	minutes,	(min.)
1 minute	= 60	seconds,	(sec.)

1 sec. = $\frac{1}{60}$ of a min.
1 min. = $\frac{1}{60}$ of an hour.
1 h. = $\frac{1}{24}$ of a day.
1 da. = $\frac{1}{7}$ of a week.
1 w. = $\frac{1}{4}$ of a month.
1 mo. = $\frac{1}{12}$ of a year.
1 da. = $\frac{4}{1461}$ of a year.

yr.	*mo.*	*wk.*	*da.*	*hr.*	*min.*	*sec.*
1 =	=	=	365¼ =	8766 =	525960 =	31557600
	1 =	4 =	28 =	672 =	40320 =	2419200
		1 =	7 =	168 =	10080 =	604800
			1 =	24 =	1440 =	86400
				1 =	60 =	3600
					1 =	60

The exact year is 365 da. 5 h. 48 min. 57 sec. This is the true solar year, which is the time measured from the sun's leaving either of the equinoxes or solstices to its return to the same again. *A siderial* or *periodical* year, is the time in which the earth makes one complete revolution round the sun, and is 365 da. 6h. 9m. 14½ sec. The civil year consists of 365 days for three successive years, but every fourth year contains 366 days. When any year can be divided by 4 without a remainder, it is *leap year*, and contains 366 days; except the centennial years, which, although divisible by 4, are not leap years, unless the hundreds are also divisible by 4; thus, the years 1800 and 1900 are not leap years, but the year 2000 will be leap year.

The year is divided into 12 months, as follows:

January (Jan.)	has 31 da.	July	has 31 da.
February (Feb.)	has 28 da.	August (Aug.)	has 31 da.
March (Mar.)	has 31 da.	September (Sept.)	has 30 da.
April (Apr.)	has 30 da.	October (Oct.)	has 31 da.
May	has 31 da.	November (Nov.)	has 30 da.
June	has 30 da.	December (Dec.)	has 31 da.

In leap year February has 29 days.

The number of days in each month may be learned by committing the following lines:

"Thirty days hath September,
April, Jûne, and November;
All the rest have thirty-one,
Except the second month alone,
To which we twenty-eight assign,
Till Leap Year gives it twenty-nine."

61. *Circular and Astronomical Measure.*

Circular Measure is used in measuring circles, latitude and longitude, and in computing the revolutions of the planets around their primaries; as the earth around the sun, the moon around the earth.

1 circumference of a circle	= 360 degrees, (°)		1″ is $\frac{1}{60}$ of a ′.
1 degree	= 60 minutes, (′)		1′ is $\frac{1}{60}$ of a °.
1 minute	= 60 seconds, (″)		1° is $\frac{1}{360}$ of a circ.

circ.	°	′	″
1 =	360 =	21600 =	1296000
	1 =	60 =	3600
		1 =	60

A *quadrant* is 90 degrees. A *sign* of the zodiac is 30 degrees. 12 signs make the circle of the zodiac.

62. *Miscellaneous Table.*

12 single things	= 1 dozen.	
12 dozen	= 1 gross.	
12 gross	= 1 great gross.	
24 sheets of paper	= 1 quire.	
20 quires	= 1 ream.	
196 lb.	= 1 bl. of flour.	
200 lb.	= 1 bl. of pork or beef.	
70 lb.	= 1 bushel of salt.	
60 lb.	= 1 " "	wheat.
56 lb.	= 1 " "	Indian corn or rye.
46 lb.	= 1 " "	barley.
30 lb.	= 1 " "	oats.

63. *Of Books.*

A sheet folded in two leaves is called *folio;* if folded in 4 leaves, *quarto*, or 4to.; if in 8 leaves, *octavo*, or 8vo.; if in 12 leaves, *duodecimo*, or 12mo.; if in 18 leaves, 18mo., &c.

QUESTIONS. What is Federal money? What is its unit? Which of the coins of the U. S. are gold? Which are silver? Which are copper? What is meant by a carat? *Ans.* By a carat is meant $\frac{1}{24}$ part of any quantity of gold. How is the purity of gold indicated? Give an example. What is the standard of the gold coin in the U. S.? Of the silver coin? Of the copper coin? What is the weight of the eagle? Of the dollar? Of the cent? Repeat the table of Federal Money.

What is English money? Was it ever used in this country? Repeat the table. What are the gold coins of Great Britain? The silver coins? What is the value of the English guinea? Of the English pound? What is the fineness of the gold coin of Great Britain? Of the silver coin? Repeat the table of French money.

For what is Troy weight used? What is its unit of weight? What is the standard of weight of the Troy lb. in the U. S.? Of what is it the standard? Repeat the table.

For what is Apothecaries' weight used? How are drugs bought and sold? What is the weight of the Troy lb. and oz.? Repeat the table.

For what is Avoirdupois weight used? Repeat the table. How is the Avoirdupois pound determined? How many grains in 1 lb. Troy? In 1 lb. Avoirdupois? In 1 oz. Avoirdupois? In 1 dr.? What is the weight of the lb. Avoirdupois compared with the lb. Troy? What is the weight of the hundred weight by the laws of the U. S.? In what cases is the ton of 2240 lb., the cwt. of 112 lb., and the qr. of

28 lb. still used? What are used in other cases? How is fish purchased? What is gross weight? Net weight?

What is long measure sometimes called? For what is it used? Repeat the table.

For what is cloth measure used? Repeat the table.

For what is square measure used? Repeat the table. What is used in measuring land? What is a square? Draw one on the slate or black-board. What is a rectangle? Draw one. What is a square inch? Draw one. A square foot? What is the rule for finding the number of square inches, feet, &c., in a square or rectangular surface?

For what is cubic measure used? Repeat the table. What is a cube? A cubic inch? A cubic foot? How may the number of cubic inches, feet, &c., in a solid whose surfaces are all squares or rectangles, be found?

For what is dry measure used? Repeat the table. How many cubic inches in a bushel? In a half peck?

For what is liquid measure used? Repeat the table. How many cubic inches in the common gallon? In the Imperial gallon of Great Britain? In a gallon of milk or malt liquors? What is said of the hogshead? Of the barrel?

Repeat the table of time. What is the exact solar year? The civil year? When does leap-year occur? How many days are there in each month?

Repeat the table for circular measure. For what is it used? What is a quadrant? A sign? Repeat the miscellaneous table. The table for books.

64. ADDITION OF COMPOUND NUMBERS.

Simple numbers are those which express things of the *same* kind. *Compound* numbers are those which express things of *different* kinds, or denominations; thus, 5 apples, 4 hats, are simple numbers; 5 cwt. 2 qr., 3 miles 5 furlongs 7 rods, are compound numbers.

Compound Addition is the addition of several numbers of different denominations.

What is the sum of £5 14s. 7d. 3qr., £13 16s. 8d. 1qr., and £17 9d. 1qr.?

£	*s.*	*d.*	*qr.*
5	14	7	3
13	16	8	1
17	0	9	1
36	12	1	1

Having placed farthings under farthings, &c., we add the column of farthings, and find the sum 5, which is equal to 1d. 1qr. over. We write the 1qr. in its place, and add the 1d. with the next column, which amounts to 25d., or 2s. 1d. Writing the 1d. in its place, we add the 2s. with the column of shillings, the sum of which is 32s., or £1 12s. We then add the £1 with the column of pounds, the whole amount of which is 36. Hence we derive the

RULE FOR COMPOUND ADDITION. *Write the numbers of the same denomination in a column under each other. Beginning with the lowest denomination, add each column separately, and divide the sum of each by as many of that denomination as make one of the next greater. Write the remainder underneath, and carry the quotient to the next higher column.*

EXAMPLES.

1. Add £7 18s. 5d. 3qr.; £6 12s. 3d. 1qr.; £25 15s. 3qr.
2. A merchant sold 2 T. 15 cwt. 3 qr. 16 lb. of hemp to one man, 5 T. 17 cwt. 18 lb. to another, and 10 T. 3 qr. to a third. How much did he sell?
3. Add 3 lb. 10 oz. 5 dwt. 17 gr.; 15 lb. 14 dwt. 5 gr.; 3 lb. 8 oz. 16 gr.; 4 lb. 3 dwt.; and 16 lb. 8 oz. 22 gr.
4. What is the sum of 6 m. 3 fur. 32 rd. 5 yd. 1 ft. 3 in.; 18 m. 7 fur. 27 rd. 2 ft.; 3 fur. 1 ft. 5 in.; and 16 m. 37 rd. 4 ft.?
5. Add 4 yd. 3 qr. 2 na.; 16 yd. 3 qr. 3 na.; 12 yd. 2 na.; and 3 qr. 3 na.
6. Add 616 A. 2 R. 37 sq. rd. 20 sq. yd. 17 sq. in.; 3 sq. m. 15 A. 1 R. 30 sq. rd. 4 sq. ft.; 17 A. 39 sq. rd. 95 sq. in.; and 25 sq. yd. 4 sq. ft. 142 sq. in.
7. Add 41 cords 7 C. ft.; 35 cords 3 C. ft.; 19 cords 6 C. ft.; 27 cords 5 C. ft.
8. Add 34 cu. yd. 18 cu. ft. 1016 cu. in.; 100 cu. yd. 25 cu. ft. 1654 cu. in.; and 5 cu. yd. 21 cu. ft.
9. Add 7 bu. 3 pk. 5 qt.; 8 bu. 7 qt.; 53 bu. 2 pk.; and 16 bu. 7 qt.
10. Add 456 gal. 3 qt. 2 gi.; 1120 gal. 1 pt.; 10 gal. 3 gi.; and 15 gal. 1 qt. 1 pt.
11. Add 4 y. 41 d. 7 h.; 15 y. 68 d. 15 h. 38 sec.; 18 y. 258 d. 23 h. 28 min.; 6 y. 360 d. 9 h.; and 1 yr. 317 d. 19 h. 31 min. 17 sec.
12. Add 346° 15′ 48″; 58° 3′ 45″; 35° 58″; and 76° 15′ 45″.

65. SUBTRACTION OF COMPOUND NUMBERS.

1. From 56 lb. 3 oz. 15 dwt. 18 gr. subtract 47 lb. 6 oz. 17 dwt. 13 gr.

lb.	*oz.*	*dwt.*	*gr.*
56	3	15	18
47	6	17	13
8	8	18	5
56	3	15	18

After writing the subtrahend under the minuend, placing numbers of the same denomination under each other, begin with the lowest, and subtract 13 gr. from 18 gr., and write the remainder, 5 gr., under the column of grains. As you cannot take 17 dwt. from 15 dwt., add 1 oz., or 20 dwt., to 15 dwt., making 35 dwt., from which subtract 17 dwt., and write the remainder, 18 dwt. Then, as you added 1 oz. to the minuend, you must also add the same to the subtrahend, as in simple subtraction. As you cannot subtract 7 oz. from 3 oz., add 1 lb., or 12 oz., to the 3 oz., making 15 oz.; 7 oz. from 15 oz. leaves 8 oz. Add 1 lb. to the subtrahend also, and subtract 48 lb. from 56 lb., and write the remainder, 8 lb. Hence the following

RULE FOR COMPOUND SUBTRACTION. *Write the numbers as in compound addition, placing the less under the greater. Beginning at the lowest denomination, subtract each number in the lower line from the number above it, and write the remainder underneath. When any number in the upper line is less than the number below it, add to it one of the next higher denomination. Then subtract, and add one to the next higher denomination in the lower line.*

EXAMPLES.

2. From £15 18s. 6d. 2qr. subtract £8 15s. 7d. 1qr.
3. From 11 T. 15 cwt. 3 qr. 12 lb. 11 oz. 15 dr. subtract 5 T. 3 qr. 15 lb. 14 oz. 6 dr.
4. From 10 m. 3 fur. 6 rd. 5 yd. 2 ft. subtract 5 fur. 10 rd. 5 yd. 9 in.
5. From 1056 gal. 3 qt. 1 pt. subtract 623 gal. 3 qt. 1 pt. 3 gi.
6. Subtract 8 bu. 3 pk. 5 qt. from 15 bu. 3 pk. 3 qt. 1 pt.
7. Subtract 3 fur. 9 rd. 5 yd. 2 ft. from 5 fur. 10 rd. 0 yd. 1 ft.

NOTE. In this example *two* rods instead of one must be added to the yards.

8. Subtract 3 A. 2 R. 15 sq. rd. 30 sq. yd. 5 sq. ft. 84 sq. in. from 5 A. 1 R. 100 sq. in.
9. From 16 d. 17 h. 18 m. 19 sec. subtract 2 d. 23 h. 25 m. 35 sec.
10. Subtract 1840 yr. 6 mo. 15 d. from 1848 yr. 3 mo. 23 d.
11. From 540 ch. 15 bu. 2 pk. 1 qt. 0 pt. subtract 158 ch. 20 bu. 3 pk. 3 qt. 1 pt.
12. From 4 S. 15° 45′ 15″ subtract 8 S. 17° 25′ 38″.

NOTE. 12 signs may be added to the minuend if it is less than the subtrahend.

66. MULTIPLICATION OF COMPOUND NUMBERS.

1. How much will 7 watches cost, at £6 5s. 8d. apiece?

Beginning with the lowest denomination, 7 times 8d. are 56d. = 4s. 8d. Write the 8d. and carry the 4s. to be added to the next product. 7 times 5s. are 35s., and 4s. are 39s. = £1 19s. Write the 19s. and carry the £1 to the next product. 7 times £6 are £42, and £1 are £43. Hence the following

£	s.	d.
6	5	8
		7
43	19	8

RULE FOR COMPOUND MULTIPLICATION. *Beginning with the lowest denomination, multiply each denomination in order, and divide and carry as in compound addition.*

NOTE. *If the multiplier is a composite number, multiply first by one factor, and then by the other.* (**28.**)

EXAMPLES.

2. Multiply 31 bu. 3 pk. 5 qt. by 8. 6 bu. 3 pk. 0 qt. 1 pt. by 5.
3. Multiply 208 gal. 3 qt. 0 pt. 3 gi. by 3; by 9; by 18.
4. Multiply 56 lb. 8 oz. 18 dwt. 17 gr. by 4; by 16; by 24.
5. Multiply 2 yr. 175 d. 6 h. 19 min. 25 sec. by 3; by 15; by 21. Call 365 days = 1 year.
6. Multiply 45 m. 7 fur. 29 rd. 3 yd. by 8; by 12; by 16.

67. DIVISION OF COMPOUND NUMBERS.

1. Divide 14 bu. 3 pk. 6 qt. of beans equally among 5 men.

	bu.	pk.	qt.	pt.
5)	14	3	6	0
	2	3	7	$1\frac{1}{5}$

Dividing 14 bu. by 5, the quotient is 2 bu., and 4 bu. remaining. We set down the 2 bu., and to the remainder, 4 bu., or 16 pecks, we add the 3 pecks in the dividend. Dividing the amount, 19 pecks, by 5, the quotient is 3 pecks, and 4 pecks over. Write down the 3 pecks, and to the remainder, 4 pecks, or 32 quarts, we add the 6 quarts in the dividend, making 38 quarts. Dividing this by 5, the quotient is 7 quarts, and 3 quarts remaining. Reducing the remainder to pints, and dividing by 5, we have $1\frac{1}{5}$ pt. for the quotient. From this example we derive the following

RULE FOR COMPOUND DIVISION. *Beginning with the highest, divide each denomination separately. When there is a remainder, reduce it to the next lower denomination, to which add the number (if any) of that denomination in the dividend, and divide the sum as before. If the divisor is more than 12, perform the work by long division, and write the quotient to the right of the dividend.*

EXAMPLES BY LONG DIVISION.

1. Divide £16 17s. 3d. by 13.

```
      £   s.  d.   £  s.  d.  qr.
13 ) 16  17   3 (  1  5  11  1 3/13.
     13
     --
     £3
     20
     --
13 ) 77s. ( 5s.
     65
     --
     12s.
     12
     --
13 ) 147d. ( 11d.
     143
     ---
       4d.
       4
       --
13 )  16 qr. ( 1 3/13 qr.
      13
      --
      3/13
```

2. Divide 28 rd. 5 yd. 2 ft. by 21.

```
     rd.  yd. ft. rd. yd. ft. in.
21 ) 28   5   2 (  1   2   0  3 15/21.
     21
     --
      7 rd.
      5½
     ---
21 ) 43½ yd. ( 2 yd.
     42
     --
      1½ yd.
      3
     --
21 )  6½ ft. ( 0 ft.
     12
     ---
21 ) 78 in. ( 3 15/21 in.
     63
     --
     15/21
```

3. Divide £16 17s. 3d. 1qr. by 5; by 7; by 13.
4. Divide 1105 gal. 3 qt. 1 pt. 3 gi. by 3; by 16.
5. Divide 15 m. 4 fur. 34 rd. 4 yd. 2 ft. 8 in. by 4; by 8; by 32.
6. Divide 118 yd. 3 qr. 2 na. by 35.
7. Divide 54 A. 2 R. 36 sq. rd. 11 sq. yd. 2 sq. ft. 8 sq. in. by 2; by 4.
8. Divide 125 lb. 11 oz. 15 dwt. 18 gr. by 8; by 7; by 15.

68. MISCELLANEOUS EXAMPLES IN COMPOUND NUMBERS.

1. If 8 yards of broadcloth cost £10 4s. 6d., what will 1 yard cost? What will 5 yards cost?

2. A merchant bought 5 cwt. 3 qr. 7 lb. of cheese, at one time; 8 cwt. 3 qr. 25 lb., at another; and 15 cwt. 2 qr. 17 lb., at another. How much did he buy?

3. How many pounds were there, reckoning the qr. at 28 pounds, and how much did it cost at 6½ cts. per lb.?

4. If the merchant sells 18 cwt. 3 qr. 24 lb., how much has he left?

5. A farmer raised 185 bu. 3 pk. of oats, 237 bu. 2 pk. of corn, 115 bu. 1 pk. of wheat, and 145 bu. of rye. How many bushels were there in all? How many pecks?

6. A grocer bought 5 firkins of butter, weighing as follows: No. 1, 47 lb. 14 oz.; No. 2, 58 lb. 13 oz.; No. 3, 38 lb. 10 oz.; No. 4, 73 lb. 8 oz.; No. 5, 46 lb. 3 oz. How much was there in all? What did he pay for it at $0.175 a lb.?

7. If the grocer should divide the butter into 10 equal portions, how much would there be in each?

8. If he should sell it at 20 cents a lb., how much would he gain?

9. A merchant bought in England 15 yds. of broadcloth, at £1 17s. 9d. per yd. How much did it come to? If he should sell it for £1 19s. 8d. more than he gave for it, what would be the price per yard?

10. A locomotive has gone 78 m. 3 fur. 15 rd. 4 yd. 2 ft. in 5 hours. How far has it run per hour?

11. How much further would another locomotive go in 7 hours, if its rate of travelling was 2 m. 2 fur. 25 rd. more per hour?

12. A merchant bought 560 bu. 3 pk. of corn, which he wishes to put in equal parcels in 6 different bins. How much must be put in each?

13. A farmer raised 35 T. 17 cwt. 3 qr. of hay; he has sold 5 loads, each weighing 2 T. 13 cwt. 1 qr. 14 lb. How much has he left?

14. He wishes to make 15 loads of the remainder, how much must he take at each load?

15. How much butter in 12 firkins, if each contains 47 lb. 7 oz.?

16. There are 848 gallons of molasses in 6 casks. How much in each cask?

17. A farmer would divide 87 A. 3 R. 15 rd. of land into 5 fields of equal size. How much shall each field contain?

18. A silversmith having 48 lb. 5 oz. 2 dwt. of silver, has used 25 lb. 8 oz. 17 dwt. How much remains on hand?

19. He wishes to make of the remainder 15 tea-pots of equal weight. How much shall each weigh, and what is its value at $6\frac{1}{4}$ cents a dwt.?

20. A man bought 2 A. 3 R. 28 sq. rd. of land. How many

sq. rods does it contain? How many sq. ft.? What is its value at 1 ct. per sq. ft.?

21. If he sells 1 A. 35 sq. rd. 6 sq. yd. 8 sq. ft., how much land has he left?

22. A farmer had 85 C. 102 cu. ft. of wood, which he carried to market in 67 loads. How much did he carry at a load?

23. A tailor bought 85 yds. of cloth, from which he has cut 21 coats, each containing 2 yd. 3 qr. 2 na. How much cloth remains?

24. If he makes the remainder into 15 coats, how much cloth will each coat contain?

25. 8 horses have eaten 47 bushels of oats in 20 days. How much has each horse eaten? How much per day?

26. A farmer carried 47 cords of wood to market in 21 loads. How much did each load contain?

69. To find the time between two different dates.

1. What is the time from April 14th to June 23d? From April 14th to June 14th is 2 months; and from June 14th to June 23d is 9 days. *Ans.* 2 mo. 9 da.

2. What is the time from Nov. 15th, 1848, to Feb. 7th, 1849? From Nov. 15th to Jan. 15th is 2 months; there are 16 days in Jan. after the 15th; adding to this the 7 days in Feb., gives 23 days. *Ans.* 2 mo. 23 da.

3. What is the time from Jan. 19th, 1845, to March 17th, 1846? From Jan. 19th, 1845, to Jan. 19th, 1846, is 1 year; from Jan. 19th to Feb. 19th is 1 month, and from Feb. 19th to March 17th is 26 da. *Ans.* 1 yr. 1 m. 26 da.

4. What is the time from June 15th, 1847, to Nov. 12th, 1849? From Dec. 15th, 1840, to July 10, 1848? From June 10, 1835, to Nov. 8, 1850? From April 6, 1832, to Oct. 15, 1845? From Aug. 4, 1847, to March 3, 1848?

5. A note was given July 17, 1845, and paid Jan. 15, 1848. How long was it on interest? (**5.**)

NOTE. The above mode of computing time, although different from the mode given in most works on arithmetic, is the mode adopted by merchants and accountants in computing the time between two different dates. It may be thus expressed:

Find the number of months from the first date to the corresponding day next preceding the last date; then count the days that intervene between that day and the last date.

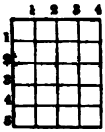
QUESTIONS. What are *simple* numbers? *Compound* numbers? What is *compound addition?* What is the *rule* for *compound addition?* What is the rule for *compound subtraction?* What is the rule

for *compound multiplication?* What may be done if the multiplier is a composite number? What is the rule for *compound division?* When is the work to be done by long division? When by short division? What is the rule for computing the time between two different dates?

SQUARE AND CUBIC MEASURE.

70. Exercises in Square and Cubic Measure.

1. A surface 1 inch long and 1 inch wide contains 1 square inch. How many square inches in a surface 2 in. long and 1 in. wide? 3 in. long and 1 in. wide? 5 in. long and 1 in. wide?



The diagram in the margin may represent a surface 5 inches or feet, &c., long, and 4 inches, &c., wide; each square representing 1 square inch or 1 square foot, &c.

2. How many square inches in a surface 5 in. long and 2 in. wide? 5 in. long and 3 in. wide? (**56.**)

3. How many square feet in a board 5 ft. long and 4 ft. wide? 8 ft. long and 5 ft. wide?

4. How many square inches in a board 216 in. long and 18 in. wide? How many sq. ft.?

5. How many square feet in a floor 18 ft. long and 13.5 ft. wide? How many of the above boards will cover it?

6. How many square inches in a board 1 ft. 4 in. wide, and 15 ft. 8 in. long? How many sq. ft.?

7. A room is 30 ft. long and 12 ft. high; how many sq. ft. on one of its sides? How many on its 2 opposite sides?

8. The room is 20 ft. wide; how many square feet in one of its ends? In both ends? How many in the ceiling? In the floor?

9. How many square feet in the four sides and the ceiling? How many square yards? What will the plastering come to, at 6 cents a sq. yd.?

10. How much will the flooring cost, at 6 cents per sq. ft.?

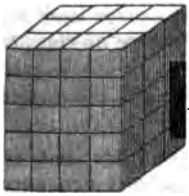
11. There are 12 windows in the room, and in each window 12 panes of glass, each pane being 16 in. by 10 in. How many square inches of glass in each window? In all the windows? How many square feet? How much did the glazing cost, at 8 cts. per sq. ft.?

12. How many square yards in the 4 sides and ceiling of a room 15 ft. long, 12 ft. wide, and 9 ft. high?

13. How many square rods in a rectangular field 48 rods long and 40 rods wide? How many acres?

14. How much land in a road 4 rods wide and 1 mile long?

15. What is the value of a rectangular field 12 rods long and 8½ rods wide, at $1.125 per square foot?



The diagram in the margin represents a solid, 5 inches or feet long, 4 inches, &c., wide, and 3 inches, &c., deep.

16. How many pieces, each containing 1 sq. inch, can be cut from a board 8 in. long and 4 in. wide, allowing nothing for waste? How many *cubic* inches are there in the board, if the board is 1 in. thick? (**57.**)

17. How many if it were 2 in. thick? 3 in. thick?

18. How many cubic feet in a block 3 ft. long, 2 ft. wide, and 1½ ft. thick?

19. A load of wood is 8 ft. long, 4 ft. wide, and 4 ft. high. How many cords does it contain?

20. How many cubic feet in 5 loads, each measuring 10 ft. long, 4 ft. wide, and 6 ft. high? How many cord feet? How many cords?

21. A pile of wood is 32 ft. long, 15 ft. wide, and 10 ft. high. How many cords does it contain? How much will it come to, at $4.50 per cord? Answer in cords and decimals.

22. How many cubic inches in a block of marble 8 ft. 3 in. long, 4 ft. 5 in. wide, and 3 ft. 7 in. thick? How many cubic feet?

23. How many tons of 2000 lb. will the above block weigh, allowing 12 cubic feet to weigh a ton?

24. How many gallons of water will a rectangular cistern hold, that is 5 ft. long, 8½ ft. high, and 4 ft. wide, allowing a cubic foot to hold 7.5 gallons?

SECTION VII.—ARITHMETICAL TERMS DEFINED AND ILLUSTRATED.

71. A *Unit* is a single thing of a kind; as one hat, one school, one nation.

An *Integer* is any whole number; as 1, 8, 15.

An *Even Number* is one whose right hand figure is 0, 2, 4, 6, or 8.

An *Odd Number* is one whose right hand figure is 1, 3, 5, 7, or 9.

A *Composite Number* is one that is composed of two or more factors. (25 and 28.)

A *Prime Number* is one that has no factors except itself and unity. Numbers are *prime to each other* that have no common factor.

Thus, 8 and 15, though composite numbers, are prime to each other, for $8 = 2 \times 2 \times 2$, and $15 = 3 \times 5$.

A Prime Factor of a number is a prime number that will divide it without a remainder. Thus, 2, 3, and 5, are prime factors of 30.

1. Is 20 an integer? Why? Is 18? Why? Is 3.8? Why? Is $7\frac{1}{2}$? Why? Is $\frac{4}{5}$? Why?
2. Is 364 an even or an odd number? Is 497? 876? 645? 3? 230?
3. Is 25 a composite or a prime number? Is 32? 37? 19? 22? 63? 51?
4. Are 9 and 12 prime to each other? Are 10 and 12? 7 and 15? 16, 24, and 25?
5. What are the prime factors of 6? Of 8? 9? 10? 11? 12? 15? 18? 23? 45?

RULE FOR FINDING ALL THE PRIME FACTORS OF A NUMBER.

Divide the number by any one of its prime factors; then that quotient by another, and so on till the quotient is a unit. The several divisors are all the prime factors of the number.

7. What are the prime factors of 1260?
Ans. 2, 2, 3, 3, 5, 7.

2	1260
2	630
3	315
5	105
7	21
3	3
	1

We see that *a number is equal to the product of all its prime factors;* for $2 \times 2 \times 3 \times 3 \times 5 \times 7 = 1260$.

NOTE. When a factor is repeated, it may be written but once, by placing a small figure above it at the right hand, called an *index*, to indicate how many times it is used as a factor. Thus, instead of writing $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$, we may write $2^3 \times 3^2 \times 5 = 360$; $5 \times 5 \times 5 \times 5 \times 2 \times 2 \times 3 \times 3 \times 3 = 67500$, may be written $5^4 \times 2^2 \times 3^3 = 67500$.

8. What are the prime factors of 35? Of 48? Of 60? Of 72? 84? 275? 864? 1084? 35952?

The following truths will aid the pupil in finding the prime factors and other divisors of numbers.

2 will divide all even numbers.

3 will divide all numbers the sum of whose figures is divisible by 3. Thus, 684 is divisible by 3, because $6+8+4=18$ is divisible by 3.

4 will divide any number whose two right hand figures are divisible by 4.

5 will divide any number whose right hand figure is 0 or 5.

6 will divide any even number that is divisible by 3.

8 will divide any number whose three right hand figures are divisible by 8.

9 will divide any number the sum of whose figures is divisible by 9.

10 will divide any number whose right hand figure is 0.

11 will divide any number in which the sum of the figures in the odd places is equal to the sum of the figures in the even places; or in which their sums differ by a number which can be divided by 11 without a remainder. Thus, 85426 is divisible by 11, because $8+4+6=5+2+11$. So is 63561894; because $6+5+1+9=3+6+8+4$.

12 will divide any number which is divisible by 3 and 4, because

A number that contains two or more factors that are prime to each other, is divisible by the product of those factors.

7, 11, and 13, are factors in numbers of 4 places in which two similar figures enclose two naughts; as 1001, 5005, 10010, 50050, &c.

For a more extended list of factors, with their signs or marks of recognition, see a little work entitled "The Plain Calculator, by Lewis Joerres, Professor of Mathematics from Prussia."

By what numbers is 348 divisible? Why?
By what is 624 divisible? 24156? 463320? Why?

72. MEASURE. COMMON MEASURE. GREATEST COMMON MEASURE.

A *Measure* of any number is a number that will divide it without a remainder. Thus, 3 is a measure of 3; of 6; 8 is a measure of 8, 16, &c.

NOTE. A number is said to measure another when it will divide it without a remainder. Thus, 8 measures 32.

What number is a measure of 24? What other? What other?

A *Common Measure* of two or more numbers is a number that will measure each of them. Thus, 3 is a common measure of 6, 18, and 24; 4 is a common measure of 24, 32, 48.

The *Greatest Common Measure* of two or more numbers is the greatest number that will measure them. Thus, 3 is a common measure of 6, 18, and 24; so is 2; but their *greatest* common measure is 6.

What is the greatest common measure of 4 and 6? Of 6 and 9? Of 12 and 18?

The common measure of two or more numbers contains no factors but those which are common to all the numbers; and their *greatest* common measure contains *all* those common factors, and no others. Hence the

RULE FOR FINDING THE GREATEST COMMON MEASURE OF TWO OR MORE NUMBERS.

Find all the prime factors of each of the numbers; then the product of all the factors, that are common to all the numbers, will be the greatest common measure.

1. What is the greatest common divisor of 18, 24, 42, and 54?

The only factors that are common to all the numbers are 2 and 3; therefore, the greatest common measure is 6.

$$18 = 2 \times 3^2$$
$$24 = 2^3 \times 3$$
$$42 = 2 \times 3 \times 7$$
$$54 = 2 \times 3^3$$

2. What is the greatest common measure of 4, 8, 10, and 12? Of 12, 15, 21, 27?

3. What is the greatest common measure of 3, 9, 14, 18? Of 5, 6, 8, and 10? Of 28 and 54?

4. Find the greatest common measure of 42, 56, and 112. Of 84 and 120.

5. What is the greatest common measure of 136 and 352? Of 72 and 162?

ANOTHER RULE. *If there are but two given numbers, divide the greater by the less, and if there is no remainder, the divisor is the greatest common measure. If there is a remainder, divide the last divisor by it, and so on till nothing remains. The last divisor is the greatest common measure.*

Find the greatest common measure of 153 and 162.

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153)162(1
    153
    ---
      9)153(17, the greatest common measure.
        153
```

6. What is the greatest common measure of 184 and 232? Of 36 and 90?

7. Find the greatest common measure of 42, 56, and 91. First find the greatest common measure of 42 and 56, which is 14; and then of 14 and 91. Do the 2d, 3d and 4th, by this rule.

73. MULTIPLE. COMMON MULTIPLE. LEAST COMMON MULTIPLE.

A *Multiple* of a number is a number that can be measured by it. Thus, 12 is a multiple of 6. What other number is a multiple of 6? Why? What other? What other?

A *Common Multiple* of two or more numbers is a number that can be measured by each of them. Thus, 24 is a common multiple of 2, 3, 4, and 6.

1. Find a common multiple of 3, 4, and 8; another; another.
2. Find a common multiple of 5, 6, 10, and 15; of 2, 3, and 7.

The *Least Common Multiple* of two or more numbers is the *least* number that can be measured by them.

Thus, 24 is a common multiple of 3, 4, and 6; but their least common multiple is 12.

3. What is the least common multiple of 2, 3, and 4? Of 4, 5, and 6? Of 3, 6, and 8? Of 4, 5, and 10? Of 8 and 12? Of 12, 15, and 30?

RULE FOR FINDING THE LEAST COMMON MULTIPLE OF TWO OR MORE NUMBERS.

Every number is a multiple of itself; and since (71) any number is equal to the product of all its prime factors, any number that contains all the prime factors of a number must be a multiple of that number. For example, if $2 \times 2 \times 3$ is a multiple of 12, any number of times $2 \times 2 \times 3$ must be a multiple of 12.

Find the least common multiple of 15 and 18. $15 = 3 \times 5$; $18 = 3^2 \times 2$. Any number that contains all these factors is a common multiple of 15 and 18, whether it contains other factors or not; and the number that contains these factors but once, and contains no other factors, is the *least* common multiple of 15 and 18. $2 \times 3^2 \times 5, = 90$, contains them all once, and only once; 90 is therefore the least common multiple of 15 and 18.

4. **What is the least common multiple of 9, 11, 15, and 24?**

$9 = 3^2$; 11 is a prime number; $15 = 3 \times 5$; $24 = 2^3 \times 3$. $2^3 \times 3 \times 5 \times 11 = 3960$, *Ans.*

5. What is the least common multiple of 18 and 35?

$18 = 3^2 \times 2$; $35 = 7 \times 5$; no factor being common to both numbers, the common multiple must be the product of all of them. $3^2 \times 2 \times 5 \times 7 = 18 \times 35 = 810$, *Ans.*

From these examples we deduce the following

RULE. *If the numbers are prime to each other, take the product of them all for the least common multiple. If they are not prime to each other, take the product of all the prime factors contained in the numbers, using each factor the greatest number of times it occurs in either of the given numbers.*

6. What is the least common multiple of 6, 8, and 10? Of 5, 12, and 15? 4, 9, and 25? 7, 21, 25, and 35?

7. Find the least common multiple of 12, 16, and 18. 2, 3, 4, 5, and 6. Of 45, 72, and 84. Of 35, 56, and 63.

CANCELLATION.

74. If, in division, both the dividend and divisor be multiplied or divided by the same number, the product or quotient will not be altered. Thus, 24 divided by 6 gives the same quotient as 48 divided by 12, or 72 divided by 18, or 12 divided by 3, or 8 divided by 2.

Therefore, when division is to be performed, if the divisor and dividend have a common factor, the operation may often be shortened by cancelling or rejecting that factor, and performing the work without it.

For example: suppose 12 times 35 is to be divided by 21. The work may be expressed thus: $\frac{12 \times 35}{21} = \frac{420}{21} = 20$. By separating 35 and 21 into their prime factors, it will be expressed thus, $\frac{12 \times 5 \times 7}{3 \times 7} = \frac{420}{21} = 20$. But as 7 is a factor of both the dividend and divisor, it may be cancelled from each, by drawing a line across it, thus, $\frac{12 \times 5 \times \cancel{7}}{3 \times \cancel{7}} = \frac{60}{3} = 20$.

The number 12 may, in like manner, be separated into factors, and the expression will be $\frac{4 \times 3 \times 5 \times 7}{3 \times 7}$; by cancelling

like factors in the dividend and divisor, it becomes $\frac{4\times\cancel{3}\times5\times\cancel{7}}{\cancel{3}\times\cancel{7}}$ $=\frac{20}{1*}=20.$

1. Multiply 165 by 36, and divide the product by 12.

Solution. $\frac{165\times\cancel{3}\times\cancel{12}}{2\times\cancel{3}\times\cancel{12}}=82\frac{1}{2}.$

2. Multiply 357 by 45, and divide the product by 126.

Solution. $\frac{357\times45}{126}=\frac{51\times\cancel{7}\times\cancel{9}\times5}{2\times\cancel{7}\times\cancel{9}}=$ ——.

3. Find the value of these expressions: $\frac{25\times36}{27}$; $\frac{45\times24}{40}$.

If any number in the dividend and divisor can be divided by the same number without a remainder, it may be done, and the quotients used instead of the numbers themselves. This method is often shorter than that of separating the numbers into prime factors.

4. Perform the 1st, 2d, and 3d, in this way.

5. Multiply together the first six digits, and divide the product by the product of the last six. The 4's, 5's, and 6's, being contained in both dividend and divisor, may be cancelled. Then the 2 and 8 may be divided by 2, and the 3 and 9 by 3.

$$\frac{1\times\cancel{2}\times\cancel{3}\times\cancel{4}\times\cancel{5}\times\cancel{6}}{\cancel{4}\times\cancel{5}\times\cancel{6}\times7\times\underset{4}{\cancel{8}}\times\underset{3}{\cancel{9}}}=\frac{1*}{84}.$$

6. Divide the product of 2, 8, 9, 10, 11, and 12, by the product of 3, 4, 5, 7, and 18.

The product of 3 and 4 in the divisor, is equal to 12 in the dividend; 18 is equal to the product of 2 and 9. Hence, the following general rule for cancelling:

$$\frac{\cancel{2}\times8\times\cancel{9}\times\overset{2}{\cancel{10}}\times11\times\cancel{12}}{\cancel{3}\times\cancel{4}\times\cancel{5}\times7\times\cancel{18}}=25\frac{1}{7}$$

If one factor, or the product of two or more factors, in the dividend, is equal to a factor, or to the product of two or more factors, in the divisor, these factors may be cancelled in both.

7. How much is $5\times28\times57$ divided by $3\times19\times7\times5$?

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* When we obtain 1 as a quotient in cancelling, we need not set it down; but if all the factors in either the dividend or divisor are cancelled, 1 must be placed in the product.

8. How much is $8 \times 49 \times 164$, divided by $4 \times 7 \times 28 \times 15$?

Perform the following examples by cancelling equal factors.

9. Divide 24×25 by 12; by 6; 8; 10; 15; 32; 65; 100; 150.

10. 36×56 is how many times 12? 18? 28? 32? 36? 42? 56? 24? 48? 84? 96? 288?

11. Divide the product of 6, 8, 9, 12, and 15, by the product of 3, 4, 6, 5, 10, and 18.

12. Divide the product of 12, 15, 18, 16, 21, and 25, by the product of 8, 5, 9, 24, and 20.

QUESTIONS. What is a *unit?* An *integer?* An *even* number? An *odd* number? A *composite* number? A *prime* number? When are numbers prime to each other? What is a prime factor of a number? A *component factor? Give an example of each.* What is the rule for finding all the prime factors of a number? To what is the product of all the prime factors of a number equal? How may a *factor that is repeated* be written? Give an example. What is called an *index?* What numbers will 2 measure? 3? 4? 5? 6? 8? 9? 10? 11? 12? *Give an example of each.* When is a number divisible by the product of 2 or more of its factors? *Ans.* When those factors are prime to each other. In what numbers are 7, 11, and 13 factors? What is a *measure* of a number? A *common measure* of two or more numbers? The *greatest common measure?* What factors does the *common measure* of two or more numbers contain? What factors does the *greatest common measure* contain? What is the rule for finding the greatest common measure of two or more numbers? Another rule? What is a *multiple? Common multiple? Least common multiple?* Of what is every number a multiple? When can a number be a multiple of another? Give an example. What is the rule for finding the *least common multiple* of two or more numbers?

Cancellation. How does multiplying or dividing both the dividend and divisor by the same number affect the quotient? Give an example. How may an operation in division often be shortened? When may numbers or factors of numbers be cancelled?

SECTION VIII.—FRACTIONS.

75. TERMS DEFINED AND ILLUSTRATED.

A *Fraction* is an expression for one or more equal parts of a unit; as, $\frac{3}{5}$, $\frac{7}{9}$, .4, .16.

Fractions are divided into two classes, *Common* and *Decimal.*

Common Fractions are expressed by two numbers, one placed above the other, with a line drawn between them; as $\frac{5}{7}$, $\frac{6}{17}$.

Decimal Fractions, or *Decimals*, are expressed by one number, with a period, called a decimal point, before it; as .106, .0075. (**12.**)

In common fractions the number below the line is called the Denominator, because it shows how many parts the unit is divided into.

The number above the line is called the Numerator, because it shows the number of parts expressed by the fraction.

Thus, if an apple were divided equally between 5 boys, one of the boys would have $\frac{1}{5}$ of the apple. If 3 apples were divided among them, each boy would have 3 times as much, or $\frac{3}{5}$ of one apple. Here the denominator, 5, shows how many parts the apple is divided into, and the numerators, 1 and 3, show how many of the parts are expressed.

76. *Fractions always express a quotient, or a division of one number by another; the numerator representing the dividend, and the denominator the divisor;* thus, the expression, $\frac{7}{8}$, although it may be read, seven divided by eight, seven eighths of one, or one eighth of seven, expresses the quotient of eight divided by seven.

The *value* of a fraction is the quotient of its numerator divided by its denominator.*

77. If both the numerator and denominator of a fraction are multiplied or divided by the same number, the value of the fraction will not be altered, (**74.**) Thus, $\frac{24}{6} = \frac{48}{12} = \frac{72}{18} = \frac{12}{3} = \frac{8}{2} = \frac{4}{1} = 4$; and $\frac{18}{24} = \frac{36}{48} = \frac{9}{12} = \frac{6}{8} = \frac{3}{4}$.

The numerator and denominator are called the *terms* of the fraction.

A fraction is said to be in its *lowest terms*, when its terms are prime to each other (**71**); as $\frac{7}{8}$, $\frac{15}{16}$.

TO REDUCE A FRACTION TO ITS LOWEST TERMS.

RULE. *Divide the terms of the fraction by their greatest common measure. Or, divide its terms by any common measure, and these quotients again in the same manner, and so on, till no number greater than unity will measure them.*

Thus, the fraction $\frac{24}{36}$ may be reduced to its lowest terms

either by dividing its terms by their greatest common measure, 12, or, by dividing its terms successively by 2, 2, and 3; $\frac{24}{36} = \frac{12}{18} = \frac{6}{9} = \frac{2}{3}$.

1. Reduce mentally these fractions to their lowest terms: $\frac{2}{4}$, $\frac{3}{9}$, $\frac{5}{15}$, $\frac{4}{12}$, $\frac{8}{10}$, $\frac{16}{18}$, $\frac{15}{18}$, $\frac{18}{24}$, $\frac{15}{24}$, $\frac{24}{32}$, $\frac{18}{27}$, $\frac{25}{35}$, $\frac{15}{25}$, $\frac{20}{30}$, $\frac{27}{36}$, $\frac{18}{33}$.

Reduce, either mentally or on the slate, the following fractions to their lowest terms.

2. $\frac{486}{792}$; $\frac{252}{420}$; $\frac{352}{384}$. (3.) $\frac{648}{3024}$; $\frac{1727}{14710}$. (4.) $\frac{615}{3690}$; $\frac{369}{1368}$.

78. PROPER AND IMPROPER FRACTIONS. MIXED NUMBERS.

A *Proper Fraction* is one in which the numerator is less than the denominator; as, $\frac{5}{6}$, $\frac{1}{3}$.

An *Improper Fraction* is one in which the numerator is either equal to or greater than the denominator; as, $\frac{8}{8}$, $\frac{8}{5}$.

NOTE 1. The value of a *proper* fraction is always less than a unit; the value of an *improper* fraction is always a unit, or more than a unit.

A *Mixed Number* is composed of an integer and a fraction, as, $4\frac{1}{3}$, 3.8.

1. What is the value, in units and parts of a unit, of $\frac{8}{5}$? $\frac{7}{4}$? $\frac{17}{6}$? $\frac{15}{3}$? $\frac{18}{7}$? $\frac{19}{4}$? $\frac{23}{5}$? $\frac{37}{8}$? $\frac{47}{9}$? $\frac{54}{6}$? $\frac{85}{10}$? $\frac{78}{9}$?

NOTE 2. *In all operations in common fractions, the result should be reduced to its lowest terms.* (**77.**)

TO REDUCE IMPROPER FRACTIONS TO WHOLE OR MIXED NUMBERS.

RULE. *Perform the division expressed by the fraction.* (**76.**)

Reduce the following improper fractions to whole or mixed numbers.

(2.) $\frac{59}{13}$; $\frac{68}{18}$. (3.) $\frac{146}{15}$; $\frac{649}{13}$. (4.) $\frac{87}{16}$; $\frac{100}{27}$.
(5.) $\frac{150}{15}$; $\frac{30016}{121}$. (6.) $\frac{4164}{17}$; $\frac{801987}{1528}$.

Reduce to decimals the following common fractions, *by the same rule;* carrying the result to 6 decimal places, if there are remainders.

(7.) $\frac{3}{4}$; $\frac{2}{5}$; $\frac{1}{8}$. (8.) $\frac{7}{8}$; $\frac{9}{10}$; $\frac{1}{3}$. (9.) $\frac{2}{3}$; $\frac{3}{5}$; $\frac{4}{7}$.
(10.) $\frac{5}{9}$; $\frac{6}{11}$; $\frac{35}{52}$. (11.) $\frac{1}{45}$; $\frac{2}{517}$; $\frac{3}{1001}$.

Reduce to integers and decimals the following.

(12.) $\frac{9}{5}$; $\frac{15}{7}$; $\frac{23}{21}$. (13.) $\frac{49}{15}$; $\frac{3617}{25}$; $\frac{5014}{35}$.

14. How many fourths are there in 1 unit? in 3 units? in $3\frac{1}{4}$ units? in $7\frac{1}{4}$ units? in $8\frac{1}{4}$ units?

15. How many sevenths are there in 1? 3? $3\frac{2}{7}$? $8\frac{4}{7}$? $5\frac{5}{7}$? $12\frac{6}{7}$?

16. Reduce these numbers to eighths: 1; 3; $3\frac{1}{8}$; $4\frac{1}{8}$; $5\frac{3}{8}$; $12\frac{5}{8}$; $20\frac{1}{8}$.

TO REDUCE MIXED NUMBERS TO IMPROPER FRACTIONS.

RULE. *Multiply the integer by the denominator of the fraction, to the product add the numerator, and write the sum over the denominator.*

17. Reduce the following mixed numbers to improper fractions.

$2\frac{1}{3}$; $3\frac{3}{4}$; $5\frac{1}{6}$; $8\frac{3}{8}$; $9\frac{4}{5}$; $10\frac{5}{9}$; $15\frac{4}{7}$; $25\frac{3}{5}$; $30\frac{6}{7}$; $44\frac{3}{8}$; $15\frac{3}{8}$.

18. Reduce to improper fractions $356\frac{12}{17}$; $649\frac{4}{15}$.

19. Reduce $1001\frac{15}{43}$; $30106\frac{156}{401}$.

Write the following in the form of common fractions, and reduce each fraction to its lowest terms.

(20.) .3; .004; .614. (21.) .01081; .05; .1085.
(22.) 3.5; 6.04; 80.46.
(23.) 14.007; 700.8001; 3.0001081.
(24.) 3; 8; 15; 145.

NOTE 3. *Integers are expressed in a fractional form by writing* 1 *for the denominator.*

79. The greater the number of parts into which any thing or number is divided, the smaller will be one, or any given number, of those parts. So the fewer the parts into which any thing or number is divided, the greater will be one, or any given number, of the parts. Therefore, as the denominator shows how many parts the thing or number is divided into, multiplying the denominator only, *divides* the fraction; and dividing the denominator only, *multiplies* the fraction. Take the expression $\frac{3}{8}$ of 32, which is equal to 12. If we multiply the denominator by 2, we have $\frac{3}{16}$ of $32 = 6$, or 12 *divided* by 2. If we divide the denominator by 2, we have $\frac{3}{4}$ of $32 = 24$, or 12 *multiplied* by 2.

If 1 bushel of corn is worth $\frac{5}{8}$ of a dollar, how much are 2 bushels worth? How much are 4 bushels worth? 8 bushels?

NOTE. Two times $\frac{5}{8}$ of a dollar are $\frac{5}{4}$ of a dollar, because fourths of a dollar are twice as large as eighths; so, four times $\frac{5}{8}$ are $\frac{5}{2} = 2\frac{1}{2}$.

2. Multiply $\frac{3}{8}$ by 2; by 4; by 8. Multiply $\frac{4}{15}$ by 3; by 5; by 15.

3. Multiply $\frac{5}{18}$ by 2; by 3; 6; 9; 18. Mulitply $\frac{17}{24}$ by 3; by 6; 8; 12; 24.

4. How much are 4 times $5\frac{3}{8}$?

Say 4 times 5 are 20; 4 times $\frac{3}{8}$ are $\frac{3}{2} = 1\frac{1}{2}$, which, added to 20, makes $21\frac{1}{2}$; therefore, 4 times $5\frac{3}{8}$ are $21\frac{1}{2}$.

5. Multiply $7\frac{3}{10}$ by 5; by 2; by 10. $8\frac{4}{15}$ by 3; by 5; by 15.

6. Multiply $9\frac{3}{16}$ by 2; 4; 8; 16.

7. Multiply $12\frac{5}{24}$ by 2; 3; 4; 6; 8; 12; 24.

8. Divide $\frac{1}{2}$ a pound of raisins equally among 3 boys; what part of a pound will each boy have?

Note. If $\frac{1}{2}$ a pound be divided into 3 equal parts, each of the parts will be $\frac{1}{6}$ of a pound. Say $\frac{1}{3}$ of $\frac{1}{2}$ is $\frac{1}{6}$. If the half pound were divided equally among 4 boys, each boy would have $\frac{1}{4}$ of $\frac{1}{2}$, or $\frac{1}{8}$ of a pound.

9. Divide $\frac{5}{8}$ of a dollar equally among 3 men. Each man would have $\frac{1}{3}$ of $\frac{5}{8}$; $\frac{1}{3}$ of $\frac{1}{8}$ is $\frac{1}{24}$; $\frac{1}{3}$ of $\frac{5}{8}$ is 5 times $\frac{1}{24}$, or $\frac{5}{24}$.

10. Divide $\frac{3}{4}$ by 4; 5; 6. $\frac{9}{10}$ by 3; 5; 8. $\frac{16}{7}$ by 9; 3; 5; 7.

11. How much is $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{4}$ of $\frac{3}{5}$? $\frac{1}{5}$ of $\frac{8}{3}$? $\frac{1}{5}$ of $\frac{9}{4}$? $\frac{1}{6}$ of $\frac{13}{7}$?

80. As the numerator shows the number of parts expressed by the fraction, multiplying the numerator multiplies the fraction, and dividing the numerator divides the fraction.

1. Multiply $\frac{3}{7}$ by 2; by 5; by 9; by 12. Say, 2 times $\frac{3}{7}$ are $\frac{6}{7}$. 5 times $\frac{3}{7}$ are $\frac{15}{7} = 2\frac{1}{7}$.

2. Multiply $\frac{4}{11}$ by 3; by 4; by 6; 8. $\frac{5}{12}$ by 4; by 7; 9; 11.

3. Multiply $\frac{15}{21}$ by 2; by 3; 5; 8. $\frac{17}{20}$ by 5; by 6; 7; 8.

4. Multiply $4\frac{3}{7}$ by 2; by 5; 9; 12.

Say 2 times 4 are 8; 2 times $\frac{3}{7}$ are $\frac{6}{7}$, which added to 8 make $8\frac{6}{7}$.

5. Multiply $5\frac{4}{9}$ by 4; by 5; 6; 7; 12. $15\frac{5}{8}$ by 3; by 5; 7; 9; 12.

6. Multiply $25\frac{3}{8}$ by 2; by 4; 7; 8; 12.

7. Divide $\frac{25}{8}$ by 5. $\frac{32}{7}$ by 4; by 8; 16.

8. Divide $\frac{21}{4}$ by 3; by 7; 21. $\frac{48}{13}$ by 3; 4; 6; 8; 12.

9. Divide $3\frac{4}{7}$ by 5. *Note.* $3\frac{4}{7} = \frac{25}{7}$.

10. Divide $4\frac{2}{3}$ by 7. $6\frac{6}{8}$ by 9. $7\frac{1}{5}$ by 12.

11. Divide $8\frac{2}{5}$ by 3. One third of $8\frac{2}{5}$ is 2, and $2\frac{2}{5} = \frac{12}{5}$ remain to be divided; $\frac{1}{3}$ of $\frac{12}{5} = \frac{4}{5}$, which added to 2 is $2\frac{4}{5}$; therefore, $\frac{1}{3}$ of $8\frac{2}{5}$ is $2\frac{4}{5}$.

12. Divide $5\frac{3}{7}$ by 2. *Note.* $5\frac{3}{7}=4+1\frac{3}{7}$.
13. Divide $5\frac{3}{7}$ by 3. $5\frac{3}{7}=3+2\frac{3}{7}$. Divide $5\frac{3}{7}$ by 4.
14. Divide $17\frac{3}{8}$ by 3; by 4; 5; 6; 7; 8; 9; 11; 12.
15. Divide $14\frac{4}{5}$ by 2; 3; 4; 5; 6; 7; 8; 9; 11; 12.
16. Multiply $3\frac{5}{8}$ by 3; 4; 5; 6; 7; 8.
17. Divide $5\frac{5}{8}$ by 2; 3; 4; 5; 6; 7; 8; 9.
18. Divide $15\frac{3}{10}$ by 2; 3; 4; 5; 6; 7; 8; 9; 12; 16.
19. Divide $497\frac{3}{7}$ by 5; by 8; by 15.

$$5\overline{)497\frac{3}{7}}$$
$$99\frac{17}{35}$$

NOTE. After dividing by 5, (see margin,) $2\frac{3}{7}$ remain, which is equal to $\frac{17}{7}$, $\frac{1}{5}$ of which is $\frac{17}{35}$.

20. Divide $31847\frac{5}{8}$ by 3; by 4; by 5; by 7.
21. Divide $576\frac{3}{8}$ by 3; by 6; by 9.
22. Divide $576\frac{3}{8}$ by 12; by 16; by 25.

81. From the last two articles we may derive the following rules:

RULE 1. *To multiply a fraction, Divide the denominator, if it can be done; if not, multiply the numerator.*
RULE 2. *To divide a fraction, Divide the numerator, if it can be done; if not, multiply the denominator.*

1. Multiply $\frac{47}{750}$ by 25; by 125; by 375.
2. Multiply $\frac{419}{640}$ by 8; by 128; by 160; by 320.
3. Multiply $378\frac{19}{24}$ by 6; by 8; by 24; by 12.
4. Multiply $\frac{37}{160}$ by 8; by 15; by 27; by 80.
5. Multiply $1847\frac{18}{57}$ by 3; by 15; by 19; by 30.
6. Divide $\frac{240}{35}$ by 3; by 12; by 16; by 48.
7. Divide $\frac{216}{317}$ by 8; by 12; by 27; by 67.
8. Divide $\frac{675}{1441}$ by 5; by 25; by 75; by 225.
9. Divide $75\frac{3}{11}$ by 12; by 36; by 15.
10. Divide $31578\frac{5}{14}$ by 15; by 37; by 6700.
11. Divide $1408071\frac{6}{17}$ by 3500; by 87100.

82. COMPOUND FRACTIONS.

A Compound Fraction is a fraction of a fraction; as, $\frac{2}{3}$ of $\frac{4}{5}$.

TO REDUCE COMPOUND FRACTIONS TO SIMPLE ONES.

How much are $\frac{2}{3}$ of $\frac{4}{5}$? Say, $\frac{1}{3}$ of $\frac{4}{5}$ is $\frac{4}{15}$ (**79**); $\frac{2}{3}$ of $\frac{4}{5}$ are twice as much, or $\frac{8}{15}$; therefore, $\frac{2}{3}$ of $\frac{4}{5}$ are $\frac{8}{15}$.

Reduce to a simple fraction $\frac{1}{3}$ of $\frac{1}{5}$; $\frac{1}{6}$ of $\frac{1}{2}$; $\frac{1}{3}$ of $\frac{1}{4}$; $\frac{2}{3}$ of $\frac{1}{5}$; $\frac{2}{3}$ of $\frac{1}{4}$; $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{1}{4}$ of $\frac{5}{7}$; $\frac{4}{5}$ of $\frac{7}{8}$; $\frac{8}{9}$ of $\frac{11}{12}$; $\frac{4}{7}$ of $\frac{9}{11}$; $\frac{3}{4}$ of $1\frac{4}{5}=\frac{3}{4}$ of $\frac{9}{5}$.

RULE. *Reduce all the numbers to a fractional form; and, after cancelling the factors which are common to the numerator and denominator, multiply all the numerators together, for a new numerator, and all the denominators, for a new denominator.*

1. Reduce the following compound fractions to simple ones. $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$; $\frac{3}{4}$ of $\frac{1}{5}$ of $\frac{2}{9}$; $\frac{3}{8}$ of $\frac{4}{9}$ of $\frac{6}{7}$ of $\frac{7}{15}$.

The last example, after cancelling, is $\frac{\cancel{3}}{\cancel{8}}\times\frac{\cancel{4}}{\cancel{9}}\times\frac{\cancel{6}}{\cancel{7}}\times\frac{\cancel{7}}{15}=\frac{1}{15}$.

2. Reduce to simple fractions $\frac{1}{4}$ of $2\frac{1}{7}$ of $4\frac{3}{8}$ of 16; $\frac{3}{8}$ of $2\frac{1}{4}$ of $15\frac{4}{5}$.
3. Reduce $\frac{3}{5}$ of $\frac{9}{10}$ of $\frac{10}{21}$ of 100; $\frac{15}{22}$ of $\frac{11}{12}$ of $15\frac{3}{14}$ of 50.
4. Reduce to simple fractions $\frac{4}{5}$ of .15 of $\frac{3}{8}$ of 40; $\frac{3}{7}$ of .7 of .08 of $5\frac{5}{12}$; $\frac{1}{8}$ of .046 of 31.07.

CIRCULATING DECIMALS.

83. In reducing common fractions to decimals, if the denominator contains any prime factors other than the factors of 10, viz., 2 and 5, the division can never be completed, the same quotient figure or figures being continually repeated. Such decimals are called *repeating*, or *circulating decimals;* and the figures repeated are called *repetends.* If only one figure is repeated, as in $\frac{1}{3}=.333$, &c., it may be expressed by writing the figure repeated once only, with a dot over; thus, $\frac{1}{3}=.\dot{3}$; $\frac{2}{3}=.\dot{6}$.

If the same figures are repeated alternately, as in $\frac{6}{11}=.5454$, &c., or in $\frac{23}{37}=.621621$, &c., the repetend is expressed by writing the figures once, with a dot over the first and last figures; thus, $\frac{6}{11}=.\dot{5}\dot{4}$; $\frac{23}{37}=.\dot{6}2\dot{1}$.

Repeating decimals are changed to common fractions by writing the repetend for the numerator, and as many 9's as

there are figures in the repetend for the denominator; thus, $.\dot{3}=\frac{3}{9}$; $.\dot{5}\dot{4}=\frac{54}{99}$; $\dot{6}2\dot{1}=\frac{621}{999}$; $.4\dot{5}=\frac{4}{10}+\frac{5}{9}$ of $\frac{1}{10}$; $38\dot{7}1\dot{7}$ $=\frac{38}{100}+\frac{717}{999}$ of $\frac{1}{100}$; $0\dot{7}=\frac{7}{9}$ of $\frac{1}{10}$.

1. Change to decimals the following common fractions: $\frac{5}{9}$; $3\frac{2}{9}$; $\frac{4}{11}$; $\frac{5}{333}$; $8\frac{5}{7}$.

2. Change to common fractions $.\dot{4}$; $\dot{4}\dot{5}$; $.\dot{4}1\dot{4}$; $5.\dot{4}3\dot{2}$.

3. Change to common fractions $.4\dot{5}$; $.38\dot{7}1\dot{7}$; $0\dot{7}$; $.00\dot{5}$.

4. Change to common fractions $.3\dot{1}\dot{8}$; $5.45\dot{1}07\dot{1}$; $30.4\dot{1}00\dot{1}$; $2.00\dot{5}10\dot{9}$; $.401\dot{0}6\dot{4}$.

84. TO REDUCE A GIVEN INTEGER OR FRACTION TO A FRACTION HAVING A GIVEN DENOMINATOR.

1. One unit is how many 4ths? *Ans.* 4 4ths. 3 units are how many 4ths? Since there are 4 4ths in 1 unit, there will be 4 times as many 4ths as units. *Ans.* 12 4ths.

2. Reduce 5 to halves. Since there are 2 halves in 1 unit, there will be 2 times as many halves as units. Reduce 5 to 3ds; to 4ths; 5ths; 6ths; 7ths; 8ths; 10ths; 12ths.

3. In 1 unit how many 3ds? In $\frac{1}{2}$ of a unit how many 3ds?

There are 3 times as many 3ds as units. 3 times $\frac{1}{2}$ of a unit are $\frac{3}{2}$ of a unit $=1\frac{1}{2}$. *Ans.* $1\frac{1}{2}$ 3ds.

4. In $\frac{1}{4}$ how many 3ds? *Ans.* $\frac{3}{4}$ of a 3d. In $\frac{3}{4}$ how many 3ds? *Ans.* $2\frac{1}{4}$ 3ds.

5. Change $\frac{3}{5}$ to 4ths. There 4 times as many 4ths as units. *Ans.* $2\frac{2}{5}$ 4ths.

6. Reduce $\frac{3}{7}$ to halves. $\frac{3}{7}\times 2=\frac{6}{7}$ of a half. Reduce $\frac{3}{7}$ to 3ds; 4ths; 5ths; 6ths; 9ths; 12ths.

RULE. *Multiply the given integer or fraction by the required denominator.*

7. Reduce $\frac{3}{4}$ to 8ths; to 12ths; 16ths; 20ths; 24ths; 36ths.

8. Reduce $\frac{5}{6}$ to 12ths; 18ths; 24ths; 30ths; 42ds; 54ths.

9. $\frac{5}{8}$ of a bushel are how many 4ths of a bushel?

$\frac{5}{8}\times 4=\frac{5}{2}=2\frac{1}{2}$. Ans. $2\frac{1}{2}$ 4ths of a bushel.

10. $\frac{7}{16}$ of a pk. are how many 8ths of a pk.?

11. Reduce $\frac{3}{5}$ of a lb. to 16ths of a lb. $\frac{3}{5}$ of an oz. Troy to 20ths of an oz.

12. Reduce $\frac{5}{8}$ of a day to 24ths of a day. $\frac{3}{8}$ of an hour to 60ths of an hour. $\frac{5}{7}$ of a minute to 60ths of a minute.

85. To reduce Fractions to their Least Common Denominator.

Fractions are said to have a common denominator when they have the same denominator. Thus, the fractions $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, have a common denominator.

1. Reduce $\frac{3}{5}$ to 10ths; to 15ths; 20ths; 25ths; 30ths; 45ths.

NOTE. In multiplying $\frac{3}{5}$ by 10, the common factor 5 should be cancelled. Thus $\frac{3}{5} \times \frac{10}{1} = \frac{3}{1} \times \frac{2}{1} = \frac{6}{1}$ or 6. *Ans.* $\frac{6}{10}$. So, $\frac{3}{5} \times \frac{15}{1} = \frac{3}{1} \times \frac{3}{1} = \frac{9}{1}$ or 9. *Ans.* $\frac{9}{15}$.

2. Reduce $\frac{4}{9}$ to 18ths; to 27ths. $\frac{5}{12}$ to 24ths; to 36ths; to 60ths.
3. Reduce $\frac{1}{2}$ and $\frac{1}{3}$ to 6ths. $\frac{1}{2}$ and $\frac{1}{5}$ to 10ths. $\frac{1}{8}$ and $\frac{1}{6}$ to 24ths.

NOTE. *The least common denominator is the least common multiple of all the denominators.*

RULE. *Reduce the fractions to their lowest terms; and multiply each fraction by the least common denominator; the several products will be the numerators to the common denominator.* (84.)

Or, which is the same, Divide the common denominator by the denominator of each of the fractions; the several quotients, multiplied by the respective numerators, will be the numerators to the common denominator.

4. Reduce $\frac{7}{8}$, $\frac{5}{12}$ and $\frac{4}{15}$, to their least common denominator. The least common multiple of 8, 12 and 15, is 120.

First Method.

$$\frac{7}{8} \times \frac{120}{1} = \frac{7}{1} \times \frac{15}{1} = 105$$
$$\frac{5}{12} \times \frac{120}{1} = \frac{5}{1} \times \frac{10}{1} = 50$$
$$\frac{4}{15} \times \frac{120}{1} = \frac{4}{1} \times \frac{8}{1} = 32$$

Ans. $\frac{105}{120}$, $\frac{50}{120}$, $\frac{32}{120}$.

Second Method.

$$\begin{array}{rl} & 120 \\ \hline 8 & 15 \times 7 = 105 \\ 12 & 10 \times 5 = 50 \\ 25 & 8 \times 4 = 32 \end{array}$$

Ans. $\frac{105}{120}$, $\frac{50}{120}$, $\frac{32}{120}$.

5. Reduce to their least common denominators $\frac{3}{8}$ and $\frac{5}{12}$; $\frac{4}{5}$ and $\frac{3}{4}$; $\frac{5}{8}$ and $\frac{2}{3}$; $\frac{7}{12}$ and $\frac{3}{4}$; $\frac{5}{6}$, $\frac{7}{8}$ and $\frac{5}{12}$; $\frac{3}{5}$, $\frac{6}{7}$ and $\frac{7}{10}$; $\frac{2}{5}$, $\frac{3}{8}$ and $\frac{7}{10}$.
6. Reduce to their least common denominators $\frac{3}{8}$ and $\frac{9}{10}$; $\frac{7}{15}$ and $\frac{9}{20}$.
7. Reduce $\frac{3}{8}$, $\frac{5}{9}$ and $\frac{4}{15}$; $\frac{3}{8}$, $\frac{4}{9}$ and $\frac{5}{12}$.
8. Reduce $\frac{7}{8}$, $\frac{7}{12}$, $\frac{7}{18}$ and $\frac{7}{24}$; $\frac{4}{15}$, $\frac{5}{24}$ and $\frac{8}{36}$.
9. Reduce $\frac{3}{15}$, $\frac{4}{17}$, $\frac{5}{24}$ and $\frac{9}{30}$; $\frac{4}{9}$, $\frac{8}{33}$, $\frac{7}{45}$ and $\frac{9}{24}$.
10. Reduce $\frac{3}{16}$, $\frac{5}{18}$, $\frac{15}{22}$ and $\frac{3}{21}$; $\frac{3}{7}$, $\frac{5}{28}$, $\frac{9}{10}$ and $\frac{49}{100}$.

86. RELATIONS OF NUMBERS. COMPOUND NUMBERS REDUCED TO FRACTIONS AND DECIMALS.

1. What part of 4 is 1? What part is 2? 3? 4? 5? What part of 5 is 3? 2? 4? 8?
2. What part of 15 is 1? 8? 7? 9? 13? 37? 26? 44?
3. What part of £1 is 1s.? 2s.? 6s.? 18s.? 15s.?
4. How many pence are there in £1? What part of a pound is 1d.? 3d.? 9d.? 3s. 5d.? 4s. 6d.? 8s. 3d.? 13s. 6d.?
5. What part of a pound Troy is 1 oz.? 8 oz.? 11 oz.? 1 dwt.? 3 oz. 5 dwt.?
6. What part of a bushel is 1 pk.? 3 pk.? 1 qt.? 3 qt.? 3 pk. 5 qt. 1 pt.?
7. What part of 75 gallons is 1 gal.? 47 gal.? 1 qt.? 8 gal. 3 qt.?

8. How many farthings are there in £1? In £1 4s. 6d.?
9. What part of £1 is 1qr.? 1d. 3qr.? 6d. 3qr.? 15s. 8d. 3qr.?
10. What part of 1 pound Troy is 1 gr.? 17 gr.? 4 oz. 16 dwt. 22 gr.?
11. What part of 1 gallon are 3 qt. 1 pt. 3 gi.?
12, 13, 14. Reduce the last three answers to decimal expressions. (78, RULE.) £$\frac{1}{960}$ = £.0010416̇; $\frac{7}{960}$ = £.0072916̇.
15. Reduce 35 rods to the decimal of a mile.
$\frac{35}{320} = \frac{7}{64} =$
16. Reduce 34 rods 3 yards to the decimal of a mile.
17. Reduce 15 rods 5 yards 2 ft. 8 in. to the decimal of a mile.
18. What part of 1 week are 1 d. 7 h.? Express it decimally.

Compound numbers may also be reduced to decimals of a higher denomination, by the following method.

19. Reduce 3 d. 15 h. 45 m. to the decimal of a week.

60	45	min.
24	15.75	h.
7	3.65625	d.
	.52232 +	wk.

We first divide 45 min. by 60, to reduce them to the decimal of an hour, and annex it to the 15 hours; we then have 15.75 hours. Dividing 15.75 hours by 24, reduces them to the decimal of a day; which added to the 3 days, gives 3.65625 days. Dividing the days by 7, reduces them to decimals of a week.

The examples in question 9 may be performed in the same manner.

4	1	qr.		4	3.	qr.
12	0.25	d.		12	8.75	d.
20	$0.0208\dot{3}$	s.		20	$15.7291\dot{6}$	s.
£	$0.00104\dot{6}$			£	$0.78645\dot{8}$	

Perform the 10th, 11th, 15th, 16th, 17th and 18th questions by this method.

20. Reduce 1 R. 15 sq. rd. 24 sq. yd. to the decimal of an acre.
21. Reduce 45 d. 17 h. 35 min. 17 sec. to the decimal of a year.
22. What part of 5 d. 13 h. 5 min. are 4 d. 6 h. 5 sec.?
23. What part of 12 cwt. 3 qr. is 1 lb.? 1 cwt. 2 qr. 18 lb.?
24. What part of 5 m. 2 fur. 8 rd. are 15 rd. 4 yd. 2 ft.?
25. What part of \$6.05 is \$3? \$4.10? \$1.005? \$0.06? \$0.105?
26. What part of \$37 is \$4.15? \$8.415? \$0.005? \$0.108?

87. To reduce the Fraction of one Denomination to the Fraction of another Denomination.

1. Reduce $\frac{9}{11}$ of an acre to the fraction of a square rod.
2. Reduce $\frac{3}{4}$ of a penny to the fraction of a pound.
3. Reduce .07 of a yard to the fraction of an inch.
4. Reduce .3 of a rod to the fraction of a mile.

NOTE 1. Specific rules for solving such problems as the above seem to be unnecessary; for to reduce $\frac{9}{11}$ of an acre to the fraction of a square rod is the same, in principle, as to reduce any number of acres to square rods. To do either, we multiply the acres or parts of an acre by 4, to reduce them to roods or parts of a rood; and these roods or parts of a rood by 40, to reduce them to square rods or parts of a square rod.

Thus, $\frac{9}{11}$ of an acre are equal to $\frac{9}{11}$ A. $\times 4 = \frac{36}{11}$ R., and $\frac{36}{11}$ R. $\times 40 = \frac{1440}{11}$ sq. rd. So, .07 of a yard are equal to .07 yd. $\times 3 =$ .21 ft.; 21 ft. $\times 12 = 2.52$ in.

NOTE 2. Again, to reduce pence or parts of a penny to pounds, we divide the pence or parts of a penny by 12, to reduce them to shillings or parts of a shilling; and these by 20, to reduce them to pounds or parts of a pound.

Thus, $\frac{3}{4}$d. $\div$ 12 $=$ $\frac{1}{16}$s., and $\frac{1}{16}$s. $\div$ 20 $=$ £$\frac{1}{320}$ (79); or, $\frac{3}{4}$d. $\times$ $\frac{1}{12}$ $\times$ $\frac{1}{20}$ $=$ £$\frac{1}{320}$. So, .3 of a rod $\div$ 40 $=$ .075 of a fur., and .075 fur. $\div$ 8 $=$ .009375 miles.

Therefore,

If the fraction is to be reduced to a *lower* denomination, MULTIPLY *it*, and if to a *higher*, DIVIDE *it*, *by the same numbers that you would use in reduction of whole numbers.* (**43.**)

5. Reduce $\frac{5}{1524}$ of a £ to the fraction of a farthing.
Solution. $\frac{5}{1524} \times \frac{20}{1} \times \frac{12}{1} \times \frac{4}{1} = \frac{400}{127}$qr.
6. Reduce .03 of a shilling to the fraction of a farthing.
Solution. $.03 \times 12 \times 4 = 1.44$qr.
7. Reduce to the fraction of a pint $\frac{3}{108}$ of a peck. $\frac{5}{38}$ of a gallon. $\frac{6}{37}$ of a bushel.
8. Reduce .4 of a pint to the fraction of a gallon.
9. What part of a lb. are $\frac{5}{108}$ of a cwt.? $\frac{3}{35}$ of a qr.?
10. What part of a £ are $\frac{4}{5}$ of a penny?
Solution. $\frac{4}{5}$d. $\times \frac{1}{12} \times \frac{1}{20} =$ £$\frac{1}{300}$.
11. What part of a lb. Troy are $\frac{4}{5}$ of a grain? $\frac{3}{16}$ of a dwt.? $\frac{15}{24}$ of an oz.?
12. Reduce $\frac{5}{11}$ of a rod to the fraction of a mile.
13. What part of an acre are $\frac{4}{5}$ of a sq. rod?
14. What part of a nail are .085 of a qr.? .17 of a yard?
15. What part of an inch are .08 of a yard? .1084 of a foot?

II. 16. What part of a mile are $\frac{4}{15}$ of a foot? $\frac{16}{25}$ of an inch?
17. What part of an acre is $\frac{1}{8}$ of a sq. yard? $\frac{4}{7}$ of a sq. ft.?
18. What part of a year are $\frac{5}{8}$ of a day? $\frac{7}{8}$ of an hour?
19. What part of a minute are .0003 of a year? .04 of a week?
20. Reduce to the fraction of a mile .35 of a yard. .106 of a foot.
21. What part of an inch are .0006 of a mile? .35 of a rod?
22. What part of 3 inches are .0006 of a mile? .35 of a rod?
23. What part of 5 miles are .35 of a yard? .106 of a foot?
24. What part of 4 years are $\frac{5}{8}$ of a day? $\frac{7}{8}$ of an hour?
25. What part of $3\frac{1}{2}$ acres is $\frac{1}{8}$ sq. yd.? $\frac{4}{7}$ sq. ft.?
26. What part of 15 minutes are .0003 of a year? .04 of a week?
27. What part of $3\frac{3}{4}$ miles are $\frac{4}{15}$ of a foot? $\frac{16}{25}$ of an inch?

88. FRACTIONAL COMPOUND NUMBERS REDUCED TO THEIR VALUES IN LOWER DENOMINATIONS.

1. Find the value of $\frac{5}{7}$ of a pound Troy in ounces, pennyweights and grains.

```
     5
    12
    ——
 7 ) 60 ( 8 oz. 11 dwt. 10 2/7 gr., Ans.
    56
    ——
     4
    20
    ——
 7 ) 80 ( 11 dwt.
    77
    ——
     3
    24
    ——
 7 ) 72 ( 10 gr.
    70
    ——
     2
```

Since there are 12 oz. in 1 lb., there will be 12 times as many sevenths of an oz. as of a lb., or $\frac{60}{7}$ = 8 oz. and $\frac{4}{7}$ of an ounce over. And since there are 20 dwt. in 1 oz., there will be 20 times as many sevenths of a dwt. as of an oz., &c. (87, NOTE 1.)

2. What is the value of .5816 of a £ in shillings, pence and farthings?

```
     £.5816
         20
     ——————
  s. 11.632
         12
     ——————
  d. 7.584
         4
     ——————
 qr. 2.336
Ans. 11s. 7d. 2qr.
```

Since there are 20 shillings in 1 pound, there will be 20 times as many ten thousandths of a shilling as of a pound, or 116320 ten thousandths, which is 11s. and 632 thousandths of a shilling. And as there are 12 pence in a shilling, there will be 12 times as many thousandths of a penny as of a shilling, &c.

Specific rules for solving such questions as the above are unnecessary.

Reduce the following fractions to their values in lower denominations:

3. £$\frac{5}{9}$; $\frac{3}{7}$s.; .3d. (4.) .67 lb. Troy; $\frac{5}{9}$ oz. Troy.
5. $\frac{3}{8}$ lb. apothecaries' weight; .14 ℥.
6. $\frac{3}{7}$ ton; $\frac{3}{14}$ cwt.; .875 qr.; $\frac{3}{16}$ lb.
7. $\frac{3}{8}$ gal.; .483 gal.; $\frac{5}{9}$ qt.

8. $\frac{1}{12}$ fur.; .35 rod; .871 yd.; $\frac{3}{7}$ foot.

9. Reduce to cents and mills $\$\frac{3}{8}$; $\$\frac{3}{4}$; $\$\frac{5}{8}$; $\$\frac{4}{5}$; $\$\frac{12}{17}$.

10. Reduce to cents and mills $\$\frac{2}{3}$; $\$\frac{15}{22}$; $\$4\frac{5}{9}$; $\$17\frac{5}{13}$; $\$20\frac{3}{13}$.

11. What is the value of .1456 of a £? .5716 of a shilling? .1847 of a square yard?

12. What is the value of $\frac{15}{23}$ of a bushel? $\frac{7}{18}$ of a peck?

II. 13. Reduce to its value in lower denominations $\frac{4}{7}$ of a mile.

14. Reduce $\frac{9}{14}$ of an acre; .3 of a rood; $\frac{3}{8}$ of a square rod.

15. Reduce to their values in lower denominations $\frac{3}{11}$ of a square mile; .5816 of an acre; $\frac{3}{8}$ of a year; .1084 of a mile; .30716 of a cord; .1516 of a cubic yard; $\frac{8}{15}$ of one deg.; $\frac{7}{18}$ of a cord.

89. ADDITION OF COMMON FRACTIONS.

If fractions have a common denominator, they are added by adding their numerators and placing the sum over the common denominator. Thus, $\frac{3}{11}+\frac{4}{11}+\frac{5}{11}+\frac{2}{11}=\frac{14}{11}=1\frac{3}{11}$.

RULE. *Having reduced compound fractions to simple ones, and all the fractions to their lowest terms, reduce them to their least common denominator, by Art.* 84. *Then add their numerators, and place the sum over the common denominator. If there are mixed numbers to be added, add the integers and the fractions separately.*

1. What is the the sum of $\frac{3}{5}$ and $\frac{4}{5}$? Of $4\frac{1}{4}+\frac{3}{4}+\frac{2}{4}$? Of $8\frac{3}{5}+5\frac{4}{5}+\frac{2}{5}$? Of $\frac{8}{9}+\frac{5}{9}+15\frac{4}{9}$?

2. What is the sum of $\frac{1}{3}+\frac{1}{2}$? Of $\frac{2}{3}+\frac{3}{4}$? *Solution.* $\frac{1}{3}=\frac{2}{6}$; $\frac{1}{2}=\frac{3}{6}$; $\frac{2}{6}+\frac{3}{6}=\frac{5}{6}$.

3. What is the sum of $4\frac{1}{2}+3\frac{2}{3}$? Say, 4 and 3 are 7; $\frac{1}{2}=\frac{3}{6}$, $\frac{2}{3}=\frac{4}{6}$; $\frac{3}{6}+\frac{4}{6}=\frac{7}{6}=1\frac{1}{6}$, which added to 7 makes $8\frac{1}{6}$.

4. Find the sum of $14\frac{3}{5}+3\frac{3}{4}$; of $\frac{1}{2}$ of $\frac{2}{7}+\frac{3}{4}$ of $\frac{2}{3}$.

5. Find the sum of $\frac{5}{8}$ and $\frac{3}{5}$; of $\frac{6}{7}$ of $\frac{7}{9}+\frac{3}{5}$ of 8; of $\frac{5}{9}+\frac{7}{12}$.

6. Add $18\frac{3}{8}+15\frac{5}{24}$; $\frac{10}{15}+\frac{20}{25}$.

7. Add $\frac{3}{4}$ of a bushel $+\frac{5}{8}$ of a peck $+\frac{1}{2}$ of a quart. *Ans.* 3 pk. 5 qt. 1 pt.

NOTE. $\frac{3}{4}$ of a bushel = 3 pecks; $\frac{5}{8}$ of a peck = 5 quarts, &c.

8. Add $\frac{3}{7}$ of a week $+\frac{3}{8}$ of a day $+\frac{3}{4}$ of an hour $+\frac{3}{10}$ of a minute.

9. What is the sum of £$\frac{3}{10}+\frac{3}{8}$s. $+\frac{1}{2}$d.? $\$\frac{3}{5}+\frac{1}{2}$ ct. $+\frac{8}{11}$ m.?

10. Add $\frac{3}{5}+\frac{2}{3}+\frac{4}{9}+\frac{8}{15}$.

SOLUTION.

45 least com. denom.

$$\begin{array}{r|l} 5 & 9\times3=27 \\ 3 & 15\times2=30 \\ 9 & 5\times4=20 \\ 15 & 3\times8=24 \end{array}$$

$$\frac{101}{45}=2\tfrac{11}{45}\ \textit{Ans.}$$

Having found the least common denominator by Art. 73, and reduced the fractions to this common denominator by Art. 84, we add the numerators thus found according to the rule.

11. Add $13\frac{5}{8}+4\frac{5}{12}+3\frac{5}{6}+\frac{2}{3}$.

SOLUTION.

24 least com. denom.

$$\begin{array}{ll} 13\frac{5}{8} & 3\times5=15 \\ 4\frac{5}{12} & 2\times5=10 \\ 3\frac{5}{6} & 4\times5=20 \\ \frac{2}{3} & 8\times2=16 \\ \hline 22\frac{13}{24}\ \textit{Ans.} & \end{array}$$

$$\frac{61}{24}=2\tfrac{13}{24}.$$

The fractions being added, are $2\frac{13}{24}$, which being added to the sum of the whole numbers is $22\frac{13}{24}$.

12. Add $\frac{5}{8}+\frac{6}{18}+\frac{5}{9}+3\frac{5}{6}$.
13. Add $\frac{4}{7}$, $\frac{3}{14}$, $\frac{7}{8}$ and $5\frac{8}{21}$ together.
14. Find the sum of $3\frac{1}{2}+5\frac{8}{17}+\frac{5}{9}+16\frac{2}{3}$.
15. Add $54\frac{1}{8}+\frac{3}{5}$ of $25+17\frac{9}{16}+4\frac{3}{32}$.
16. Add $28\frac{1}{15}+75\frac{8}{25}+105\frac{9}{65}$.

17. What is the sum of £$\frac{2}{3}$ + £$\frac{5}{8}$ + $\frac{4}{9}$s. + $\frac{3}{8}$d.?

SOLUTION.

£		s.	d.	qr.
$\frac{2}{3}$	=	13	4	0
$\frac{5}{8}$	=	12	6	0
$\frac{4}{9}$s.	=	0	5	$2\frac{2}{5}$
$\frac{3}{8}$d.	=	0	0	$1\frac{1}{2}$

£1 6s. 3d. $3\frac{9}{10}$qr. *Ans.*

We find the value of each fraction, (**87**,) and then add the quantities as in Compound Addition, (**64**.)

18. Add 3 yr. + $\frac{3}{7}$ da. + $\frac{4}{9}$ h. + $\frac{5}{12}$ min. + $\frac{7}{12}$ sec.
19. Add $\frac{5}{12}$ bu. + $\frac{3}{10}$ pk. + $\frac{4}{5}$ qt.
20. Add 18.9 gal. + .45 gal. + .17 qt. + .416 pt.
21. Add $\frac{3}{8}$ yd. + $\frac{4}{9}$ qr. + $\frac{5}{12}$ na.
22. Add $\frac{3}{10}$ cwt. + $\frac{4}{5}$ qr. + $\frac{3}{8}$ lb. + $\frac{4}{5}$ oz.
II. 23. Add $\frac{3}{7}$ mile + $\frac{4}{5}$ rd. + $\frac{8}{15}$ ft.

24. Bought 4 house-lots; the first contained $\frac{3}{11}$ of an acre, the second $\frac{4}{5}$ of an acre, the third .275 of an acre, and the fourth .8706 of an acre. How much land was there in the whole?

90. SUBTRACTION OF COMMON FRACTIONS.

RULE 1. *Prepare the fractions as in addition: subtract the numerator of the subtrahend from that of the minuend, and write the remainder over the common denominator.*

1. How much is $\frac{7}{9} - \frac{2}{9}$? $\frac{8}{11} - \frac{5}{11}$? $\frac{18}{25} - \frac{11}{25}$? $\frac{45}{62} - \frac{17}{62}$? $\frac{39}{80} - \frac{25}{80}$? $\frac{875}{1000} - \frac{349}{1000}$? $\frac{75}{94} - \frac{18}{94}$? $5\frac{7}{15} - 2\frac{4}{15}$? $16\frac{13}{20} - 8\frac{5}{20}$?
2. How much is $7\frac{3}{8} - 3\frac{5}{8}$?

RULE 2. *If the fractional part of the minuend is less than that of the subtrahend, add to the fractional part of the minuend a unit reduced to the same denominator; then subtract and add a unit to the subtrahend.* (21.)

$$\begin{aligned} \text{Thus, } 7\tfrac{3}{8} + \tfrac{8}{8} &= 7\tfrac{11}{8}; \\ 3\tfrac{5}{8} + 1 &= \underline{4\tfrac{5}{8}}; \\ &\quad 3\tfrac{6}{8} = 3\tfrac{3}{4} \text{ remainder.} \end{aligned}$$

3. How much is $8\frac{3}{11} - 5\frac{7}{11}$? $8\frac{4}{9} - 6\frac{7}{9}$? $15\frac{4}{15} - 4\frac{11}{15}$? $4\frac{18}{25} - 1\frac{23}{25}$? $16\frac{5}{13} - 8\frac{10}{13}$?
4. Subtract $3\frac{5}{8}$ from 10. *Solution.* $10 + \frac{8}{8} = 10\frac{8}{8}$; $3\frac{5}{8} + 1 = 4\frac{5}{8}$; $10\frac{8}{8} - 4\frac{5}{8} = 6\frac{3}{8}$.
5. From 25 subtract $5\frac{7}{8}$; $3\frac{4}{9}$; $15\frac{3}{7}$; $20\frac{3}{4}$; $18\frac{1}{2}$; $22\frac{5}{16}$; $12\frac{4}{25}$; $16\frac{1}{10}$.
6. How much is $\frac{3}{4} - \frac{1}{3}$? *Solution.* $\frac{3}{4} = \frac{9}{12}$; $\frac{1}{3} = \frac{4}{12}$; $\frac{9}{12} - \frac{4}{12} = \frac{5}{12}$.
7. How much is $\frac{2}{3} - \frac{3}{5}$? $\frac{4}{5} - \frac{1}{2}$? $\frac{6}{8} - \frac{5}{9}$? $\frac{3}{4}$ of $\frac{5}{6} - \frac{1}{12}$? $\frac{3}{8}$ of $\frac{4}{9} - \frac{1}{9}$ of $\frac{5}{8}$?
8. How much is $4\frac{3}{8} - 1\frac{3}{4}$? $16\frac{3}{4} - 11\frac{5}{8}$? $25\frac{5}{12} - 16\frac{1}{6}$? $25\frac{1}{6} - 16\frac{5}{12}$? $8\frac{4}{15} - 5\frac{3}{5}$? $8\frac{3}{5} - 5\frac{4}{15}$?

9. How much is $\frac{5}{8} - \frac{4}{5}$ of $\frac{8}{15}$? $5\frac{8}{19} - 2\frac{1}{15}$?
10. How much is $3\frac{1}{7} - \frac{4}{25}$? $348\frac{5}{24} - 167\frac{3}{74}$?

11. Subtract $\frac{5}{8}$ of $\frac{3}{7}$ of $\frac{8}{15}$ from $\frac{6}{7}$ of $\frac{14}{25}$ of $\frac{5}{8}$.
12. Subtract $\frac{3}{8}$ of $\frac{6}{17}$ of $8\frac{1}{2}$ from $\frac{3}{7}$ of $28\frac{5}{19}$.

13. What is the difference between $\frac{3}{11}$ of a pound and $\frac{3}{7}$ of a shilling?

14. Subtract .13d. from £.17; .354 lb. from .257 cwt.
15. From 100 subtract $25\frac{9}{21}$; $18\frac{11}{15}$; $74\frac{9}{13}$.
16. Subtract $\frac{5}{7}$ of a yd. from $\frac{6}{7}$ of a mile.
17. Subtract $\frac{4}{11}$ of a min. from $\frac{4}{11}$ of a wk.

MULTIPLICATION OF FRACTIONS.

91. The learner has already had, in the preceding articles, some exercises in multiplication of fractions. The following GENERAL RULE will apply to all possible cases.

Reduce the multiplier and multiplicand to the form of common fractions, and cancel like factors in the numerators and denominators; then multiply the numerators together for the numerator of the product, and the denominators together for the denominator of the product.

1. Multiply $\frac{3}{8}$ by $\frac{4}{9}$. *Solution.* $\dfrac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{8}}} \times \dfrac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{9}}} = \frac{1}{6}$ *Ans.*

2. Multiply $3\frac{1}{8}$ by $\frac{6}{7}$.

Solution. $3\frac{1}{8} = \frac{25}{8}$; $4\frac{6}{7} = \frac{34}{7}$; $\dfrac{25}{\underset{4}{\cancel{8}}} \times \dfrac{\overset{17}{\cancel{34}}}{7} = \frac{425}{28} = 15\frac{5}{28}$ *Ans.*

3. Multiply $\frac{5}{8}$ by $\frac{1}{7}$; $\frac{3}{16}$ by $\frac{8}{9}$.
4. Multiply $\frac{4}{7}$ of $\frac{3}{8}$ by $\frac{5}{9}$ of $3\frac{3}{7}$; $5\frac{3}{8}$ by $6\frac{2}{5}$.
5. Multiply 5 by $\frac{3}{11}$; 17 by $\frac{3}{8}$.
6. How much is $46 \times \frac{24}{25}$? $25\frac{4}{17} \times 15\frac{3}{16}$? $35\frac{1}{4} \times \frac{3}{8}$ of $27\frac{1}{3}$?
7. What cost $\frac{7}{8}$ of a bushel of corn, at $\frac{7}{8}$ of a dollar per bushel?
8. What will $3\frac{5}{8}$ lb. of butter cost, at $15\frac{3}{8}$ cts. per lb.?
9. At \$$10\frac{4}{7}$ per barrel, what will $15\frac{3}{8}$ barrels of beef come to?
10. If 8 bushels of potatoes cost \$$4\frac{3}{7}$, what will 15 bushels cost?

NOTE. Fifteen bushels will cost $\frac{15}{8}$ as much as 8 bushels.

11. Multiply $\frac{3}{8}$ of $\frac{4}{9}$ of $6\frac{3}{8}$ by $\frac{3}{4}$ of $\frac{8}{17}$ of $9\frac{3}{4}$.

NOTE. Cancel before performing the multiplication.

12. How much is $4\frac{3}{8} \times 5\frac{3}{7} \times 18\frac{7}{12} \times 3\frac{5}{7}$?
13. How much is $5\frac{9}{11} \times \frac{6}{25} \times 3.8 \times .05$?
14. If $\frac{3}{8}$ of $\frac{4}{9}$ of $\frac{8}{11}$ of an acre of land is worth \$50, how much can be bought for \$15.75?
15. How much is $\frac{3}{4}$ of $\frac{5}{8}$ of 3.07×15.008?

92. DIVISION OF FRACTIONS. COMPLEX FRACTIONS.

Divide 8 by $\frac{1}{5}$. 1 is contained in 8 eight times; $\frac{1}{5}$ being $\frac{1}{5}$ as large, must be contained in 8, 5 times as many times as 1, or 5 times 8 times. Therefore, dividing by $\frac{1}{5}$ is the same as multiplying by $\frac{5}{1}$.

Again. Divide $\frac{3}{8}$ by $\frac{4}{5}$. $\frac{3}{8}$ divided by 4 is $\frac{1}{4}$ of $\frac{3}{8}$, or $\frac{3}{32}$; but if $\frac{3}{8}$ be divided by $\frac{1}{5}$ of 4, or $\frac{4}{5}$, the quotient must be 5 times as large, or $\frac{5}{4}$ of $\frac{3}{8} = \frac{15}{32}$. Hence the following

GENERAL RULE. *Reduce the dividend and divisor to the form of common fractions, then* INVERT THE DIVISOR, *and proceed as in multiplication of fractions.*

1. Divide $\frac{4}{5}$ by $\frac{8}{9}$. *Solution.* $\frac{\overset{1}{\cancel{4}}}{5} \times \frac{9}{\underset{2}{\cancel{8}}} = \frac{9}{10}$.

2. Divide $3\frac{7}{8}$ by $5\frac{1}{4}$. *Solution.* $\frac{31}{8} \times \frac{4}{21} = \frac{31}{42}$.

3. Divide $\frac{3}{8}$ by $\frac{5}{7}$; $\frac{4}{9}$ by $\frac{6}{15}$; $1\frac{4}{5}$ by $8\frac{4}{5}$.

4. Divide $9\frac{3}{16}$ by $3\frac{4}{7}$; $41\frac{3}{8}$ by $\frac{1}{7}$; $\frac{3}{4}$ of $\frac{6}{7}$ of $\frac{5}{8}$ by $\frac{3}{16}$ of $4\frac{2}{5}$.

5. How much is $34\frac{5}{7} \div 16\frac{3}{8}$? $16 \div 1\frac{3}{7}$? $6\frac{1}{8} \div 9$?

6. How much is $3\frac{1}{8} \div 4\frac{9}{16}$? $\frac{5}{16} \div \frac{3}{25}$? $\frac{4}{7}$ of $25 \div \frac{3}{8}$ of $17\frac{1}{3}$?

7. How much is $\frac{4\frac{5}{8}}{3\frac{5}{7}}$?

A fraction that has a fraction for its numerator or denominator is called a *Complex fraction.*

Complex fractions are reduced to simple fractions by performing the division expressed by them. (76.)

Reduce the following complex fractions to simple ones:

8. $\frac{\frac{3}{5}}{\frac{5}{7}}$; $\frac{\frac{8}{9}}{\frac{2}{3}}$; $\frac{4\frac{5}{7}}{3\frac{1}{8}}$. (9.) $\frac{4}{3\frac{1}{7}}$; $\frac{5}{\frac{8}{9}}$; $\frac{\frac{3}{4}}{5}$; $\frac{8}{\frac{15}{7}}$.

10. Reduce $\frac{\frac{4}{9}}{7\frac{3}{4}}$; $\frac{5\frac{1}{7}}{13\frac{1}{2}}$; $\frac{5}{23\frac{1}{8}}$; $\frac{6\frac{1}{15}}{4}$.

11. Divide $\frac{7}{8}$ of $\frac{5\frac{1}{2}}{7\frac{1}{2}}$ by $\frac{4}{9}$ of $\frac{15}{14\frac{7}{8}}$.

93. PARTICULAR RULE. *To divide a whole number by a mixed number, reduce both the divisor and dividend to the denomination denoted by the fractional part of the divisor, and then divide as in whole numbers; the remainder, if any, will be of the same denomination as that to which the dividend was reduced.*

1. In 34572 yards how many rods?

$5\frac{1}{2}$	34572
2	2
11	69144
	6285 rd. 4 yd. 1 ft. 6 in.

First reduce the divisor and dividend to halves; the quotient will be whole numbers, (**74**,) or rods. The remainder, 9, is half yards = 4 yd. 1 ft. 6 in.

2. In 381770 feet how many rods? Divide by $16\frac{1}{2}$.
3. In 5847 square yards how many square rods?
4. In 5146 inches how many nails?
5. In 814768 days how many years?

NOTE. $365\frac{1}{4}$ days = 1 yr.

6. In 81641807 square feet how many square rods?
$272\frac{1}{4}$ sq. ft. = 1 sq. rod.

7. Divide 1400700 by $4\frac{1}{4}$; by $8\frac{7}{8}$; by $358\frac{15}{16}$; by $2758\frac{8}{15}$.

To divide a mixed number by a whole number, see Arts. **79, 80, 81.**

94. MISCELLANEOUS EXERCISES IN FRACTIONS.

1. Reduce to their lowest terms $\frac{28}{54}$; $\frac{60}{72}$; $\frac{80}{96}$; $\frac{75}{80}$; $\frac{33}{55}$.

2. Express in common fractions and reduce to their lowest terms .6; .8; .04; .25; .75; .28; .85; .002; .405; .816; .15; .45.

3. Reduce these improper fractions to whole or mixed numbers: $\frac{3}{2}$; $\frac{9}{4}$; $\frac{25}{6}$; $\frac{35}{4}$; $\frac{48}{7}$; $\frac{67}{8}$; $\frac{49}{15}$; $\frac{87}{12}$; $\frac{108}{9}$; $\frac{208}{8}$; $\frac{516}{7}$.

4. Change to decimals $\frac{1}{2}$; $\frac{3}{4}$; $\frac{2}{5}$; $\frac{3}{5}$; $\frac{7}{20}$; $\frac{9}{25}$; $\frac{28}{50}$; $\frac{13}{20}$; $\frac{17}{25}$; $\frac{46}{50}$.

5. Express in a fractional form $4\frac{7}{8}$; $9\frac{3}{7}$; $14\frac{2}{5}$; $75\frac{1}{8}$; $116\frac{3}{7}$; $584\frac{4}{5}$; $1016\frac{2}{3}$; 4; 8; 25; 10085; 4.5; 16.3; 5.16; 80.101; 30.106; 4.54.

6. Reduce $\frac{3}{8}$ to 5ths; $\frac{5}{7}$ to 9ths; $\frac{12}{5}$ to 8ths; $\frac{3}{8}$ to 7ths; $\frac{2}{10}$ to 25ths.

7. Change $\frac{3}{5}$ to 10ths; to 25ths; $\frac{7}{8}$ to 24ths; $\frac{5}{9}$ to 36ths; $\frac{5}{12}$ to 48ths.

8. Add $5\frac{3}{4} + 6\frac{3}{8} + 4\frac{1}{2} + 6\frac{1}{4}$; $3\frac{5}{6} + 2\frac{2}{3} + 1\frac{5}{6} + 2\frac{5}{12}$.

9. What is the value, in lower denominations, of £$\frac{3}{4}$? £$\frac{5}{8}$? $\frac{3}{4}$s.? $\frac{3}{4}$ lb. Avoirdupois? $\frac{3}{4}$ qr.? $\frac{5}{12}$ oz.?

10. What is the sum of $\frac{3}{8}$ bu. $+ \frac{3}{4}$ pk. $+ \frac{2}{5}$ qt.?

11. Add $5\frac{3}{4}$, $6\frac{1}{8}$ and $5\frac{7}{12}$. (12.) Add $3\frac{2}{3} + 6\frac{4}{5} + 4\frac{3}{15} + 8\frac{3}{10}$.

13. Change to integers or mixed numbers $\frac{352}{15}$; $\frac{486}{25}$; $\frac{1984}{37}$; $\frac{10484}{18}$.

14. Change to integers and decimals $\frac{84}{7}$; $\frac{1680}{74}$; $\frac{510401}{16}$; $\frac{40169}{257}$.

15. What are the sum and difference of $3\frac{5}{8}$ and $6\frac{11}{12}$?

16. Of $15\frac{1}{18}$ and $24\frac{5}{24}$? (17.) Of $\frac{3}{8}$ of $5\frac{1}{2}$ and $\frac{4}{7}$ of $18\frac{7}{8}$?

18. Of $\frac{2}{5}$ of $\frac{7}{8}$ and $\frac{5}{18}$ of $\frac{27}{35}$?

19. What is the product of $9\frac{3}{4} \times 8\frac{2}{5}$? Of $15\frac{4}{7} \times 9\frac{16}{37}$? Of $3\frac{4}{45} \times 15\frac{4}{5}$?

20. Add $\frac{3}{7}$ cwt., $\frac{4}{9}$ lb., $\frac{3}{5}$ oz. and $\frac{5}{8}$ dr.

21. Add $15\frac{3}{8}$ gal., $3\frac{3}{5}$ qt. and $\frac{5}{8}$ pt.

22. What is the quotient of $9\frac{5}{8} \div 3\frac{8}{9}$? Of $6\frac{5}{8} \div 15\frac{4}{7}$?

23. Divide $41\frac{6}{7}$ by $53\frac{1}{9}$; $\frac{3}{8}$ of $\frac{5}{9}$ by $4\frac{7}{8}$.

24. Divide $\dfrac{3}{4\frac{1}{8}}$ by $\dfrac{4\frac{1}{8}}{3}$; $\dfrac{17\frac{1}{2}}{5\frac{1}{3}}$ by $\dfrac{8\frac{7}{8}}{23\frac{1}{4}}$.

25. Multiply $\dfrac{3\frac{1}{2}}{\frac{7}{8}}$ by $\dfrac{4\frac{1}{4}}{1\frac{2}{9}}$; $\dfrac{8\frac{3}{4}}{\frac{1}{2}}$ by $\dfrac{\frac{3}{7}}{5\frac{1}{4}}$.

26. Add $4\frac{5}{8} + \frac{3}{10} + 5\frac{7}{12} + \frac{3}{5}$ of $\frac{10}{21} + \dfrac{\frac{5}{8}}{7\frac{1}{2}}$.

27. Add $3\frac{5}{6} + \dfrac{5\frac{1}{2}}{5} + \dfrac{28\frac{1}{2}}{2\frac{1}{2}} + \frac{3}{8}$ of $12\frac{3}{16}$.

28. What cost 204 gal. 2 qt. of molasses, at \$0.23$\frac{3}{14}$ per gal.? (**86.**)

29. What is the value of 1 lb. 3 oz. 15 dwt. of silver, at \$12.50 per lb.?

30. What cost 4 cords 1 C. ft. 3 cu. ft. of wood, at \$4$\frac{7}{8}$ per cord?

31. What will 6 A. 1 R. 7 sq. rods of land come to, at \$$75\frac{3}{8}$ per acre?

32. What will 3 yds. 3 qrs. 2 na. of cloth cost, at £1 2s. 6d. = £$1\frac{1}{8}$ per yard?

NOTE. The pupil, after performing the last five questions by common fractions, should solve them by decimals, (86 and 87.)

33. Same as 28. $\$14.625 \times 3.246 = \$47.473$, *Ans.*

34, 35, 36, 37 — See 29, 30, 31, 32.

The remaining questions of this article may be solved by both methods.

38. How many square feet in a board 3 ft. 9 in. long, and 2 ft. 6 in. wide?

Solution. $3\frac{3}{4} \times 2\frac{1}{2} = 9\frac{3}{8}$ sq. ft. — $3.75 \times 2.5 = 9.375$ sq. ft.

39. What is a board 25 ft. 3 in. long, and 1 ft. 8 in. wide, worth, at $2\frac{1}{2}$ cts. per sq. ft.?

Solution. $\frac{101}{4} \times \frac{\overset{1}{\cancel{5}}}{3} \times \frac{1}{\underset{8}{\cancel{40}}} =$; $25.25 \times 1.667 \times .025 =$

40. How many sq. ft. are there in 1 yd. of carpeting that is 3 ft. 3 in. wide?

41. How many yards of such carpeting will it take to cover a floor 15 ft. 6 in. long and 12 ft. 3 in. wide? What will it cost, at \$$0.87\frac{1}{2}$ per yd.?

II. 42. How many sq. ft. in a roll of paper hangings which is $1\frac{5}{12}$ ft. wide and 27 ft. long? How many such rolls will it take to paper the above room, the walls being $8\frac{1}{2}$ ft. high, after deducting 140 sq. ft. for the doors and windows?

43. How many times will a wheel which is $15\frac{3}{8}$ ft. in circumference turn round in going $15\frac{3}{8}$ ft.? In going $40\frac{5}{8}$ miles?

44. How many bottles, each containing $1\frac{3}{4}$ pt., can be filled from $43\frac{5}{7}$ gallons of wine?

45. How many bushels of potatoes at \$$\frac{5}{8}$ per bushel will pay for $75\frac{1}{2}$ gallons of molasses at \$$\frac{3}{8}$ per gallon?

46. How many acres of land in a rectangular field $45\frac{3}{8}$ rods long and $32\frac{3}{4}$ rods wide? What will it come to, at \$25 per acre?

47. If $\frac{5}{8}$ of $25\frac{1}{2}$ bushels of flax seed are worth $\$10\frac{3}{8}$, what is 1 bushel worth? $7\frac{5}{8}$ bushels?

48. Bought flour at $\$5\frac{3}{8}$ per bbl.; at what price per 7 lbs. must I sell it to gain $\$\frac{1}{2}$ per bbl.?

NOTE. A barrel of flour contains 196 lbs.

49. How many solid feet in a box 8 ft. 3 in. long, 3 ft. 4 in. wide, and 2 ft. 9 in. high? How many bushels of corn will it hold, if 1 cubic foot will hold $\frac{45}{56}$ of a bushel?

50. How many cubic feet in a pile of wood 18 ft. 6 in. long, 4 ft. 3 in. wide, and 3 ft. 9 in. high? How many cords? What will it come to, at $\$4\frac{5}{8}$ per cord?

$$\frac{37}{2} \times \frac{17}{4} \times \frac{15}{4} \times \frac{1}{128} \times \frac{37}{8} =$$

51. How many acres in a piece of land that is 31 rd. $2\frac{1}{2}$ yd. long, and 28 rd. 5 yd. 1 ft. wide?

NOTE. Reduce the dimensions to feet, and get the number of sq. ft. in the area; then reduce the sq. ft. to yards, rods, roods, acres. *Ans.*, 248082 sq. ft. = 5 A. 2 R. 31 rd. 6.25 sq. yd. 6 ft. = 5 A. 2 R. 31 sq. rd. 6 sq. yd. 8 sq. ft. 36 sq. in.

52. How many acres in a field 7 rd. $4\frac{3}{4}$ yd. wide, and 57 rd. 5 yd. .5 ft. long?

53. Reduce 16232 feet to yards, rods, furlongs, miles.

54. Reduce 607858 inches to miles, &c.

55. Multiply 2 A. 3 R. 15 sq. rd. 7 sq. yd. 5 sq. ft. by 8; by 12.

56. Reduce $\frac{4}{7}$ of an acre to lower denominations.

57. Multiply 3 m. 7 fur. 8 rd. 3 yd. 2 ft. 9 in. by 16; by 24.

58. Multiply 6 A. 3 R. 14 sq. rd. 25 sq. yd. 4 sq. ft. 109 sq. in. by 8; by 24.

59. Reduce .27 rd. to lower denominations.

60. Reduce 415.4766 sq. ft. to sq. rds. and lower denominations.

61. Reduce 113.0976 sq. ft. to sq. yards and lower denominations.

62. Reduce 852.0866 sq. rods to acres and lower denominations.

63. Reduce 1841.3751 cu. ft. to cubic yards and lower denominations.

QUESTIONS. What is a *Fraction?* How are fractions *divided?* How are *common* fractions expressed? What is the *denominator?* — the *numerator?* What does each show? Give an example. What do fractions always express? What is meant by the *value* of a fraction?

What are the *terms* of a fraction? What if both of the terms of a fraction be multiplied or divided by the same number? Why? Give an example. When is a fraction in its lowest terms? Repeat the rule for reducing fractions to their lowest terms. Give an example.

What is a *proper* fraction? — an *improper* fraction? What is said of the *value* of proper and improper fractions? Give examples. What is a *mixed* number? What should be done with the results of all operations in fractions? Repeat the rule for reducing improper fractions to whole or mixed numbers — for reducing mixed numbers to improper fractions — for reducing common fractions to decimals. How are decimals reduced to the form of common fractions? *Ans. By writing the given decimal as a numerator, and* 1 *with as many naughts as there are places in the decimal for a denominator.* How are integers written in a fractional form?

How is a fraction affected by *multiplying* the *denominator?* How by *dividing* the *denominator?* Why? Give an example.

How is a fraction affected by *multiplying* the *numerator?* How by *dividing* the *numerator?* Why? Give an example.

In how many ways can you *multiply* a fraction? In what ways can you divide a fraction?

What is a *compound* fraction? What is the rule for reducing *compound* fractions to *simple* ones?

What are *repeating* or *circulating* decimals? What are repetends? How are they expressed? Give examples. How are repeating decimals changed to common fractions?

What is the rule for reducing an integer or fraction to a fraction having a given denominator? When have fractions a *common denominator?* What is the *least common denominator* of several fractions? Repeat the rule for reducing fractions to their lowest terms. Give an example.

How are fractions of one denomination reduced to the fraction of another? Give examples. How may fractional compound numbers be reduced to their values in lower denominations? Give examples.

Repeat the rule for the *addition* of fractions. What is the rule for the *subtraction* of fractions? The rule, if the fractional part of the minuend is less than that of the subtrahend? Give an example.

What is the rule for the *multiplication* of fractions? Give examples. Repeat the rule for the *division* of fractions. What are *complex* fractions? How are they reduced to simple fractions? How may you divide by a mixed number?

95. ANALYSIS. (See 47.)

The pupil may perform the questions in this Art., and similar questions, as he did those in Art. 47; or, by a shorter method, though perhaps less analytical, he may only *express* the value of the unit, and then *express* the value of the whole; and after cancelling equal factors in the dividend and divisor, *perform* the operation.

1. If 4 lb. of veal cost 28 cts., what will 8 lb. cost?

Solution. Since 4 lb. cost 28 cts., 1 lb. will cost $\frac{1}{4}$ of 28 cts. If 1 lb. costs $\frac{1}{4}$ of 28 cts., 8 lb. will cost $\frac{8}{4}$ or $\frac{2}{1}$ of 28 cts. $\frac{2}{1}$ of 28 cts. $=$ 56 cts.

2. A man bought a cow, and paid \$25 cash, which was $\frac{5}{8}$ of the price of the cow. What did the cow cost him?

If 25 is $\frac{5}{8}$ of some number, $\frac{1}{8}$ of that number will be $\frac{1}{5}$ of 25, and the whole number will be $\frac{8}{5}$ of 25.

Obs. Let the pupil review Art. 47, analyzing the questions in this way.

3. If a family consume $\frac{4}{7}$ of a bbl. of flour in 8 weeks, how long will a barrel last them?

4. A merchant bought a quantity of butter, and paid $\frac{3}{4}$ of it, which was \$33, in goods. How much did the butter cost?

5. 36 is $\frac{4}{5}$ of what number? 48 is $\frac{3}{5}$ of what number? $\frac{4}{9}$ of what? $\frac{6}{7}$ of what? $\frac{8}{3}$ of what? $\frac{12}{15}$ of what? $\frac{16}{9}$ of what?

6. 25 is $\frac{3}{4}$ of what number? $\frac{5}{6}$ of what? $\frac{8}{9}$ of what? $\frac{5}{8}$ of what?

7. 36 is $\frac{5}{8}$ of what? $\frac{6}{7}$ of what? $\frac{7}{8}$ of what? $\frac{10}{3}$ of what? $\frac{8}{5}$ of what?

8. If $2\frac{1}{2}$ pounds of coffee cost 25 cts., what is 1 lb. worth?

NOTE. If $\frac{5}{2}$ of 1 lb. cost 25 cts., $\frac{1}{2}$ lb. will cost $\frac{1}{5}$ as much. If $\frac{1}{2}$ lb. cost $\frac{1}{5}$ of 25 cts., 1 lb. would cost $\frac{2}{5}$ of 25 cts., or 10 cts.

9. If $3\frac{1}{4}$ pounds of raisins cost 26 cts., what is 1 lb. worth?

NOTE. One lb. is worth $\frac{4}{13}$ as much as $3\frac{1}{4}$ lbs. Why?

10. If $\frac{3}{8}$ of a bushel of corn is worth 24 cts., what is 1 bushel worth? 1 bushel is worth $\frac{8}{3}$ as much as $\frac{3}{8}$ of a bushel. Why?

11. If $4\frac{1}{4}$ yards of cloth cost \$34, what is 1 yd. worth? 3 yards? $3\frac{1}{4}$ yards?

NOTE. $3\frac{1}{4}$ yards will cost $\frac{13}{4}$ of $\frac{4}{17}$ of \$34. Why?

12. If $4\frac{1}{6}$ cords of wood cost \$25, what are $2\frac{1}{2}$ cords worth? $4\frac{1}{4}$ cords? $10\frac{3}{4}$ cords?

13. If $3\frac{3}{4}$ barrels of flour cost \$25, what is 1 bbl. worth? 4 barrels? $7\frac{1}{2}$ barrels?

Solution. $\frac{15}{2}$ of $\frac{4}{15}$ of \$25 = *Ans.* Why?

14. $\frac{4}{9}$ of 36 are $\frac{8}{11}$ of what number? $\frac{15}{4}$ of 24 are $\frac{9}{13}$ of what number?

15. $\frac{3}{4}$ of 13 are $\frac{1}{2}$ of what number? $\frac{3}{8}$ of 21 are $\frac{2}{3}$ of what number? $\frac{5}{9}$ of 12 are how many times 4?

16. $2\frac{2}{5}$ times 15 are how many times 8?

17. If 6 barrels of flour cost \$32, what will 15 barrels cost?

NOTE. 15 barrels will cost $\frac{15}{6}$ as much as 6 barrels.

$$\frac{\overset{5}{\cancel{15}}}{\underset{1}{\cancel{6}}} \text{ of } \frac{\overset{16}{\cancel{32}}}{1} = 80.$$

18. If 17 barrels of flour cost \$60.35, what will 10 barrels cost? 19 barrels? 25 barrels? $30\frac{1}{4}$ barrels.

19. If a cistern discharge 35 gallons of water in 24 minutes, in what time will it discharge 18 gallons?

20. If 10 men do a piece of work in 8 days, how long will it take 8 men to do it? 12 men? 23 men? 25 men? How many men will do it in 1 day? 5 days? 10 days? 16 days?

21. If by working 10 hours a day it would take 5 days to do a certain piece of work, how many days would it take at 1 hour a day? 10 hours? 6 hours? $12\frac{1}{2}$ hours?

22. If $\frac{7}{9}$ of a bushel of oats cost 35 cents, what will $3\frac{7}{8}$ bushels cost?

NOTE. 1 bushel will cost $\frac{9}{7}$ of 35 cts., and $3\frac{7}{8}$ bu. will cost $\frac{31}{8}$ of $\frac{9}{7}$ of 35 cts. Why? *Ans.*, \$1.743$\frac{3}{4}$.

23. If $\frac{4}{5}$ of a ton of coal cost $\$4\frac{4}{7}$, what will $\frac{5}{8}$ of a ton cost?

24. If 6 yds. of cloth cost $\$5\frac{1}{4}$, what will $6\frac{3}{7}$ yds. cost?

25. If 8 cords of wood cost $\$25\frac{3}{7}$, what cost $15\frac{7}{8}$ cords? $25\frac{1}{8}$ cords?

26. If $\frac{3}{16}$ of a tub of butter cost $\$5\frac{3}{4}$, what cost 3 tubs? $8\frac{3}{4}$ tubs?

27. If $\$20\frac{5}{8}$ furnish provision for 8 men 3 weeks, how much

will supply them for $7\frac{1}{2}$ weeks? $10\frac{5}{7}$ weeks? How long will $\$\frac{1}{8}$ furnish them? \$1? $\$30\frac{1}{4}$?

NOTE. $\$30\frac{1}{4}$ will furnish them $\frac{121}{4}$ of $\frac{8}{165}$ of 3 weeks. Why?

28. If $\$4\frac{1}{6}$ pay for building 5 rods of wall, how many rods will $\$15\frac{1}{8}$ pay for?

29. How many bushels of potatoes at \$.45 a bushel must be paid for 35 bushels of corn at 65 cts. a bushel?

$\frac{65}{45}$ of one bu. of potatoes for 1 bu. of corn. Why?

30. How much tea at 40 cts. a lb. will pay for 15 bushels of apples at 75 cts. per bushel? At 80 cts. per bushel? At $\$1.12\frac{1}{2}$ per bushel?

31. How much cotton cloth at $\$\frac{3}{16}$ per yard will come to as much as $115\frac{1}{2}$ lb. of pork at $\$\frac{2}{25}$ per lb.?

32. If a post $6\frac{1}{2}$ ft. high casts a shadow $7\frac{1}{2}$ ft. long, how high is the spire which at the same time casts a shadow 140 ft.? 127 ft.? $108\frac{1}{8}$ ft.?

33. How many men in 12 days will do the work that 8 men will do in 18 days? How many will do it in 4 da.? 8 da.? 24 da.? 36 da.?

34. A boy having 72 cts., has spent $\frac{3}{8}$ of it; what part of his money remains? How many cents? If he had spent $\frac{1}{4}$ and $\frac{3}{8}$ and $\frac{2}{9}$ of it, how many cents would remain?

35. A man having spent $\frac{1}{8}$ of his money for coffee, $\frac{5}{12}$ of it for sugar, $\frac{1}{6}$ of it for rice, and $\frac{5}{24}$ of it for molasses, has 20 cts. left. How much had he at first? How much did he pay for each article?

36. In a certain school $\frac{3}{10}$ of the scholars are 8 years old, $\frac{5}{6}$ as many are 9 years old, $\frac{8}{9}$ as many are 10 years old, and the remaining 22 are 11 years old. How many scholars belong to the school?

37. $\frac{3}{8}$ and $\frac{2}{7}$ and $\frac{3}{4}$ of a certain number are equal to 79. What is the number?

38. In an orchard $\frac{3}{8}$ of the trees are apple trees, $\frac{4}{9}$ as many bear cherries, $\frac{1}{4}$ as many quince trees as cherry trees, 5 times as many peach trees as quince trees, and the remaining 30 are pear trees. How many trees are there in the orchard? How many of each kind?

SECTION IX.—PROBLEMS.

96. The sum and difference of two numbers being given, to find the numbers.

RULE. *Add half the difference to half the sum for the greater number, and subtract half the difference from half the sum for the smaller.*

1. The sum of two numbers is 40, and their difference 10; what are the numbers?

2. What two numbers are they whose sum and difference are 65 and 35? 40 and 14? 8 and 1? 13 and $3\frac{1}{4}$? $21\frac{1}{4}$ and $8\frac{1}{4}$?

97. The product of two factors and one of the factors given, to find the other factor?

RULE. *Divide the given product by the given factor; the quotient will be the other factor.*

1. The product of two factors is 75; one of the factors is 15, what is the other?

2. 44 is the product of two factors, one of which is 8; what is the other? If one factor be 12, what will be the other?

3. $40\frac{1}{2}$ is the product of two factors; if one of them is 7, what will the other be? If one be 9, what is the other?

4. 754 is a dividend, and 8 the quotient; what is the divisor? (**33.**)

5. $8+12$ is the product of two factors, one of which is 2; what is the other? *Ans.*, $4+6$; because $(4+6)\times 2=8+12$.

NOTE. A parenthesis ( ) or bracket [ ], enclosing 2 or more numbers, or a line called a vinculum drawn over them, as $(4+6)$, or $[4+6]$, or $\overline{4+6}$, implies that the total result of these numbers is to be considered as one quantity. Thus, $(4+6)\times 2$ implies that both the 4 and the 6 are to be multiplied by 2; and $(6+3+9)\div 3$ implies that each of these quantities, or their sum, is to be divided by 3.

6. $8+12$ is the product of two factors, one of which is 4; what is the other factor? *Ans.*, $2+3$; because $(2+3)\times 4=8+12$.

II. 7. $(2\times 6)+\overline{7\times 6}$ is the product of two factors, one of which is 6; what is the other factor? *Ans.*, $2+7$; for, $(2+7)\times 6=\overline{2\times 6}+\overline{7\times 6}$.

8. $25+35+125$ is the product of two factors, one of which is 5; what is the other factor?

9. $5^2+\overline{7\times 5}+5^3$ is the product of two factors, one of which is 5; what is the other factor?

10. $(4^2\times 3\times 7)+(4\times 3\times 7^2)+7^3$ is the product of two factors, one of which is 7; what is the other factor?

98. The product of three factors and two of the factors being given, to find the other factor.

RULE. *Divide the given product by the product of the two given factors; the quotient will be the third factor.*

1. 120 is the product of 3 factors, two of which are 3 and 5; what is the other?

2. 127 is the product of 3 factors, two of which are 4 and 6; what is the other? What is the 3d factor, if the given factors are 3 and 5? 5 and 7? 15 and 3?

MISCELLANEOUS EXAMPLES IN THE FOREGOING PROBLEMS.

99. 1. If a piece of board is 12 inches wide, how long must it be to contain 1 sq. ft.? How long must it be, if it is 6 inches wide? 4 in.? 1 in.? 8 in.? 18 in.?

2. The floor of a room, 15 feet long, contains 180 sq. ft.; how wide is it?

3. If a board is $2\frac{1}{2}$ feet wide, how long must it be to contain 30 sq. ft.? ($\frac{60}{2} \div \frac{5}{2} = 12$.) 12 sq. ft.? 18 sq. ft.? $48\frac{1}{2}$ sq. ft.?

4. How long must a piece of land be, that is 10 rods wide, to contain an acre? 20 rods wide? 40 rods wide? 4 rods? 2 rods? 1 rod?

5. How much cloth $\frac{1}{2}$ yard wide will it take to line one yard that is yard wide? How much will it take that is $\frac{1}{4}$ yd. wide? $\frac{3}{4}$ yd. wide? $\frac{3}{8}$ yd. wide? $1\frac{1}{2}$ yd. wide?

6. How much cloth that is $\frac{3}{8}$ wide will it take to line a cloak containing 9 yards that is $\frac{7}{8}$ wide? $1\frac{1}{4}$ yd. wide? $\frac{3}{4}$ yd. wide?

7. If a board is $18\frac{1}{2}$ inches wide, how long must it be to contain $3\frac{1}{2}$ sq. ft.? $5\frac{1}{2}$ sq. ft.? $28\frac{3}{4}$ sq. ft.?

8. How long must a piece of land be to contain $2\frac{1}{2}$ acres, if it is 12 rods wide? $15\frac{3}{4}$ rods wide? $18\frac{1}{8}$ rods wide?

9. $85\frac{3}{4}$ is a dividend, the quotient is $45\frac{1}{8}$; what is the divisor?

10. The sum of two numbers is $48\frac{1}{8}$, their difference is $22\frac{1}{4}$; what are the numbers?

11. What numbers are they whose sum and difference are $85\frac{7}{8}$ and $33\frac{3}{4}$? $48\frac{5}{6}$ and $17\frac{5}{14}$?

12. A box, the content of which is 160 cubic feet, is 8 feet long and 5 feet wide; how deep is it?

13. A pile of wood containing $5\frac{1}{2}$ cords is 4 feet wide and 5 feet high; how long is it?

100. SECTION X.—PRACTICE.

In **Practice**, questions frequently occur which can be performed more expeditiously than by either of the foregoing methods. Most business operations which contain compound numbers may be abbreviated, by first finding the value for the highest denomination in the question, and then taking aliquot parts of this for lower denominations.

Note. The *aliquot parts* of a number are all the numbers that will measure it. Thus, 1, 2, 3, 4, 6, 8 and 12 are aliquot parts of 24.

1. What cost 18 bushels of wheat, at $1.37½ a bushel?

Assuming the price at $1 per bushel, the cost of 18 bushels is $18. At 25 cts. a bushel, the cost would be ¼ of $18 = $4.50; and at 12½ cts. per bushel, it would be ½ of the cost at 25 cts. = $2.25.

	$				
¼)	18.00	at	$1.00	per	bushel.
½)	4.50	"	.25	"	"
	2.25	"	.12½	"	"
	$24.75	"	$1.37½	"	"

2. What cost 148 bushels of potatoes, at 12½ cts. a bushel? at 18¾ cts.? at 25 cts.? 31¼ cts.? 37½ cts.? 50 cts.? at 56¼ cts.? $0.62½? $0.75? $0.87½? $1.12½?

3. What will 37 yards of cloth cost, at 15s. 10d. per yard?

	£	s.	d.		
½)	37	0	0	price of 37 yds., at £1	per yd.
½)	18	10	0	" " " " " 10s.	" "
⅙)	9	5	0	" " " " " 5s.	" "
	1	10	10	" " " " " 10d.	" "
	£29	5	10	price of 37 yds., at 15s.10d.	per yd.

4. What will 37 yds. of cloth cost, at £1 12s. 8¼d. per yard?

	£	s.	d.		
½)	37	0	0	price at £1	per yd.
⅕)	18	10	0	" " 10s.	" "
⅓)	3	14	0	" " 2s.	" "
$\frac{1}{32}$)	1	4	8	" " 8d.	" "
	0	0	9¼	" " ¼d.	" "
	£60	9	5¼	price at £1 12s.8¼d.	per yd.

Tables showing the aliquot parts of money, weights and measures, may be prepared by the pupil, if the teacher thinks it best. The following may serve as a model:

PRACTICE TABLE.

Parts of a dollar.

4	cents	is	$\frac{1}{25}$	20	cents	is	$\frac{1}{5}$	$56\frac{1}{4}$	cents	is	$\frac{9}{16}$
5	"	"	$\frac{1}{20}$	25	"	"	$\frac{1}{4}$	$62\frac{1}{2}$	"	"	$\frac{5}{8}$
$6\frac{1}{4}$	"	"	$\frac{1}{16}$	$31\frac{1}{4}$	"	"	$\frac{5}{16}$	$66\frac{2}{3}$	"	"	$\frac{2}{3}$
$8\frac{1}{3}$	"	"	$\frac{1}{12}$	$33\frac{1}{3}$	"	"	$\frac{1}{3}$	$68\frac{3}{4}$	"	"	$\frac{11}{16}$
10	"	"	$\frac{1}{10}$	$37\frac{1}{2}$	"	"	$\frac{3}{8}$	75	"	"	$\frac{3}{4}$
$12\frac{1}{2}$	"	"	$\frac{1}{8}$	40	"	"	$\frac{2}{5}$	$81\frac{1}{4}$	"	"	$\frac{13}{16}$
$16\frac{2}{3}$	"	"	$\frac{1}{6}$	$43\frac{3}{4}$	"	"	$\frac{7}{16}$	$87\frac{1}{2}$	"	"	$\frac{7}{8}$
$18\frac{3}{4}$	"	"	$\frac{3}{16}$	50	"	"	$\frac{1}{2}$	$93\frac{3}{4}$	"	"	$\frac{15}{16}$

Parts of a year.

6 months is $\frac{1}{2}$	2 months 12 d. is $\frac{1}{5}$	1 month 10 d. is $\frac{1}{9}$
4 " " $\frac{1}{3}$	2 " " $\frac{1}{6}$	1 " 6 " " $\frac{1}{10}$
3 " " $\frac{1}{4}$	1 " 15 d. " $\frac{1}{8}$	1 " " $\frac{1}{12}$

5. What cost 3 yd. 3 qr. 2 na. of broadcloth, at $3.25 per yard?

6. At $6.50 per ton, what cost 5 ton 8 cwt. 3 qr. 16 lb. of coal? 1 ton 4 cwt. 2 qr. 21 lb.? 15 cwt. 1 qr. 20 lb.?

7. At $87.50 per acre, what cost 3 A. 3 R. 24 rd. of land? 5 A. 1 R. 15 rd.? 3 A. 75 rd.?

For more examples in Practice, see Art. 94, ques. 27—32.

101. CONTRACTIONS IN MULTIPICATION AND DIVISION.

PROB. I. TO MULTIPLY BY 5. *Take $\frac{1}{2}$ of the number to be multiplied, and multiply that by* 10. This will give ten halves of the number, which is equal to 5 times the number. Or,

Annex a naught, and halve it.

1. How much are 5 times 24? One half of 24 is 12; and ten times 12 are 120. (**28**, page 30.)

2. How much are 5 times 35? $\frac{1}{2}$ of 35 is $17\frac{1}{2}$; and 10 times $17\frac{1}{2}$ are 175.

3. How much are 5 times 44? 55? 248? $355 = 340 + 15$? $677 = 660 + 17$? Say 677 is equal to 660 and 17. $\frac{1}{2}$ of 660 is 330; $\frac{1}{2}$ of 17 is $8\frac{1}{2}$, which added to 330 makes $338\frac{1}{2}$; 10 times $338\frac{1}{2} = 3385$.

4. Multiply these numbers by 5. 548; 874; 957; 1684; 2570 = 2400 + 160 + 10; 2590; 3376; 4087; 5359.

PROB. II. TO MULTIPLY BY 25. *Take* $\frac{1}{4}$ *of the number, and multiply that by* 100. This will give $\frac{100}{4}$ of the number, which is equal to 25 times the number. Or,

Annex two naughts, and take $\frac{1}{4}$ *of it.*

5. Multiply the following numbers by 25. 8; 12; 20; 17; 21; 22; 34; 47; 144; 275 = 240 + 35; 575 = 400 + 160 + 15; 576; 648; 185; 668; 754; 3216; 4036.

PROB. III, TO MULTIPLY BY 50. *Take* $\frac{1}{2}$ *of the number, and multiply that by* 100. This will give $\frac{100}{2}$, which is equal to 50. Or,

Annex two naughts, and halve it.

6. Multiply the numbers in Prob. II. by 50.

PROB. IV. TO MULTIPLY BY $12\frac{1}{2}$. *Take* $\frac{1}{8}$ *of the number, and multiply that by* 100. This will give $\frac{100}{8}$, which is equal to $12\frac{1}{2}$.

7. Multiply the numbers in Prob. II. by $12\frac{1}{2}$.

PROB. V. TO MULTIPLY BY 125. *Take* $\frac{1}{8}$ *of the number, and multiply that by* 1000. Or,

Annex three naughts, and take $\frac{1}{8}$ *of it.* NOTE. $\frac{1000}{8} = 125$.

8. Multiply the numbers in Prob. II. by 125.

PROB. VI. TO MULTIPLY BY $33\frac{1}{3}$. *Take* $\frac{1}{3}$ *of the number, and multiply that by* 100. Or,

Annex two naughts, and take $\frac{1}{3}$ *of it.* NOTE. $\frac{100}{3} = 33\frac{1}{3}$.

9. Multiply the numbers in Prob. II. by $33\frac{1}{3}$.

PROB. VII. TO DIVIDE BY 25. *Multiply by* 4, *and divide the product by* 100. (**39.**) This will give four one hundredths, or $\frac{1}{25}$ of the number.

10. Divide these numbers by 25. 150; 225; 400; 500; 675; 270; 576; 384; 990; 1575; 1640; 3248; 4564; 8167.

PROB. VIII. TO DIVIDE BY 125. *Multiply by* 8, *and divide the product by* 1000. (**39.**) This will give $\frac{8}{1000}$, or $\frac{1}{125}$, of the number.

11. Divide the numbers in the last Prob. by 125.

PROB. IX. TO DIVIDE BY $12\frac{1}{2}$. *Multiply by* 8, *and divide the product by* 100.

12. Divide the numbers in Prob. VII. by $12\frac{1}{2}$.

PROB. X. TO DIVIDE BY 50. *Multiply by* 2, *and divide the product by* 100.

13. Divide the numbers in Prob. VII. by 50.

PROB. XI. TO DIVIDE BY $33\frac{1}{3}$. *Multiply by* 3, *and divide the product by* 100.

14. Divide the numbers in Prob. VII. by $33\frac{1}{3}$.

NOTE. The pupil should perform all the exercises in the above problems *mentally*, as well as on the slate.

PROB. XII. TO MULTIPLY BY ANY NUMBER OF 9'S. Annex as many naughts to the multiplicand as there are nines in the multiplier, and from this number subtract the given multiplicand; the remainder will be the product.

15. Multiply 841685 by 99999.

SOLUTION.
$$\begin{array}{r} 84168500000 \\ 841685 \\ \hline 84167658315 \end{array}$$

16. Multiply 51487 by 9; by 99; by 999; by 99999.
17. Multiply 371548 by 5; by 25; by 50; by $12\frac{1}{2}$; by $33\frac{1}{3}$; by 125.
18. Divide 419684 by 25; by 125; by $12\frac{1}{2}$; by 50; by $33\frac{1}{3}$.
19. Multiply 51487 by 50; by $12\frac{1}{2}$; by $33\frac{1}{3}$; by 25; by 125; by 5.
20. Divide 610840 by $33\frac{1}{3}$; by 50; by $12\frac{1}{2}$; by 125; by 25.

102. SECTION XI. — PERCENTAGE.

The term *percentage* is used to express some number of hundredths of a given sum or quantity. *Per cent.* signifies hundredths, from the Latin *per centum*, *by the hundred*. Therefore, any given per cent. of a quantity is so many hundredths of that quantity. Thus, if a man buys goods for \$100, and sells them for \$108, we say that he gains 8 per cent. of the cost of the goods. So 1 per cent. of any number, or quantity, is .01 of that quantity; 3 per cent., .03; 100 per cent. of a

quantity is 100 hundredths of it; that is, the quantity itself. 500 per cent. of a number is 5 times that number.

2	per cent.	is	.02		$\frac{3}{8}$	per cent.	is	.00375
12	"	"	"	.12	$5\frac{1}{2}$	"	"	"	.055
20	"	"	"	.20	$125\frac{5}{8}$	"	"	"	1.25625
100	"	"	"	1.00	$6\frac{3}{4}$	"	"	"	.0675
120	"	"	"	1.20	$5\frac{2}{3}$	"	"	"	$.05\frac{2}{3}$
500	"	"	"	5.00	$4\frac{2}{7}$	"	"	"	$.04\frac{2}{7}$
$\frac{1}{2}$	"	"	"	.005	$\frac{1}{3}$	"	"	"	$.00\frac{1}{3}$

1. A man collects bills for a merchant to the amount of $15, and is to receive 3 per cent. for collecting them. How much shall he receive?

Note 1. 3 per cent. is 3 cents for every 100 cents, $3 for every $100, &c. If he receives 3 cents for collecting $1, he must receive 3 times as many cents as there are dollars collected. *Ans.*, 45 cents.

2. What is 3 per cent. of $1? Of $20? $40? $25? $55? $75? $517? What is 5 per cent. of $1? Of $8? $28? $87? $185? What is 8 per cent. of $1? $12? $25? $354? $1025? $3108?

3. A man borrowed $20 for a year, and is to pay 6 per cent. for the use of it. How much shall he pay? What is 6 per cent. of $1? $17? $75? $584?

4. What is 12½ per cent. of $1? Of $8? $24? $44? $50? $75? $354?

Note 2. 12½ per cent. is ⅛ of 100 per cent. 100 per cent. of $8 is $8; and 12½ per cent. is ⅛ as much. *Ans.*, $1. Again: 100 per cent. of $44 is $44; 12½ per cent. will be ⅛ as much = $5½. See Practice Table.

5. What is 20 per cent. of $40? $35? $54? $48? $357? $485? 20 per cent. is ⅕ of 100 per cent.

6. What is 25 per cent. of 12 lb.? Of 40 lb.? Of 50 lb.? 60 yards? 100 yards? 35 yards?

7. What is 33⅓ per cent. of 18? 24? 30? 75? 240 oz.? 270 acres?

8. What is 50 per cent. of $1? $10? $25? $87? 84 cwt.? 75 bushels?

Rule for computing Percentage. *Multiply by the rate per cent. expressed decimally; or assume the rate at* 100 *per cent., and take the aliquot parts as in Practice.*

9. What is 23 per cent. of $3? Of $7? $15? $25? $187? $1016? $10.16?

10. What is 15 per cent. of \$3.25? \$5.75? \$1010.085?

11. What 18¾ per cent. of \$34.50? Of \$748.20? \$1010.15?

Assume the rate at 100 per cent., and take the aliquot parts. (**100.**)

12. What is 37½ per cent. of 5 casks of raisins, each weighing 140 lb.?

13. What is 100 per cent. of \$75? 125 per cent. of \$116.75? 300 per cent. of \$84?

NOTE. If the per cent. contains a fraction that cannot be exactly expressed in decimals, multiply first by the hundredths, and then by the common fraction, thus:

14. Find 3⅓ per cent. of \$48.75.

48.75		
.03⅓		
14625	at 3	per cent.
1625	" ⅓	" "
\$1.6250	" 3⅓	" "

15. Find 5$\frac{3}{7}$ per cent. of \$100.84. Of \$1008.45.

16. How much is ⅚ per cent. of \$851.61? Of \$651.67?

NOTE. 1 per cent. of \$851.61 is \$8.5161, ⅚ of which will be the answer.

17. A ship, whose cargo is valued at \$4518, throws overboard, in a storm, 15¾ per cent. of her cargo. What value of property was thrown overboard?

18. Two men commence business with equal sums of money, viz., \$5000. The first year, one adds to his property 12½ per cent. of it; the other loses 10 per cent. of his. How much more property has one at the end of the year than the other?

19. A man having \$5000, has spent 56¼ per cent. of it; how much has he left?

20. A merchant bought 5060 lb. of butter, at 15½ cents a pound; at what price must he sell it per lb. to gain 12½ per cent. of the cost?

21. What part of 5 is 3? (**44**, 31—35; and **86.**) What per cent. is equivalent to ⅗?

NOTE. Reduce ⅗ to hundredths. (**78**, Ex. 7—11.)

22. What part of $3.05 is $1.75? What per cent.?
23. What part of $100.75 is $6.05? What per cent.?
24. What per cent. of $516.25 is $5.25? What per cent. of it is $15.105? $2.75? $501.25?
25. What per cent. of 8164 lb. are 204.1 lb.?

103. COMMISSION AND BROKERAGE.

Commission and Brokerage are compensations made to agents and brokers for the purchase, sale, or care of the property of others. Their compensation is usually estimated at so much per cent.

An agent who resides in another place, and transacts business for another, is called a factor.

1. A factor bought for his employer 3000 bushels of corn, at $0.62½ per bushel, and charged 2½ per cent. commission. How much did he receive for his services?

2. An agent sold 2184 gallons of oil, at $1.08 per gallon. What will his commission amount to, at 1¾ per cent.?

3. A broker purchased for another 8 shares of railroad stock, at $109¾ per share. How much will his brokerage amount to, at ½ per cent.?

4. My factor in London has purchased cloths amounting to £107 8s. 6d. How much is his commission, at 1¾ per cent.?

£107 8s. 6d. = £107.425; 1¾ per cent. = .01¾, or .0175. (86 and 88.)

5. What is the commission on £500 3s. 4d., at 4⅝ per cent.? at 3½ per cent.?

104. INSURANCE.

Insurance is security given against losses by fire, shipwreck, and other accidents. The amount paid for the insurance is called the *premium*. The instrument which contains the contract is called the *policy*. The insurer is sometimes called the *underwriter*.

1. What amount of premium must be paid for insuring my house against fire, the house being insured at $2500, at a premium of 1⅝ per cent.?

2. A merchant shipped 500 barrels of beef, valued at $12.50 per bbl., to S. America, and effected insurance on the amount at 1¾ per cent. What did the premium amount to?

3. My factor in Liverpool informs me that he has shipped

to me goods valued at £300 15s. 9d. I have effected insurance for this amount at a premium of $2\frac{1}{4}$ per cent. How much must I pay, including 1 dollar for the policy, the pound being worth $4.87?

105. STOCKS.

By *Stocks* is meant the capital of moneyed institutions, such as Banks, Manufactories, Railroads, Insurance Companies, United States Bonds, &c.

Stocks are divided into portions called *shares*, the owners of which are called *stockholders*.

The *par* value of a share is its original cost or valuation.

Premium is the market price paid above par, and *Discount* is the market price paid below par.

If $100 is the par value of a share in a bank or other institution, and the market price for a share is $108, the shares are said to be worth 8 *per cent. premium*. So, on the contrary, if the shares are in the market selling at $92 each, they are said to be at 8 *per cent. discount*.

The profits arising from stocks are divided among the stockholders annually, semi-annually, or at other regular periods, each receiving a certain per cent. of the par value of his shares. The sum thus divided is called a *dividend*.

1. What is the cost of 5 shares of Eastern Railroad stock, at $1\frac{3}{8}$ per cent. above par, the par being $100? At $2\frac{5}{8}$ per cent. premium? At $1\frac{1}{8}$ per cent. below par? At $1\frac{5}{8}$ per cent. discount?

2. A stock broker bought 8 shares of Western Railroad stock, at $3\frac{1}{2}$ per cent. discount, and sold them at $1\frac{3}{4}$ per cent. premium. How much did he gain by the transaction?

3. What are 7 shares of Boston and Lowell Railroad stock worth, at $8\frac{3}{4}$ per cent. premium, the par value of the shares being $500?

4. If the above railroad should declare a dividend of $4\frac{3}{4}$ per cent., how much should he receive who owns 4 shares?

5. The par value of the shares in a certain bank is $\$28\frac{1}{4}$; what will 8 shares come to, at $1\frac{3}{8}$ per cent. advance? At $2\frac{5}{8}$ per cent. advance? At $\frac{3}{4}$ per cent. discount?

6. If the above bank should pay a semi-annual dividend of $3\frac{1}{2}$ per cent., how much will the holder of 5 shares receive?

106. Taxes.

Taxes are sums of money paid by owners of property for the support of government. They are usually laid at a certain per cent. of the property possessed. In some states, however, every male citizen, over a certain age, is required to pay a certain tax without regard to his property, called a *poll* tax. Each person so taxed is called a *poll*. In some of the states the poll taxes are more than in others.

In assessing taxes, the first thing to be done is to take an inventory of all the taxable property in the town. If there is a poll tax, make a full list of the polls, and multiply the number by the tax on each poll, and subtract the amount from the whole sum to be raised by the town; the remainder will be the amount to be raised on the property. Then divide the whole tax to be raised on the property by the amount of all the taxable property; the quotient will be the tax on $1. Multiply each man's property by this quotient, and the product will be the amount of his taxes.

1. A certain town is to be taxed $1887.50; the property on which the tax is to be laid is valued at $350,000. There are 250 polls, each of which is taxed $1.25. What will be the tax on $1? If A's property is valued at $1500, and he pays for 3 polls; B's at 3500, and he pays for 2 polls; and C's at $4000, and he pays for 4 polls; what shall be the tax of each?

NOTE. 250 polls at $1.25 = $312.50. $1887.50 — $312.50 = $1575 to be raised on the property.

Having found, as in the last example, the per cent., or the amount to be raised on $1, form a table showing the amount which $2, $3, &c., would produce, at the same rate per cent.

TABLE.

$		$	$		$	$		$
1	gives	0.0045	20	gives	0.09	300	gives	1.35
2	"	0.009	30	"	0.135	400	"	1.80
3	"	0.0135	40	"	0.18	500	"	2.25
4	"	0.018	50	"	0.225	600	"	2.70
5	"	0.0225	60	"	0.27	700	"	3.15
6	"	0.027	70	"	0.315	800	"	3.60
7	"	0.0315	80	"	0.36	900	"	4.05
8	"	0.036	90	"	0.405	1000	"	4.50
9	"	0.0405	100	"	0.45	2000	"	9.00
10	"	0.045	200	"	0.90	3000	"	13.50

2. What is D's tax, by the above table, whose real estate* is valued at $2550, and his personal estate† at $1375, and who pays for 2 polls?

OPERATION.

Tax on	$3000	is	$13.50	
" "	900	"	4.05	
" "	20	"	.09	
" "	5	"	.02	
2 polls,			2.50	
Valuation,	$3925		$20.16	tax.

3. What is E's tax, whose valuation is $2549. and who pays for 1 poll? F's, whose valuation is $4716, and who pays for 3 polls? G's, whose property is valued at $3186, and who pays for 1 poll?

4. When the tax is laid at .56 per cent., how much is his tax whose property is valued at $5,018, and pays for 1 poll? $2148, and 2 polls?

5. The state tax being .01 per cent., the county tax .04 per cent., and the town tax .37 per cent., how much is the whole tax of him who pays for 3 polls at $1.50 each, and whose property is valued at $50,000?

6. A man's town tax is $20.35; what is his property valued at, the tax being laid at $\frac{37}{100}$ per cent.? *Divide his tax by the per cent. at which the tax is laid.* Why?

7. The tax on my real estate is $5.70, personal estate $4.56; what are my real and personal estates valued at, if the tax is laid at .38 per cent.?

107. DUTIES.

Duties are taxes imposed by government on most articles of merchandise imported from foreign countries.

A *specific* duty is a specified sum on a square yard, gallon, pound, &c.

Ad valorem duty is a specified per cent. on the *cost* of the goods in the country from which they were imported.

By the tariff of 1846, all duties are levied on the *ad valorem*

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*By *real estate* is meant, "all lands, and all buildings and other things erected on or affixed to the same."

† *Personal estate* includes all other taxable property.

principle. In estimating the cost, *every expense in the foreign port* is reckoned, including that of the cask, box, or bag, (if of foreign manufacture,) which contains the articles.

The custom-house officers are required to make correct returns to the government of all merchandise imported. For this purpose, all merchandise is weighed or measured as formerly, when specific duties were levied, the usual legal allowances for draft, tare, leakage, &c., being made.

Gross weight and *net* weight, see Art. **53.** *Draft* is an allowance for waste. *Tare* is an allowance for the weight of the box, cask or bag, that contains the article.

The allowance formerly made for draft and scaleage are entirely dispensed with in buying and selling among merchants. There is no rule among them for estimating damage, such as waste, leakage, &c., except by agreement or appraisal.

1. What is the net weight of 5 boxes of sugar, weighing gross 2142 lb., tare 15 per cent.; also the duty, at 30 per cent. ad valorem, the cost of the sugar in Havana being $4\frac{3}{4}$ cts. per pound?

Solution. 15 per cent. of 2142 lb. is 321 lb.; gross weight 2142 lb. — tare 321 lb. = 1821 lb., the net weight. The value of this, at $4\frac{3}{4}$ cts., is \$86.50; 30 per cent. of which is \$25.95, the duty.

2. Required the cost of an invoice of 19 boxes of sugar, imported from Havana, gross weight 9611 lb., tare 15 per cent., the price there being $2\frac{1}{2}$ cents per lb. The boxes containing the sugar being charged at \$3.25 each. Export duty, \$.37½ per box; weighing and drayage, 33 cts. per box; wharfage, watching and tarpaulins, (to cover the boxes,) 87½ cts.; brokerage, \$1.25; samples, \$0.25. Commission at $2\frac{1}{2}$ per cent. on the amount of the other charges. What would the duties amount to, at 30 per cent.?

3. What is the duty on 20 barrels of cloves, costing in Cayenne \$252.80, exclusive of commissions, which are charged at $2\frac{1}{2}$ per cent., the duty being 40 per cent. ad val.?

4. What is the duty on 100 hogsheads of molasses, containing 12500 gallons, costing in Cayenne, exclusive of commissions, $9\frac{3}{10}$ cents per gallon; commission being $2\frac{1}{2}$ per cent., and the duty at 30 per cent. ad val.? What will the duty average per gallon?

5. What is the net weight of 10 bags of pepper, weighing

gross 1379¼ lb., tare 2 per cent.? What is the duty, at 30 per cent. ad valorem, the cost being 1¾ cts. per lb.?

6. What is the duty on 115 tons 13 cwt. 3 qr. hemp, valued at $145.50 per ton, at 30 per cent. ad valorem?

108. INTEREST.

Interest is a certain percentage of a sum of money paid by the borrower to the lender for the use of it.

Principal is the money lent, or on which the interest is to be paid.

Rate is a certain per cent. agreed upon. It is generally fixed by law at 6 per cent. per annum.

Amount is the principal and interest added together.

NOTE. In the following questions in interest, the annual rate is understood to be 6 per cent. unless some other rate is named.

1. What is the interest of $1 for 1 yr. at 6 per cent.? $10? $15? $35?
2. What is the interest of $15 for 1 year? 2 years? 3 years? 5 years?
3. What is the interest of $18 for 1 year? For 2 months = $\frac{1}{6}$ yr.? 3 mo.? 6 mo.?
4. What is the interest of $12 for 1 yr. 2 mo. = 1$\frac{1}{6}$ yr.? 1 yr. 3 mo. = 1¼ yr.? 1 yr. 6 mo.? 1 yr. 4 mo.? 2 yr.? 2 yr. 2 mo.? $20 for 2 yr. 4 mo.? 2 yr. 1 mo.? 3 yr. 6 mo.?
5. What is the interest of $30 for 1 yr. at 5 per cent.? For 1 yr. 3 mo.? 1 yr. 6 mo.?
6. What is the interest of $6 for 1 yr. at 7 per cent.? For 1 yr. 2 mo.? 2 yr. 4 mo.?

If the rate of interest is 6 per cent. per annum, for 2 months it will be $\frac{1}{6}$ of 6 per cent., or 1 per cent., of the principal.

7. What is the interest of $1 for 2 mo.? How much for 1 mo.?
Ans. ½ of a cent, or 5 mills. For 3 months? *Ans.* 1½ cent, or 1 cent 5 mills. For 4 mo.? 5 mo.? 6 mo.? 7 mo.? 8 mo.? 9 mo.? 1 yr. 3 mo. = 15 mo.?

Since 6 days are $\frac{1}{10}$ of 60 days, or 2 months, if the interest of $1 for 2 months is 1 cent, for 6 days it will be $\frac{1}{10}$ of 1 cent, or 1 mill; for 12 days, 2 mills, &c.

8. What is the interest of $1 for 6 days? 24 days? 18 days? 1 day? *Ans.* $\frac{1}{6}$ of a mill? For 5 days? *Ans.* $\frac{5}{6}$ of 1 mill. For

8 days? *Ans.* 1⅓ mill. For 12 days? 17 days? 27 days? Hence,

To find the interest of $1 for any given time, when the rate per annum is 6 per cent.,

RULE 1. *Reduce the years and months to months, and call ½ the number of months cents, and ⅙ of the number of days mills.*

9. What is the interest of $1 for 2 months and 12 days? *Ans.* $0.012. For 3 mo. 18 da.? *Ans.* $0.018. Why? For 8 mo. 15 d.? *Ans.* $0.042½. For 7 mo. 16 d.? *Ans.* $0.037⅔.

10. What is the interest of $1 for 9 mo. 25 d.? For 1 yr. 3 mo. 28 d. = 15 mo. 28 d.? For 3 yr. 9 mo.?

11. For 5 yr. 7 mo. 23 d.? 3 yr. 11 mo. 29 d.? 4 yr. 1 mo. 21 d.?

To find the interest on any given principal, at 6 per cent.,

RULE 2. *Find the interest of $1 for the given time by the last rule, and multiply the interest thus found by the given principal.*

12. Compute the interest on $561.14 for 2 yr. 7 mo. 20 d.

NOTE. The interest of $1 for 31 mo. is $0.155; for 20 da. it is $0.003⅓, which, being added, gives $0.158⅓. Multiplying this by the principal gives the interest on $561.14. *Ans.* $88.847.

13. Find the interest on $48.17, for 1 yr. 3 mo. 8 d; for 3 yr. 7 mo. 12 d.; 4 yr. 8 mo. 9 d.; for 5 yr. 7 mo. 27 d.

14. What is the interest on $1418.46, from Jan. 15, 1841, to Nov. 25, 1846? (60.) from Dec. 14, 1845, to May 6, 1848? from July 4, 1776, to March 23, 1781? from Aug. 25, 1837, to Nov. 7, 1848?

15. What is the *amount* of $2007.81, from Feb. 21, 1844, to Jan. 15, 1849? from Oct. 17, 1832, to April 12, 1850? from Nov. 25, 1845, to Jan. 20, 1849?

To compute the interest when the rate per annum is either more or less than 6 per cent.,

RULE 3. *Having found the interest at 6 per cent. by Rule 2, make a proportional addition or subtraction; for 7 per cent. add ⅙; for 5 per cent. subtract ⅙; for 5½ per cent. subtract $\frac{1}{12}$; for 7½ per cent. add ¼, &c.*

Take the 12th example. The interest at 6 per cent. is \$88.847; at 7 per cent. it would be 103.656; at $5\frac{1}{2}$ per cent., \$81.443.

16. What is the interest of \$48.17, for 1 yr. 3 mo. 8 d., at $5\frac{1}{2}$ per cent.? at $4\frac{1}{2}$ per cent.? 7 per cent.? $7\frac{1}{2}$ per cent.? 9 per cent.? $12\frac{1}{2}$ per cent.? $3\frac{1}{4}$ per cent.?

17. Compute the interest on \$35.125, at $6\frac{1}{4}$ per cent., for 3 mo. 15 d.; at 7 per cent., for 1 yr. 5 mo. 20 d.; at $6\frac{3}{4}$ per cent., for 8 yr. 9 mo. 15 d.; at $5\frac{3}{4}$ per cent., for 3 yr. 4 mo. 24 d.

Some practical men prefer to compute interest by the following rule. It is not commonly so short a method as that before given, but as it is sometimes desirable to compute the interest by different methods, in order to test the correctness of a result, the pupil should learn to compute interest by it.

RULE. *Multiply the principal by the rate per annum, expressed decimally* (**102**); *the product will be the interest for* 1 *year. From this, calculate the interest for any number of years, months or days, as in Practice.* (**100.**)

NOTE. The pupil may do the following questions by both methods.

18. What is the interest of \$416, for 1 year, at 6 per cent.? at $5\frac{1}{2}$ per cent.? at $6\frac{1}{2}$ per cent.? at 7 per cent.? at $8\frac{1}{4}$ per cent.?

19. What is the interest on \$517.84, for 1 yr. 5 mo. 12 d., at $7\frac{1}{2}$ per cent.?

$$
\begin{array}{rl}
517.84 & \\
.07\frac{1}{2} & \\
\hline
36.2488 & \\
2.5892 & \\
\hline
\frac{1}{3})\ 38.838 = & \text{interest for 1 yr.} \\
\frac{1}{4})\ 12.946 = & \text{" " 4 mo.} \\
3.236 = & \text{" " 1 mo.} \\
{}^{*}1.295 = & \text{" " 12 d.} \\
\hline
\$56.315 = & \text{interest for 1 yr. 5 mo. 12 d.}
\end{array}
$$

* 12 d. $= \frac{1}{10}$ of 4 mo.

20. What will $52.87 amount to in 8 mo. 27 d., at 7 per cent.?

	$52.87		
	.07		
½)	3.7009	= interest for	1 yr.
⅓)	1.850	= " "	6 mo.
⅓)	.617	= " "	2 mo.
¼)	.206	= " "	20 d.
	.051	= " "	5 d.
	.021	= " "	2 d.
	$2.745	= interest for 8 mo. 27 d.	
	$5.287	= principal.	
	$55.615	= amount.	

21. What is the interest of $461.75, at 5½ per cent., for 2 years? for 2 yr. 3 mo.? for 3 yr. 4 mo.? for 4 yr. 5 mo.?

22. What is the interest of $923.50, at 4½ per cent., for 5 years 6 mo.? for 6 yr. 7 mo.? for 7 yr. 8 mo.?

23. What is the interest of $1385.25, for 6 yr. 9 mo., at 7½ per cent.? for 7 yr. 10 mo.? for 9 yr. 11 mo.?

24. Compute the interest on $35.125, at 6½ per cent., for 3 mo. 15 d.; 8 mo. 24 d.; 1 yr. 5 mo. 20 d.

25. What is the amount of $70.25, at 6 per cent., for 3 yr. 2 mo. 27 d.; for 8 yr. 9 mo. 15 d., at 7 per cent.; for 3 yr. 4 mo. 24 d., at 5¼ per cent.?

To find the interest on pounds, shillings, pence, &c., for any given time,

RULE. *Reduce the shillings, &c., to the decimal of a pound* (86); *annex it to the pounds, and then multiply by that decimal of a £ which expresses the interest of £1 for the given time, and reduce the decimal in the result to shillings, pence, &c.*

NOTE. Shillings, pence and farthings, may be reduced to the decimal of a pound by the following

RULE. *Write half the shillings as tenths, and if there is an odd shilling, call it 5 thousandths; write the farthings in the given pence and farthings as so many thousandths, increasing their number by 1 when it exceeds 12, and by 2 when it exceeds 36; and add the numbers.*

26. What is the interest of £54 17s. 9½d., for 1 yr. 5 mo. 15 d., at 6 per cent.?

Solution. 17s.=£.85; 9½d.=£.040; .85+.040=£.89.

£54 17s. 9½d.=£54.89
.087½ rate per cent. for 1 yr. 5 mo. 15 d.

38423
43912
2744

£4.80287=£4 16s. 0d. 2.7 qr., the answer.

27. Find the interest, at 6 per cent. per annum, of £150 6s. for 8 mo. 15 d.; of £75 15s. 6d. for 3 mo. 18 d.; of £950 19s. 7¾d. for 3 yr. 7 mo. 25 d.; of £105 0s. 9¼d. for 1 mo. 14 d., at 4½ per cent. per annum.

NOTE. Get the shillings, &c., to the nearest thousandths of a £, and the answer to the nearest qr.

109. PARTIAL PAYMENTS. LEGAL RULE.

When partial payments have been made and endorsed on notes or bonds, the following is the rule adopted by the United States courts, and most other courts, for computing the interest.

LEGAL RULE FOR PARTIAL PAYMENTS.

1. *Compute the interest on the given principal up to the time of the first payment, and if it is less than the payment, add it to the principal. From this amount subtract the first payment; the remainder will be a new principal.*

2. *Compute the interest on this new principal from the time of the first payment to the time of the second payment, and if it is less than the payment, subtract the second payment from the amount, for another new principal, as before, and thus proceed till the time of settlement.*

3. *If any payment is less than the interest due at the time the payment is made, subtract the payment from the interest, (not from the amount,) and add the excess to the next interest, to be computed on the same principal as before, to the time of the next payment;* SO THAT NO NEW PRINCIPAL SHALL BE GREATER THAN THE PRECEDING ONE; *as in the following example.*

NOTE. Merchants and bankers usually reject mills in their computations; but when the fraction of a cent is ½ or more, they add one to the number of cents. The pupil may do so in the next nine articles.

Salem, May 14, 1845.

$740.00.

1. For value received, I promise to pay William Hodges, or order, seven hundred and forty dollars, on demand, with interest from date.

JAMES PAYWELL.

*Endorsements.**

Nov. 20, 1845, received eighty-seven dollars.
April 3, 1846, received fifty-five and $\frac{20}{100}$ dollars.
Oct. 9, 1846, received ten dollars.
Jan. 15, 1847, received twelve dollars.
Apr. 15, 1847, received one hundred and forty-five dollars.
What was due Nov. 17, 1847?

METHOD OF OPERATION.

Principal bearing interest from May 14, 1845,		$740.00
Interest to time of first payment, (6 mo. 6 da.,)		22.94
Amount due Nov. 20, 1845,		762.94
First payment, to be deducted from the amount,		87.00
Balance for a new principal, due Nov. 20, '45,		675.94
Int. on bal. from first payment to 2d payment, (4 mo. 14 da.,)		15.10
Amount due April 3, 1846,		691.04
2d payment, to be deducted from the amount,		55.20
Balance for new principal, due April 3, 1846,		635.84
Int. on bal. to 3d payment, Oct. 9, '46, (6 mo. 6 da.,)	$19.71	
3d payment, which is less than the interest due,	10.00	
Excess of interest still due,	9.71	
Int. on the same principal to 4th payment, (3 mo. 6 da.,)	10.17	
Int. due Jan. 15, '47,	19.88	
4th payment, less than interest due,	12.00	
Excess of interest still due,	7.88	
Int. on same principal to time of 5th payment, (3 mo.,)	9.54	
	$17.42	
Int. due April 15, 1847, to be added to the principal, because less than the 5th payment,		$17.42
Amount due April 15, 1847,		653.26

* Endorsements should be made in *words*, not in *figures*.

Amount due April 15, 1847,	653.26
5th payment, to be deducted from the amount due,	145.00
Balance due April 15, 1847,	508.26
Int. on balance to Nov. 17, '47, (7 mo. 2 d.,)	17.95
Balance due at the time of settlement,	526.21

2. *Boston, July* 1, 1846.

$450.00.

For value received, I promise to pay James King, or order, four hundred and fifty dollars, on demand, with interest from date.

JONATHAN CROSS.

Endorsements.

April 1, 1847, received seventy dollars.
Oct. 1, 1847, received five dollars.
Feb. 1, 1848, received fifty dollars.
What was due July 1, 1848?

3. *Hartford, April* 5, 1845.

$200.00

For value received, we promise to pay Johnson and Mills, or order, two hundred dollars, in two months, with interest after.

J. STEVENS & Co.

Endorsements.

July 5, 1847, received seventy-five dollars.
Feb. 20, 1848, received one hundred and fifteen dollars.
What was due Sept. 5, 1848?

4. *New York, June* 16, 1845.

$200

For value received, I promise to pay William Johnson, or order, two hundred dollars, on demand, with interest at 7 per cent. per annum.

SOLOMON PAYWELL.

Endorsements.

Nov. 16, 1845, received fifty-five dollars.
Aug. 16, 1846, received six dollars.
March 1, 1847, received ten dollars.
June 16, 1847, received seventy-five dollars.
What was due Nov. 16, 1847?

5. *Cincinnati, April* 1, 1848.

$500.

For value received, we promise to pay Joseph Emerson, or order, five hundred dollars, with interest, at 6 per cent. per annum.

J. BRYANT & Co.

Endorsements.

Nov. 15, 1848, received twelve dollars.
May 23, 1849, received seventy-five dollars.
Sept. 20, 1849, received ten dollars.
What will be due April 1, 1850?

II. 6. A note was given June 10, 1844, for $1750. The endorsements were as follows: Sept. 15, 1844, $150. April 1, 1845, $50. Sep. 12, 1845, $40. Jan. 7, 1846, $500. May 12, 1846, $15. July 17, 1846, $500. What was due March 25, 1847, computing interest at 6 per cent. per annum?

110. PARTIAL PAYMENTS. COMMON METHOD.

The following easier rule is generally adopted for computing interest where the note on which partial payments have been made is settled within a year of the date of the note.

RULE 1. *Find the* AMOUNT *of the note from the time interest is to commence to the time of settlement.*

2. *Find the* AMOUNT *of each payment from the time it was paid to the time of settlement.*

3. *Subtract the total amounts of the payments from the amount of the principal; the remainder will be the amount due.*

1. A note of $375 is dated Aug. 16, 1848. Endorsements: Sept. 1, 1848, $50. Oct. 16, 1848, $75. Nov. 1, 1848, $80. Jan. 16, 1849, $35.43. What was due April 16, 1849?

NOTE. The amount of $375 from Aug. 16, 1848, to April 16, 1849, is $390. The amount of $50 from Sept. 1, 1848, to April 16,

1849, is $51.875; of $75 from Oct. 16, 1848, to April 16, 1849, is $77.25, &c.

2. A note for $1000 is dated Nov. 17, 1848. Endorsements: Dec. 12, 1848, $100. Jan. 20, 1849, $75. March 3, 1849, $150. July 15, 1849, $200. What was due Sept. 25, 1849?

3 and 4. Perform the 4th and 5th examples of the last article by this rule.

5. Find the balance due on the following account, settled Jan. 1, 1849, by getting the amount of each item from its date to Jan. 1, and then the difference between the total amount of the Dr. items and that of the Cr. items.

Simon Smith's account with William Burnham.

Dr.			Cr.		
1848.			1848.		
Apr. 1,	To Goods,	$500.00	Mar. 15,	By Corn,	$150.00
June 15,	" "	350.00	May 8,	" Flour,	375.00
Sept. 27,	" "	175.00	Oct. 15,	" Butter,	350.00

6. What is due on the following account, settled July 1, 1849; interest at 8 per cent.?

Timothy Merchant's account with William Farmer.

Dr.			Cr.		
1848.			1848.		
Dec. 10,	To Cash,	$75.00	Nov. 17,	By Beans,	$100.00
1849.			Dec. 15,	" Pork,	200.00
Jan. 15,	" Cash,	210.00	" "	" Cheese,	150.00
Apr. 1,	" Cash,	100.00	1849		
June 3,	" Cash,	100.00	Jan. 20,	" Wheat,	250.00
			Apr. 7,	" Oats,	100.00

7. Find the balance of the following account Jan. 1, 1850, reckoning interest on each item at $7\frac{1}{2}$ per cent.

David Stetson's account with William R. Downing.

Dr.			Cr.		
1849.			1849.		
Jan. 15,	To Goods,	$75.00	Mar. 25,	By Cash,	$200.00
Mar. 20,	" "	150.00	June 6,	" Pork,	150.00
July 15,	" "	248.54	Aug. 9,	" Hay,	75.00
Sept. 6,	" "	84.75	Oct. 15,	" Potatoes,	50.50
Oct. 27,	" "	189.48	Dec. 10,	" Sundries,	148.75

111. Compound Interest.

When interest becomes due, and is not paid, it is sometimes added to the principal, and interest computed on that amount, as on the principal before. This is called *Compound Interest.*

Note. When the interest is reckoned upon the principal only, as in the preceding articles, it is called *Simple Interest.*

1. What is the compound interest of $300 for 3 years, interest at 6 per cent., payable annually?

$300.00	Principal.
18.00	Interest for the 1st year.
318.00	Amount for 1 year.
19.08	Interest for the 2d year.
337.08	Amount for 2 years.
20.22	Interest for the 3d year.
357.30	Amount for 3 years.
300.00	Deduct the principal.
57.30	Compound interest for 3 years.

2. What will $1000 amount to in 1 year 8 months, interest payable semi-annually?

$1000.00	Principal.
30.00	Interest for the 1st ½ year.
1030.00	Amount for ½ year.
30.90	Interest for the 2d ½ year.
1060.90	Amount for 1 year.
31.83	Interest for the 3d ½ year.
1092.73	Amount for 1½ year.
10.93	Interest for 2 months.
1103.66	Amount for 1 year 8 months.

Rule. *Compute the interest on the principal for the specified time, and add it to the principal for a new principal. Then compute the interest on this new principal as on the first principal, and add the interest to it; and so on for each successive period of time. Finally, subtract the given principal from the last amount; the remainder will be the Compound Interest.* See also table II. page 262.

3. Compute the compound interest on $1000.00, at 6 per cent., for 3½ years, payable annually. Semi-annually. Quarterly.

4. What will $500.00 amount to, at 5½ per cent., in 3½ years, the interest being added to the principal in a new note at the end of every 6 months? What will be the amount if the rate of interest be 6 per cent.?

5. What will £100 10s. 6d. amount to in 3 years, at 4 per cent. per annum?

112. PROBLEMS IN INTEREST.

PROB I. *Principal, interest and time given, to find the rate.*

1. At what rate per cent. must $200 be put on interest for 5 years, to gain $50?

$200 in 5 years, at 1 per cent., will gain $10; in order to gain $50, the rate must be as many times 1 per cent. as there are times $10 in $50. *Ans.* 5 per cent.

RULE. *Divide the given interest by the interest of the principal for the given time at 1 per cent. per annum; the quotient will be the rate required.*

2. At what rate per cent. must $200 be put on interest, to gain $45.83 in 2 yr. 3 mo. 15 da.? in 4 yr. 3 mo. 27 d.?

PROB. II. *Principal, interest and rate given, to find the time.*

3. In what time will $300, at 7 per cent., yield $87.50 interest? $300 will, in 1 year, yield $21; in order to yield $87.50, it must be on interest as many years as there are times $21 in $87.50. *Ans.* 4⅙ yrs. = 4 yr. 2 mo.

RULE. *Divide the given interest by the interest of the given principal for 1 year.*

4. A note of $340 amounted to $381.65; how long was it on interest, the rate being 6½ per cent.?

Deduct the principal from the amount to get the interest.

5. In what time will $500 amount to $1000, at 6 per cent.? at 8 per cent.? at 7½ per cent.?

6. In what time will any sum of money double itself, at 6

per cent.? 8 per cent.? 7½ per cent.? at 4¼ per cent.? 15 per cent.?

NOTE. Assume $1 as the principal.

PROB. III. *Time, rate and interest given, to find the principal.*

7. What principal will yield $50 a year, at 6 per cent.? One dollar will yield six cents in one year. To yield $50 a year, it will require as many dollars for the principal as there are times 6 cents in $50. *Ans.* $833.33⅓.

RULE. *Divide the given interest by the interest of $1 for the given time.*

8. What sum of money on interest at 5½ per cent. will yield $75 semi-annually?

9. What sum of money at 6 per cent. will yield $650 in 8 yr. 3 mo.?

PROB. IV. *Amount, time and rate, to find the principal.*

10. What principal at 6 per cent. will amount to $371 in 1 year? $1 in 1 yr., at 6 per cent., will amount to $1.06. The principal which will amount to $371 must be as many dollars as there are times $1.06 in $371. *Ans.* $350.

RULE. *Divide the given amount by the amount of $1 for the time.*

11. What sum of money at 5½ per cent. will amount to $1000 in 1 yr. 6 mo.?

12. What principal at 6 per cent. will amount to $854 in 3 yr. 4 mo.?

13. What is the present worth of $150 due in 2 years without interest, money being worth 6 per cent.? It is worth a sum which being put on interest for two years would amount to $150. The question is, therefore, the same as this. What principal at 6 per cent. will amount to $150 in two years? See PROB. IV.

14. What is the present worth of $500 due in 3 years 3 mo.? in 5 years? in 4½ years? in 2 years 3 mo. 15 da.?

15. What is the present worth of $100 due in 3½ years, interest at 6 per cent.? 6½ per cent.? 5½ per cent.? 8 per cent.?

Although this mode of calculating the present worth of a

sum of money due at a future time is strictly equitable, and, *for a long term of time*, should always be used; *banks, always, and merchants, almost or quite universally, deduct* BANK DISCOUNT (**115**) *from the face of a note payable at a future time, to get the present worth.*

For the mode of reckoning discount on purchases of goods, &c., see the next article.

DISCOUNT.

113. Discount is an allowance made for the payment of a sum of money or a debt before it becomes due.

RULE FOR COMPUTING DISCOUNT.

Multiply the given amount by the per cent. expressed. (**102.**)

This is the rule universally adopted by merchants and traders in all business transactions, except in computing the discount on notes and drafts payable at a future day; for which, see BANK DISCOUNT. (**115.**)

1. A owes B $500, to be paid in 6 months without interest; what is the discount for present payment, money being worth 6 per cent. per annum?

2. Bought goods in New York to the amount of $350, to be paid in 8 months. What discount may the merchant make for present payment, money being worth 7 per cent. per annum? How much do the goods amount to, after deducting the discount?

3. Sold goods amounting to $1000, payable one half in 3 months, the other half in 6 months. What sum will pay for them now, after deducting a discount of 1 per cent. per month?

4. C owes D $578, to be paid Nov. 16, 1849; C wishes to pay it on the 10th of Aug., 1849. How much ought D to receive, interest being $\frac{3}{4}$ per cent. per month?

5. Bought a horse for $125, payable in 3 months; for cash the seller would make a discount of $1\frac{1}{2}$ per cent. per month; how much cash will pay for the horse?

6. The price of a book is 0.37\frac{1}{2}$; what will be the cost of it, after making a discount of 8 per cent.?

7. Sold 500 lb. of butter, at 0.18\frac{3}{4}$ per lb., on 6 months. What cash will pay for it, discounting at the rate of 9 per cent. per annum?

114. BANKING.

By Banking is meant the business usually transacted at Banks.

Banks are joint stock companies, that is, associations of individuals, under a charter, or an act of incorporation, who contribute each a certain portion of money, the aggregate of which, when paid into the bank, is called the *Capital Stock*.

Formerly, that is, for many years after the first establishment of banks, they were restricted to the mere business of receiving money, which was then almost exclusively specie, on deposit for safe keeping, paying it out to the owner when called for, and charging for the service a certain per centage. But gradually enlarging their agencies, with the increasing wants of modern times, they have come to be almost universally Banks of *Discount*, *Deposite*, and *Circulation*, and many of them Banks of *Exchange* also. As such they are presented to the beginner in business in our community. We will therefore endeavor, as briefly as possible, to explain to him these several business operations.

1. DISCOUNT. The main business of a bank, that which is its chief source of income, is to buy or take notes of hand, or Bills of Exchange payable at some future day, and pay the money for them, after deducting therefrom the discount or interest for the time which these notes and bills have to run, that is, from the time at which they are discounted till they become due. When a bank discounts a note or bill drawn in one place and payable in another, it is customary to charge, in addition to the discount upon it, a certain per centage, called Exchange, determined by the rate current at the time of discount.

2. DEPOSITES. Another source of income to a bank is its deposites, that is, the money placed in the bank by its customers, or the people doing business with the bank, and who call for it as they may want it. The person who places the money in the bank is called the depositor, and receives as a voucher a small book in which are entered his deposites as he makes them from time to time, and his checks, with which he draws out his money as he wants it. Thus daily receiving and paying out sums of money from many persons, there is all the time on deposite a certain average amount which forms a fund that can be used to advantage.

3. CIRCULATION. By their charters, the banks, in consideration of a tax which they pay the state in which they are located, have the privilege of supplying the currency of the country; that is, the circulating medium through which almost all the business of the country is carried on. This is a very important privilege, as the quantity of

money in the country, whether composed of paper or specie, regulates the price of all kinds of property. Now when banks pay out money, whether in discounting notes, or to depositors, or otherwise, they do not pay specie, that is gold or silver, but their own notes, called *Bank Notes*, or *Bank Bills*. These bank notes go out into the community and come back according to the wants of business men. The banks are continually paying out these notes, and taking them back again, or redeeming them, as it is called, as money. By this process going on continually, a certain amount of these bills remains out in the community. This amount is called the *circulation* of the bank, and constitutes a very considerable source of profit, by enabling the bank to discount upon it.

4. EXCHANGE. The *exchange business* is the buying and selling, in one place, of drafts which are to be paid in another. A bank professing to carry on this business can, on application, usually furnish drafts on all the principal commercial cities in the country. A man in Boston, for example, owes a debt which he is bound to pay in New York or New Orleans. Now, if there were no facilities of banks or other offices of exchange, he either would be obliged to go to A, B, and C, to find some one who had funds or money in the place where he owed the money, or he would have to suffer the inconvenience and delay of buying and sending the specie to pay his debt. Instead of this, he goes to a bank, pays his money, and receives for it an order or draft for the amount he owes, on the desired place. This exchange business forms an indispensable agency in all commercial countries.

115. BANK DISCOUNT.

The *Bank Discount* of a note is the interest of the note from the time it is discounted to the time when it becomes due; and differs from the discount of a debt not due, only in the fact that in the case of the discount of *a note of hand*, whether by a bank or an individual, interest is to be taken, not only for the time the note has to run, but also for 3 additional days, called *days of grace*.

It was the custom in former times among merchants, when the time expressed in the note or bill had expired, and the debtor could not pay, to give him, as a *favor*, three days more to pay it. Hence the term grace, or favor. From being a *favor* merely to the promissor of a note, it has got to be his right; for now, all notes and drafts, payable at a future day, are by law entitled to three days of grace. When the last day of grace expires on a Sunday, or some holiday, the note must be paid on the day previous.

1. *Form of a common Note of Hand, such as is usually discounted at Banks.*

Salem, June 30, 1849.

For value received, I promise to pay to A. B., or order, five hundred dollars in four months.

$500. C. D.

The person who signs the note is called the *signer*, or *maker*, or *promissor;* and he to whom it is payable is called the *payee*, before he puts his name on the back of the note; afterwards he is called the *endorser*. If there are two or more names on the back, they are called 1st, 2d, &c., endorsers. He who asks the bank to discount the note is called the *applicant*.

2. *Form of a Joint or Several Note.*

Albany, June 30, 1849.

For value received, we jointly and severally promise to pay to A.B., or order, seven hundred and eighty-nine dollars and $\frac{89}{100}$, in four months from date, with interest afterwards.

C. D.
$789.89.* E. F.

This form of a note is generally used when it is the understanding of the parties that it is not to be paid when due. The payee having the security of two names without notification.

The *face* of a note is the sum specified in it. The *proceeds* of the note is the amount received from the bank after deducting from it the *Bank Discount*.

116. TO FIND THE PROCEEDS OF A NOTE DISCOUNTED AT A BANK.

RULE. *Add three days to the time that will elapse before the note becomes due, and deduct from the face of the note its interest for the time.*

1. What is the bank discount of a note of $500, payable in 30 days, at 6 per cent. per annum? What sum shall be received for the note?

2. A merchant sold goods amounting to $575, for which he received a note payable in 4 months. What are the proceeds of the note, if discounted on the day the note is dated?

* When the cents on a note or draft amount to 50 or more, they are, *in computing the interest*, called $1; when less, *nothing*.

3. *Salem, Nov.* 18, 1848.

For value received, I promise to pay O. P., or order, three hundred dollars, in thirty days.

$300.00. Q. R.

What would be the proceeds of the above note if discounted Nov. 25, 1848?

4. What are the proceeds of a note of $150, dated Nov. 20, payable in 3 months, if the note is discounted Dec. 12th?

5. *Boston, Dec.* 15, 1848.

William Jackson bought of Caleb Thornton,

500	bushels	of Corn,	at	$.62½	per	bushel.
75	"	" Rye,	"	.87½	"	"
200	"	" Oats,	"	.45	"	"

If the above articles are sold at 3 months, and Jackson gives his note for the amount, what is the value of the note Jan. 1, 1849, discounting at 8 per cent. per annum?

6. *Boston, Nov.* 5, 1848.

For value received, I promise to pay to my own order, four hundred dollars in three months, four shares in the Western Railroad being pledged as collateral security.

$400. JOSEPH WINTER.*

What are the proceeds of the above note Dec. 3, 1848, discount at 6 per cent.?

117. TO FIND THE AMOUNT FOR WHICH A NOTE MUST BE WRITTEN, PAYABLE AT A FUTURE TIME, THAT SHALL HAVE A KNOWN PRESENT VALUE.

1. For what amount must a note at 60 days be written, that its present value at the bank may be $500, discount at 6 per cent.?

NOTE. The present value of $1, after deducting the interest for 63 days, is $0.9895. The face of the note must therefore be as many dollars as there times $0.9895 in $500. Hence, to find the amount for which a note due at a future time must be made, in order to obtain a given sum at the bank, we have the following

* Joseph Winter must endorse this note.

RULE. *Divide the sum you wish to obtain by the present value of $1, at the given rate.**

2. I wish to obtain $350 on a note at 3 months. What shall be the face of the note, money being 6 per cent.? 7 per cent.? 8 per cent.?

3. What must be the face of a note at 2 months, the proceeds of which shall be $1000, the rate of discount being 6 per cent.? 5 per cent.?

4. For what sum must a note be drawn, which has 2 months and 18 days (including grace) to run, to pay a debt of $450?

118. BANK EXCHANGE.

A bank usually discounts not only notes payable in the place where it is located, but also notes and drafts made payable in other places, charging a certain per centage, called *Exchange*, in addition to the interest, to pay for the trouble and expense of collecting them.

Form of a Bill of Exchange.

Dollars 500$\frac{00}{100}$. *Salem, July* 1, 1848.

Six months from date, please pay to James Perkins or order five hundred dollars, value received, and charge the same to the account of

JONATHAN TOLMAN.

To John Ingersoll, Esq., }
Merchant, New York. }

NOTE. This draft is payable in New York, because it is drawn on a person living or doing business there. The signer of the draft, viz., Tolman, is called the *drawer;* the person to whom it is made payable, viz., Perkins, is called the *payee*, and also the *endorser*, when he has written his name on the *back* of the draft. The person on whom the draft is drawn, viz., Ingersoll, is called the *drawee*, and also the *acceptor*, when he has written his name on the *face* of the draft, which is then called an *acceptance*. The acceptor is bound to pay the draft when due.

When the draft or bill is payable *within* the country in which it is drawn, it is called an *Inland* Bill of Exchange; when it is payable out of the country, it is called a *Foreign* Bill of Exchange.

1. What would be the proceeds of a draft on New York for $500, dated July 1, 1848, payable 6 months after date, and

* See table I. page 261.

discounted in Salem or Boston, August 6, 1848, the rate of exchange on New York being $\frac{1}{4}$ per cent.?

It is due Jan. 4, 1849. Interest 4 mo. 29 d., $12.42. Exchange, $\frac{1}{4}$ per cent., $1.25, which added, give $13.67. $500 — $13.67 = $486.33, the proceeds of the draft.

2. Find the proceeds of an acceptance for $259.60, dated Sept. 1, 1848, payable in New Orleans sixty days from date, and discounted the same day, the rate of exchange on New Orleans being $1\frac{1}{4}$ per cent.

3. Find the proceeds of a draft on Philadelphia for $3518.75, dated Feb. 21, 1849, payable in 3 months after date, and discounted in New York, March 8, 1849, the rate of exchange being $\frac{1}{4}$ per cent.

In many cases, drafts are not made payable at a future day, but *at sight*, as it is called; that is, to be paid when the person on whom the draft is drawn *sees* it, on presentation to him for payment. No interest therefore, can be taken; they are not in fact discounted, but when payable at a distant place, exchange is charged to pay for collection. This is the general rule, though there are many exceptions in practice.

Form of a Draft at Sight.

4. *Bangor*, *June* 20, 1848.

$200.

At sight, please pay to A. B., or order, in New York, two hundred dollars, value received, and charge to

C. D.

To E. F., Brooklyn, N. Y.

What will be the proceeds of the above draft, exchange being $\frac{1}{2}$ per cent.?

119. PROFIT AND LOSS.

Profit and Loss are terms used to express the gain or loss in mercantile transactions. The gain or loss is usually estimated at a certain percentage of the cost of the article. For example: A merchant bought a barrel of flour for $6.00. At what price must he sell it to gain 10 per cent.? 10 per cent. of $6 is 60 cents. This added to $6, gives $6.60, the price at which he must sell it.

1. A farmer bought a yoke of oxen for \$80. What must he sell them for to realize a profit of 10 per cent.? $12\frac{1}{2}$ per cent.? 20 per cent.?

2. A boy bought a knife for 40 cents, and sold it at a loss of 10 per cent. How much did he sell it for? For how much must he have sold it to gain 10 per cent.? to lose 20 per cent.?

3. If I buy cloth at \$5 per yard, at what price must I sell it in order to gain 8 per cent.? 12 per cent.? 15 per cent.?

4. A merchant buys flour at \$5 per bbl., and sells it at \$6. What per cent. does he gain?

NOTE. His gain is $\frac{1}{5}$ of the cost. *Ans.* 20 per cent. (**78** and **102**, Ex. 21, &c.) If he sells it for \$4, what per cent. does he lose?

5. Bought a watch for \$25. What per cent. shall I lose if I sell it for \$24? for \$20? for \$15? What per cent. shall I gain by selling it for \$26? \$27? \$30?

6. A man paid \$500 for railroad stock. What per cent. does he gain by selling it for \$520? for \$540? What per cent. would he lose by selling it for \$490? for \$475?

7. A drover sold an ox for \$42, by which he gained 20 per cent. of what the ox cost. What was the cost of the ox?

If he sold it so as to gain 20 per cent. of the cost, he must have sold it for $\frac{120}{100} = \frac{6}{5}$ of the cost. Therefore, \$42 is $\frac{6}{5}$ of the cost, and the cost is $\frac{5}{6}$ of \$42 = \$35, the answer.

8. A trader sells molasses at 27 cents a gallon, by which he gains $12\frac{1}{2}$ per cent. What did the molasses cost?

27 cents is $\frac{112\frac{1}{2}}{100} = \frac{9}{8}$ of what?

9. A miller sells meal at 63 cents per bushel, by which he loses 10 per cent. of the cost. What did the meal cost him per bushel?

NOTE. 63 cents is $\frac{90}{100} = \frac{9}{10}$ of the cost.

10. At what price per bushel must a miller sell meal that cost 70 cents per bushel, so as to lose 10 per cent.?

11. Sold cloth for \$3.50 per yard, and by so doing lost $12\frac{1}{2}$ per cent. What was the cost per yard? \$3.50 is $\frac{87\frac{1}{2}}{100}$ of what?

12. Bought cloth at \$4 per yard, and sold it at a loss of $12\frac{1}{2}$ per cent. At what price did I sell it?

13. Bought cloth at $4.00 per yard, and sold it at $3.50. What per cent. did I lose?

14. A merchant sold flour for 50 cents a bbl. more than it cost him, by which he gained 10 per cent. What was the cost of the flour, and at what price did he sell it?

50 cents must be $\frac{10}{100}$ of the cost.

15. Sold cloth at 50 cents per yard less than the cost, by which I lost $12\frac{1}{2}$ per cent. What did the cloth cost?

16. A merchant, with a capital of $10,000, has lost in one year $1500. What per cent. of his capital has he lost?

17. A stock broker bought railroad stocks at $$105\frac{1}{4}$ per share, which he is obliged to sell at $$102\frac{1}{2}$. What per cent. does he lose by the transaction?

18. Bought 75 bbl. of flour at $5 cash, and sold it at $5.50 on a note of 3 months. What per cent. do I gain, if I get my note discounted at 8 per cent.?

19. Bought 3000 gallons of molasses for $700. At what price per gallon must I sell it to gain 15 per cent.?

20. A bankrupt has property amounting to 3500 dollars. What per cent. do his creditors lose, if his debts amount to $5000?

21. Sold corn for 65 cents a bushel, by which I gained 12 per cent. What was the corn per bushel? (See quest. 8.)

22. A merchant, by selling sugar at 8 cents a pound, loses 8 per cent. Will he gain or lose by selling it at 10 cents per pound? How much per cent?

NOTE. He will sell it for $\frac{10}{8} = \frac{5}{4}$ of 92 per cent. of the cost. Why? How much if he sell it at $10\frac{3}{4}$ cts. per pound?

23. If, by selling flour at $$5.87\frac{1}{2}$ per bbl., I lose 6 per cent., how much per cent. shall I gain or lose by selling it at $6.25 per bbl.? How much if I sell it at $6.50 per bbl.?

24. A merchant buys rice at $4.50 per 100 lb., and it has risen while in his hands to $5.25 per 100 lb. What per cent. is the rise?

25. Sold 300 bushels of corn for $14.40 more than the cost, by which I gained 8 per cent. of the cost. What was the cost per bushel?

26. A merchant bought goods, which he marked at 25 per cent. above the cost. If he should sell them at $12\frac{1}{2}$ per cent. below the marked price, what per cent. does he gain on their first cost?

120. Equation of Payments.

Equation of Payments is the process of finding an average time for the payment of several debts due at different times, without loss to either debtor or creditor.

1. A man owes $500 to be paid in 2 months, $350 to be paid in 4 months, and $275 to be paid in 6 months. At what time may he pay the whole so that neither party may lose any interest?

The interest of	$500 for 2 mo.	= the interest of	$1000 for 1 mo.
The interest of	$350 for 4 mo.	= the interest of	$1400 for 1 mo.
The interest of	$275 for 6 mo.	= the interest of	$1650 for 1 mo.
The interest of	$1125 for — mo.	= the interest of	$4050 for 1 mo.

The interest on $1125, the whole debt, is equal to the interest of $4050 for 1 month. The interest on the whole debt, for the *average* time, should equal the interest of $4050 for 1 month. The average or equated time must therefore be as many months as there are times 1125 in 4050, which is $3\frac{3}{5}$. *Ans.* 3 mo. 18 d.

Rule. *Multiply each debt by the time to elapse before it is due, and divide the sum of the products by the sum of the debts; the quotient is the equated time.*

2. A man owed $20 to be paid in 3 months, $50 in 4 months, and $80 in 5 months. What is the average time for the payment of the whole sum?

3. A man purchased 2000 dollars' worth of goods; $500 of which he is to pay now, $400 in 60 days, $500 in 90 days, and the rest in 120 days. What is the equated time for the payment of the whole?

Note 1. As one of the payments is to be made on the day from which the equated time is to be reckoned, the product of that payment by its time will be nothing; as the time is nothing; but that payment must be added to get the amount of the payments.

Note 2. Any fraction of a day which is less than one half is not counted; if one half, or more than one half, it is reckoned as 1 day.

Operation.

$500	— days		——
400	× 60	=	24000
500	× 90	=	45000
600	× 120	=	72000
2000		)	141000

Ans. 71 days. 70½ da.

4. What will be the average time for the payment of the following account without loss to either party: $200, due Nov. 1st; $350, due Nov. 16th; $150, due Dec. 4th; and $100, due Dec. 16th?

Nov. 1,	$200	days ——
" 16,	350	15 = 5250
Dec. 4,	150	33 = 4950
" 16,	100	45 = 4500
	800	) 14700

18 d. from Nov 1. *Ans.*, Nov. 19.

5. Average the following account: $75, due April 1; $80, due April 21; $45, due May 1; and $30, due May 16.

6. The following purchases of goods were made on a credit of 4 months: June 1, $175; June 26, $250; July 21, $450; Aug. 12, $100. What is the equated time for the payment of the whole?

NOTE. It will be 4 months and — days after Oct. 1. The several sums were due Oct. 1st, Oct. 26th, Nov. 21st, and Dec. 12th.

7. William Davidson to Joseph Wilkins, Dr.,

Aug. 10, 1848,	To a bill of goods at 3 mo.,	$200.00.
" 15,	" " " " " 4 mo.,	110.00.
Sept. 25,	" " " " " 2 mo.,	150.00.

What is the average time for the payment of the whole amount?

NOTE. In this account $200 is due Nov. 10, $110 Dec. 15, and $150 Nov. 25. The question is, therefore, just like the preceding examples.

Make out in form, accounts of the transactions named in No. 8, 9, and 10, and find the average time for the payment of each account.

8. Caleb A. Moore, of Concord, purchases of J. P. Williams, of Boston, as follows: Jan. 15, 1849, a quantity of merchandise, amounting to $354.87, on a credit of 4 months; Jan. 25th, another quantity, amounting to $608.50, at 3 months; Feb. 20th, another, amounting to $150, at 4 months; March 25th, another quantity, to the amount of $175.18, at 2 months.

9. I have bought of Andrew Jackson as follows: May 16, 1848, 875 lb. of sugar, at 8¼ cts., on a credit of 3 months: July 7, 350 gal. of molasses, at 24 cts., on 4 months, and 500 lb. of rice, at 5¼ cts., on 2 months; July 18, 175 lb. of tea, at 37½ cts., on 3 months; Aug. 3, 50 barrels of flour, at $5.75, cash (that is no credit is to be allowed on it); Aug. 8, 850 yds. of cotton cloth, at 12½ cts., on 6 months.

10. I have sold to James Robinson as follows: Feb. 6, 1849, 1525 lb. of pork, at 7¼ cts., on 3 months; Feb. 20, 825 lb. beef, at 8¼ cts., on 2 months; March 10, 1000 bushels of corn, at 62½ cts., on 2 months; Apr. 3, 470 bushels of oats, at 37½ cts., on 3 months; Apr. 10, 1548 lb. of cheese, at 8 cts., and 285 lb. of butter, at 16 cts., cash; May 1, 75 bushels of wheat, at $1.15, on 2 months.

More exercises in Equation of Payments occur on pages 161, 162, 164, and 165.

121. EXCHANGE OF CURRENCIES.

Exchange of Currencies (**15**, margin) is the process of changing the currency of one country to its equivalent value in the denominations of another country.

Although the Federal currency is the currency of the United States, the English denominations of money are still employed, to some extent, in the ordinary reckonings of the people. The estimated value of these denominations differs in different places, as follows:

The English or Sterling pound is equal to $\$4\frac{4}{9}$; $\$1 = £\frac{9}{40} = 4s.\ 6d.$

In Canada and Nova Scotia, $£1 = \$4$; $\$1 = £\frac{1}{4} = 5s.$

In New England, Indiana, Illinois, Missouri, Virginia, Kentucky, Tennessee, Mississippi, Alabama, Florida, and Texas, $£1 = \$3\frac{1}{3}$; $\$1 = £\frac{3}{10} = 6s.$

In New York, Ohio and Michigan, $£1 = \$2\frac{1}{2}$; $\$1 = £\frac{2}{5} = 8s.$

In Pennsylvania, New Jersey, Delaware and Maryland, $£1 = \$2\frac{2}{3}$; $\$1 = £\frac{3}{8} = 7s.\ 6d.$

In Georgia and South Carolina, $£1 = \$4\frac{2}{7}$; $\$1 = £\frac{7}{30} = 4s.\ 8d.$

EXAMPLES. **Reduce £15 7s. 6d. New England currency to Federal money.**

Since $£1 = \$3\frac{1}{3}$, there will be $\frac{10}{3}$ as many dollars as there are pounds. Reducing the shillings and pence to a decimal, £15 7s. 6d. = £15.375. (Page 132, bottom.) $15.375 \times \frac{10}{3} = 51.25 = \$51.25$, the answer.

Reduce $25.35 to the Pennsylvania currency.

Since $\$1 = £\frac{3}{8}$, there will be $\frac{3}{8}$ as many pounds as dollars. $25.35 \times \frac{3}{8} = 9.50625 = £9\ 10s.\ 1\frac{1}{2}d.$

1. **Reduce $540.50 to each of the six currencies above named.**
2. Reduce £52 7s. 8d. of each of the above currencies to Federal money.
3. Reduce $1.25 to each of the above currencies.
4. Reduce 7s. 6d. of each of the above currencies to Federal money.

122. Although the English pound is usually estimated at 4\frac{4}{9}$, which is its *par* value, its *real* value, by an act of Congress, passed in 1837, is 4.86\frac{2}{3}$ nearly; so that its real value is at a certain percentage above its par value. This percentage is called the *premium of exchange;* and were it not for the fluctuations in trade, and other accidental circumstances, it would be about 8 per cent. Thus, when it is said that the premium of exchange on England is 9 per cent., that is, 9 per cent. above par, it is meant that its real or commercial value is 109 per cent. of its par value; that is, £1 = 4\frac{4}{9} \times 1.09$; and that $1 = £$\frac{9}{40} \div 1.09$. If the premium is 9$\frac{1}{2}$ per cent., £1 = 4\frac{4}{9}$ $\times$ 1.095; and $1 = £$\frac{9}{40} \div 1.095$. Therefore,

TO REDUCE ENGLISH MONEY TO FEDERAL MONEY.

Add 1 *unit to the premium expressed decimally, and* MULTIPLY *its par value in Federal money* (**121**) *by the sum.*

TO REDUCE FEDERAL MONEY TO ENGLISH MONEY.

Add 1 *unit to the premium expressed decimally, and* DIVIDE *its par value in English money* (**121**) *by the sum.*

1. When exchange on London is at a premium of 8$\frac{1}{2}$ per cent., what is the value of £100 17s. 6$\frac{3}{4}$d. in Federal money? What is the value of $500.50 in English money?

.8 = one half of 16s.
.05 = the odd shilling.
.028 = 6$\frac{3}{4}$d.

£.878 = 17s. 6$\frac{3}{4}$d.
£100.878 $\times$ 4$\frac{4}{9}$ $\times$ 1.085 = $486.46, 1st *Ans.*
$500.50 $\times \frac{9}{40} \div$ 1.085 = £103.79 = £103 15s. 9$\frac{1}{2}$d., 2d *Ans.*

2. When exchange on London is at a premium of 9 per cent., what is the value in Federal money of £1? £17? 16s. 8d.? £16 17s. 9d.? £8 0s. 5$\frac{3}{4}$d.?

3. When exchange on London is at 10 per cent. premium, what is the sterling value of $1? $15.50? $175.83?

4. When the rate of exchange is at a premium of 8$\frac{3}{4}$ per cent., what is the sterling value of $100? $54.75? $1575?

5. What is the Federal value of £100? Of £75 18s. 6$\frac{1}{4}$d.? £1500 8s. 9d.?

QUESTIONS. How may most business operations be abbreviated? What are the aliquot parts of a number? Repeat the table of "Parts of a dollar." "Parts of a year." How may you multiply a number by 5? Why? By 25? Why? By 50? Why? By 12½? Why? By 125? Why? By 33⅓? Why? How may you divide by 25? Why? By 125? Why? By 12½? Why? By 50? Why? By 33⅓? Why? By any number of 9's? Why?

For what is the term *percentage* used? What does per cent. mean? Give examples. What is the rule for computing percentage? What, if the per cent. contains a fraction that cannot be exactly expressed in decimals?

What are *commission* and *brokerage?* What is an *agent* sometimes called? *When?* How is his compensation usually estimated?

What is *insurance?* What is the *premium?* The *policy?*

What are *stocks?* What are *shares?* *Stockholders?* What is *par value?* *Premium?* *Discount?* Give examples. How are the profits arising from stocks divided? What is a *dividend?*

What are *taxes?* How are they levied? What is a poll tax? A *poll?* In making taxes, how is the amount to be raised on *property* to be ascertained? How do you find the tax on $1? How is the amount of each man's tax found? What is real estate? Personal estate?

What are *duties?* What is a *specific duty?* An *ad valorem duty?* What expenses are included in estimating the *cost?* How are duties levied in the United States by the tariff of 1846? Why is merchandise, imported into the country, weighed or measured? What is *gross* weight? *Net* weight? *Draft?* *Tare?* What is said of the practice of merchants in respect to draft, scalage, damage, &c.?

What is *interest?* *Principal?* *Rate?* *Amount?* What is the rule for finding the interest of $1 for a given time, when the rate is 6 per cent. per annum? What is the reason for this rule? What is the rule for finding the interest on any given principal? How do you find the interest for any other rate than 6 per cent.? Give examples.

What is the rule for finding the interest on pounds, shillings, pence, &c.? What is the rule for reducing shillings, pence and farthings, to the decimal of a pound? Give an example.

What is the *legal* rule for *partial payments?* How should endorsements be expressed? What easier rule is generally adopted? What is *compound interest?* What is the *rule* for compound interest?

Repeat the rule when the principal, interest and time are given, to find the *rate*. Principal, interest, and rate given, to find the *time*. Time, rate, and interest given, to find the *principal*. Amount, time, and rate given, to find the *principal*. How is the present worth of sums of money due at a distant future day to be estimated? What is the practice of banks and merchants in computing the present worth of a note payable at a future time?

What is *discount?* What is the rule for computing discount?

What is *banking?* What are banks? What is *bank discount?*

How does the *bank discount of a note* differ from the discount of a debt not yet due? What is the origin of the term "days of grace?" What is to be done if the last day of grace expires on a Sunday or holiday? Repeat the form of a common note of hand, such as is discounted at banks. Who is the promissor? The payee? The endorser? The applicant? Repeat the form of a joint or several note. When is this form used? How is discount computed when the cents on a note or draft amount to 50 or more? To less than 50? What is meant by the *face* of a note? The *proceeds* of a note?

What is the rule for finding the proceeds of a note discounted at a bank? The rule for finding the amount for which a note must be written, in order to obtain a given sum from a bank?

What is meant by *bank exchange?* Repeat the form of a bill of exchange. Who is the drawer of a draft? The endorser? The payee? The acceptor? What is an acceptance? What is an inland bill of exchange? A foreign bill of exchange? What is meant by a draft being *payable at sight?*

What is meant by *profit and loss?* How is gain or loss usually estimated? Give an example.

What is *equation of payments?* What is the rule?

What is *exchange of currencies?* What is the value of a dollar in English or sterling money? In Canada? What is its value in shillings and pence in the different states of the Union? What is the *par* value of a pound sterling expressed in Federal money? What is the real value of the English pound in Federal money? What is meant by the premium of exchange? Give an example. What is the rule for reducing English or sterling money to Federal money? For reducing Federal money to English money?

123. ANALYSIS.

Review Art. 95.

1. If $5\frac{1}{4}$ yards of cloth cost 63 cents, what will $7\frac{3}{4}$ yards cost?

NOTE. One yard will cost $\frac{4}{21}$ of 63 cents. Why? And $7\frac{3}{4}$ yards will cost $\frac{31}{4}$ of $\frac{4}{21}$ of 63 cents. Why?

2. If $3\frac{1}{4}$ bushels of corn cost $1.95, what will $15\frac{7}{8}$ bushels cost? $19\frac{5}{16}$ bushels?

3. If $25\frac{1}{8}$ lb. of coffee cost $2.55, what will $3\frac{3}{4}$ lb. cost? $5\frac{7}{8}$ lb.?

4. If $3\frac{1}{16}$ lb. of butter cost 63 cents, what will $15\frac{7}{8}$ lb. cost? $25\frac{1}{10}$ lb.?

5. If $4\frac{1}{8}$ acres of land cost $55, how much will $100 buy? $150.25? $238\frac{1}{8}$?

6. If $\frac{3}{8}$ of a barrel of flour cost $\$2\frac{3}{4}$, what is the cost of $5\frac{3}{8}$ barrels? $8\frac{7}{8}$ barrels?

NOTE. $5\frac{3}{8}$ barrels will cost $\frac{43}{8}$ of $\frac{8}{3}$ of $\frac{11}{4}$ of one dollar. Why?

7. Bought $4\frac{5}{8}$ bushels of corn for $\$3\frac{3}{8}$; what will $8\frac{7}{8}$ bushels come to, at the same rate? $3\frac{5}{8}$ bushels? $10\frac{4}{5}$ bushels?

8. If $5\frac{3}{10}$ tons of hay cost \$70.50, what is the price of $3\frac{7}{10}$ tons? $14\frac{1}{8}$ tons? $25\frac{1}{20}$ tons?

9. Sold $3\frac{5}{8}$ bushels of apples for $\$3\frac{1}{3}$; what should be paid for $4\frac{5}{8}$ barrels? $3\frac{3}{4}$ barrels? $8\frac{9}{10}$ barrels?

10. If $\frac{3}{8}$ of a ship is worth \$3582, what is the whole ship worth?

11. If $\frac{3}{8}$ of $\frac{5}{6}$ of a ship cost \$3000, what is the whole ship worth? $\frac{3}{4}$ of the whole? $\frac{5}{6}$ of the whole?

12. If $\frac{3}{8}$, $\frac{5}{12}$ and $\frac{4}{5}$ of an acre of land cost \$375, what is the cost per acre?

13. A farmer sold $7\frac{1}{2}$ bushels of corn at $62\frac{1}{2}$ cents per bushel, and took his pay in molasses at 25 cents per gallon; how many gallons did he receive?

NOTE. Such exchanges of articles of commerce are called *Barter*. Specific rules for solving such questions are useless; the general rule of common sense being sufficiently specific for the purpose. The same remark may apply to many arithmetical operations for which particular rules have been framed.

14. How many bushels of wheat, at $\$1.12\frac{1}{2}$ per bushel, must be given in exchange for $25\frac{1}{2}$ yards of cloth, at $\$3\frac{7}{8}$ per yard?

15. How many pounds of sugar, at $8\frac{3}{4}$ cents per lb., must be given for $28\frac{1}{2}$ bushels of potatoes, at $62\frac{1}{2}$ cents per bushel?

16. A farmer bought 34 sheep, at $\$1\frac{7}{8}$ apiece; how many bushels of oats will pay for them, at $37\frac{1}{2}$ cents a bushel?

17. What will 950 bushels of rye come to, at $\frac{3}{4}$ of a dollar per bushel? (**100.**)

18. What will 856 bushels of potatoes come to, at 25 cents per bushel? at $33\frac{1}{3}$ cts.? at $37\frac{1}{2}$ cts.? at 50 cts.? at $12\frac{1}{2}$ cts.? at 75 cts.? at $87\frac{1}{2}$ cts.? (**100** and **101.**)

19. What will 250 acres of land cost, at £1 8s. 6d. per acre? (**100.**) at £3 9s. 5d.?

20. What will 35 cords 7 C. ft. of wood come to, at $\$4\frac{7}{8}$ per cord?

21. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$ of a certain number is 285. What is the number?

22. $\frac{4}{7}$ of a certain number exceeds $\frac{3}{8}$ of it by 60. What is the number?

23. Two persons start from the same place and travel in the *same* direction, one at the rate of $4\frac{5}{8}$ miles per hour, and the other at the rate of $5\frac{1}{6}$ miles per hour; how far apart will they be in 6 hours? in $5\frac{1}{4}$ hours? How far apart will they be in the same time, if they travel in *opposite* directions?

24. In how many hours will the above travellers be 50 miles apart, if they travel in the *same* direction? In how many hours, if they travel in opposite directions?

NOTE. Reduce the fractional parts of the answers to the following questions to the lowest denomination.

25. A cistern has 2 pipes; the first can fill it in 2 hours, and the second in 3 hours; what part of it will each pipe fill in one hour? What part of it will both pipes fill in one hour? How long will it take them both to fill it?

26. A cistern has 3 pipes; the first will fill it in 2 hours, the second in 3 hours, and the third in 4 hours; how long will it take them all to fill it?

27. If, in the last example, the first will fill the cistern in 2 hours, the second in 3 hours, and the third will *empty* it in 4 hours, in what time will the cistern be filled, if all are set running together?

28. A reservoir has 4 pipes; the first will fill it in 3 hours, the second in 2 hours, the third in $\frac{1}{2}$ an hour, and the fourth will empty it in $\frac{3}{4}$ of an hour. If all were left open, in what time would the reservoir be filled?

29. A cistern has 4 pipes; the first will fill it in $\frac{1}{2}$ an hour, the second in $\frac{1}{3}$ of an hour, the third in $\frac{1}{4}$ of an hour, and the fourth will empty it in $\frac{1}{8}$ of an hour. In what time will the cistern be filled, if all are left open together?

30. What sum of money at simple interest will amount to $1000 in 5 years, at 6 per cent.? at $5\frac{1}{2}$ per cent.? (**112.**)

31. At what rate per cent. simple interest will $1000 amount to $1500 in 5 years? in 8 years?

32. In what time will $500 amount to $750, at 6 per cent. simple interest?

33. Bought a box of sugar for $40, but not proving so good as I expected, I sold it for $35; what per cent. did I lose?

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34. Sold a quantity of merchandise for $20 more than it cost, by which I gained 8 per cent.; what was the cost of the merchandise? What was it sold for?

35. William can do a piece of work in 3 hours; with the assistance of John, he can do it in $1\frac{1}{5}$ hours; in what time can John do it alone?

II. 124. Miscellaneous Examples in Percentage.

The following abbreviations are used in merchants' accounts, viz. — *Bo't* for bought; *Co.* for company; *Mdse.* for merchandise; *B. P.* for bill of parcels; *Recd. payt.* for received payment; *gro.* for gross; *ea.* for each; @ for at. Shillings are usually expressed by an oblique line, thus /; as 6/, for 6 shillings; $^5/_3$ for 5 shillings and 3 pence; the time at which a sum of money falls due, on which the days of grace are allowed, is frequently expressed thus, March $^3/_6$, meaning by it that the time and grace expired March 6th. Per cent. is often expressed thus, $^o/_o$.

Examples.

1. What is the interest of $500.75 for 2 yr. 4 mo. 18 da., at 6 per cent.? at $6\frac{1}{2}$ per cent.? at 7 $^o/_o$? at $7\frac{1}{2}$ $^o/_o$? at $8\frac{1}{2}$ per cent.? at $4\frac{1}{2}$ per cent.? at $5\frac{1}{4}$ $^o/_o$?

2. *Concord, Feb.* 27, 1848.

For value received, I promise to pay William Coggswell, or order, $275.45, in 4 months, with interest afterward.

$275.45. Solomon Paywell.

Endorsements.

Aug. 15, 1848, recd. one hundred dollars.
Oct. 8, 1848, recd. fifty dollars.
Jan. 17, 1849, recd. five dollars.
April 12, 1849, recd. fifty dollars.

What was due on the above note May 15, 1849? Perform it by both the legal rule and the common business rule.

3. A merchant bought 5000 gallons of molasses at 20 cts. per gallon cash, and sold it immediately at $21\frac{1}{2}$ cents. He received in pay a note payable in 4 months, for one half of the amount, and one at 6 months, for the remainder. Does he

gain or lose by the transaction, if he gets his notes discounted at the rate of 6 per cent. per annum? How much per cent.? How much does he gain or lose, if his notes are discounted at 9 per cent. per annum?

4. A person disposes of 13 shares of bank stock, the par value of which is $100, at 2¾ % premium. How much does he realize for them, after paying the broker 25 cents per share for selling them? How much, if they are sold at a discount of 4½ %? How much, if at a premium of 12⅛ per cent.?

5. How much railroad stock, that is 10 per cent. above par, the par being $100, will pay a debt of $500? How much that is at a discount of 3 per cent.? 4½ %? 15 %? 23¼ %? How much that is at a premium of 2¼ per cent.? 5⅜ per cent.?

6. William Nichols, of Baltimore, purchased of Stephen Goodhue, Boston, 25 hhds. of molasses, containing 3286 gallons, at 25 cents per gallon, cash. The charges are as follows: Freight, $15.30; Drayage, $6.87; Storage, $5; Gauging, $1.50; Cooperage and Labor, $4.20; Advertising and Postage, $2; Insurance, $2.50; other expenses, $5.42. At what cash price per gallon must he sell it, in order to gain $100 on the whole? to gain $75? to gain 5 per cent.? 7½ per cent.?

7. Does the merchant gain or lose by selling his molasses at 28 cents per gallon, and how much? How much at 24 cents per gallon? at 26½ cents?

8. At what price per gallon, on a credit of 4 months, must the merchant in the last question sell his molasses to gain $100? At what price to gain 5 per cent.?

9. What must I pay for insuring my house for $800, and my barn and its contents for $1000, the premium being 1¾ per cent., one dollar additional being charged for the policy?

10. Sold corn at 60 cents per bushel, and by so doing lost 10 per cent.; at what price must I sell it in order to *gain* 10 per cent.? 15 per cent.? 12½ per cent.?

11. A Boston merchant buys in Baltimore 1000 barrels of flour at $4.50 per barrel, cash; he pays for freight 25 cts. per barrel, other expenses $40.00, and sells it 30 days after the purchase at $5.25 per barrel, receiving in payment a note payable in 4 months. How much per cent. does he gain, reckoning interest on the cost, and discount on the note at the rate of 7 per cent. per annum?

12. Bought 100 boxes of sugar, weighing 42,853 pounds gross, less 15 per cent. tare, at \$8.75 per 100 lb., less $2\frac{1}{2}$ per cent. for cash. What was the cash price? How much per 100 lbs.?

13. At what price per lb. must the purchaser sell the above sugar, in order to gain 5 % on the purchase money?

14. What will 100 bags of pepper come to, weighing gross 13,975 lb., tare 2 per cent., at \$8.75 per 100 lb., less 4 % for cash?

15. My agent at Philadelphia informs me that he has sold for me 10 barrels of cloves, viz.:

2 barrels,	weighing	gross	272 lb.,	tare	46	lb.,	at	$25\frac{1}{2}$	cts.
3	"	"	437 "	"	87	"	"	25	"
5	"	"	730 "	"	$127\frac{1}{2}$	"	"	24	"

What are the net proceeds of the sale, after deducting $2\frac{1}{2}$ per cent. on the amount of sales for commission, and other charges amounting to \$2.50?

16. What will be the net proceeds of a draft payable at sight (**118**) for the amount of the above sales, exchange on Philadelphia being $\frac{1}{2}$ per cent.?

17. Sold 26,987 lb. of hemp, at \$187.50 per ton of 2000 pounds, less $3\frac{1}{2}$ % for cash. What did it come to, after the discount? What would it come to, reckoning the ton at 2240 pounds?

18. If the above hemp were sold on a note payable in 4 months, what would be the cash proceeds of the sale, if the note were discounted at the rate of 8 per cent. per annum, reckoning the ton at 2000 pounds? at 2240 pounds?

19. A merchant in Boston owes £500 15s. 8d. in Liverpool; how much Federal money will purchase a bill for this amount, exchange being at a premium of $9\frac{1}{2}$ per cent.? How much if the premium be $8\frac{1}{2}$ per cent.? $8\frac{3}{4}$ per cent.? $9\frac{5}{8}$ per cent.? (**122.**)

20. A person in New York wishes to remit to London \$1000; for how many pounds, shillings and pence must his bill be drawn, exchange being $8\frac{1}{4}$ per cent. premium? $7\frac{3}{4}$ %? $9\frac{1}{4}$ %? 10 %?

21. *Philadelphia, March* 5, 1849.

For value received, I promise to pay Moses Austin, or order, five hundred dollars in three months?

$500.00. JOSEPH PERKINS.

What would be the proceeds of the above note, if discounted March 20th, at the rate of 1 per cent. per month? at $1\frac{1}{4}$ per cent. per month? at $1\frac{3}{4}$ °/₀? at 2 °/₀?

22. *Boston, April* 12, 1849.

Messrs. ALLEN & THOMPSON,

Bo't of WILLIAM MARSHALL,

5060 lb. Pork,	@ $6\frac{3}{4}$ cts., @ 2 mo.,		$
3075 lb. Beef,	@ $7\frac{1}{4}$ " " 2 "		
1575 lb. Bacon,	@ 8 " " 3 "		
			$
	Discount at 9 per cent. per annum,		$

Recd. Payt.,

WILLIAM MARSHALL,

Per DAVID R. BREWSTER.

NOTE. The *time* for which the discount of the above bill is to be made is the time which will elapse before the equated time at which the whole bill is payable; or, discount may be made on each item separately. Do it by both methods.

23. *Cincinnati, March* 10, 1849.

WILLIAM GORDON, Esq.,

Bo't of JONATHAN FARMER,

340 bush.	Corn,	@ 25	cts., on a credit of 2 mo.,	$
375 "	Wheat,	" 60	" " " " " " "	
600 "	Potatoes,	" 12	" " " " " 3 "	
2587 lbs.	Pork,	" $3\frac{1}{4}$	" " " " " 4 "	

What cash will pay the above bill, discounting at the rate of 10 per cent. per annum?

Make the discount, and discharge the bill; that is, receipt it.

24. *St. Louis, April* 3, 1849. Thomas Williams buys of Theodore Smith 27 casks of Bacon, @ $10.50; 10 barrels of Beef, @ $7; 18 barrels of Lard, @ $10; 250 barrels of flour, @ $3.25. Smith allows $5\frac{1}{2}$ per cent. discount for cash.

Write a bill of parcels, and discharge it.

25. Boston, March 1, 1849. Joseph Johnson buys of Wm. Daniels, 3 hhds. Havana Sugar, 3840 lb., @ 5½ cts.; 6 bags Java coffee, 720 lb., @ 12½ cts.; 10 casks raisins, @ $9.75; 5 chests Y. H. tea, 450 lb., @ 65 cts., and gives in payment his note at 6 months.

Write a bill of parcels, and receipt it. Write a note for the amount, and find what the proceeds of it will be, discounting at the rate of 6 per cent. per annum.

26. Samuel Taylor, Esq., to James Merchant, Dr.,

1848.								
Sept. 28,	For	2	pieces	broadcloth,	50	yards,	@	$3.50.
"	"	"	"	"	47	"	"	4.25.
Oct. 8,	"	2	"	cassimere,	32	"	"	1.75.
" 15,	"	6	"	sheetings,	125½	"	"	.08½.
Nov. 2,	"	3	"	linen,	75½	"	"	.70.
" "	"	2	"	"	49¾	"	"	.62½.
" "	"	2	"	broadcloth,	48½	"	"	4.50.

The above goods were purchased on a credit of 4 months. The amount was settled by a note payable in 4 months from date, without interest till that time. What was the equated time for the date of the note? Write the note, and upon the back of it write the following endorsements. (**109**, marginal note.)

The following endorsements were made on the above note; viz., Oct. 28, 1848, $145.00. Nov. 20, 1848, $75. Jan. 10, 1849, $200. March 12, 1849, $75. The balance was paid March 25, 1849. How much was it, allowing interest at the rate of 8 per cent. per annum?

NOTE. Get the amount of all the payments to the time of settlement; then subtract the amount of all the payments at the time of settlement, from the amount of the note at the same time.

125. ACCOUNTS CURRENT. INTEREST ACCOUNTS.

II. An *Account Current* is a statement of the mercantile transactions between two persons, arranged in the form of debtor and creditor, and exhibiting the state of their affairs up to any given date.

The term Dr. is used to indicate that the person *with* whom the account is kept is debtor for the sums on the left, and the term Cr., to indicate that he is creditor for sums on the right.

The word *To* is employed to denote debtor, and the word *By* to denote creditor. The word *current*, as here used, indicates the present state of an account of continuous or successive transactions from one period to another, while other accounts, as bills of parcels, &c., embrace only *particular* transactions.

Accounts Current are usually made up every six or twelve months, and it is the practice to *charge* interest on sums on the debtor side, that fall due before the time of making up the account, and to *allow* interest on sums on the creditor side, that fall due before the time of making up the account.

Accounts Current may be arranged and settled in different ways; viz., 1. The equated time may be found, at which the balance should be paid without loss of interest to either party. 2. This balance may be paid at some other specified time, interest being added, if paid *after*, or discount deducted, if paid *before*, the equated time. Or, 3. The interest on each item, from the time it falls due to the time to which the account is made up, may be placed opposite to it, and the difference between the Dr. and Cr. sides carried to the proper sides of the Account Current.

The examples that follow will illustrate these different methods.

1. If A lends B \$50 for 2 months without interest, how long should B lend A 25, to repay the favor?

NOTE. The interest of \$50 for 2 months = the interest of \$100 for 1 month; therefore, A must have the use of \$25 of B's money as many months as there are times \$25 in \$100. *Ans.* 4 months. How long may A keep \$75, to balance the favor? How long \$200? \$150? \$100? \$125? \$500?

2. A has had the use of \$200 of B's money for 3 months without interest. How long may B keep \$50 of A's money, that neither party may lose any interest? How long may B keep \$75 of A's? \$150? \$500? \$1000?

3. A owes B \$10, payable June 15th; B owes A \$15, which falls due June 18th. When should B pay the balance of \$5, that neither may lose any interest?

NOTE. On the 18th, A will have had the use of \$10 of B's money for 3 days = \$30 for 1 day. B may therefore keep the balance, \$5, 6 days *after* the 18th. *Ans.* June 24th.

4. A owes B \$15, which will be due June 15th; B owes

A $10, payable June 18th. When ought A to pay the balance without interest?

NOTE. If it is not paid before the 18th, A will have had the use of $15 of B's money for 3 days = 45 for $1 day, or $5 for 9 days. B, therefore, should either receive the balance of $5 and 9 days' interest, on the 18th, or should receive the balance 9 days *previous* to the 18th. *Ans.* June 9th.

From these examples we derive the following rule for equating an account so that neither debtor nor creditor shall be entitled to any balance of interest.

RULE. *Find the equated time of the Dr. and Cr. sides; then multiply the sum first due by the number of days between the dates, and divide the product by the balance of the account; the quotient will be the equated time in days from the* LATEST DATE; *to be reckoned* BACKWARD, *if the sum last due is the smaller; to be reckoned* FORWARD, *if the sum last due is the larger.*

5. A owes B $50, due Jan. 25th; B owes A $75, due Feb. 4th. When ought the balance to be paid?

6. A owes B $75, due Jan. 25th; B owes A $50, due Feb. 4th. When ought the balance to be paid?

7. The Dr. side of an account is $800, due April 1, 1849; the Cr. is $1000, due March 1, 1849. When should the balance be paid? When, if the Cr. were due May 1, 1849?

8. The Dr. side of an account is $10050.75, due July 8, 1850; the Cr. is $5582.50, due Sept. 12, 1850. When should the balance be paid? When, if the Dr. were due Oct. 8, 1850?

9. *William Symonds in account current with Joseph Watson.*

Dr.					*Cr.*
1848.			1848.		
Jan. 16,	To mdse. at 8 mo.,	$300.00	June 1,	By cash,	$200.00
" 27,	" " " 4 mo.,	250.00	" 25,	" "	300.00
Mar. 12,	" " " 6 mo.,	175.00	July 18,	" "	250.00
Apr. 1,	" " " 3 mo.,	200.00			

At what time should the balance of the above account be paid? *Ans.* Dec. 23, 1848.

NOTE. Find the equated time at which the Dr. items fall due, and also the equated time of the Cr. items, (**120**); then find the equated time for paying the balance, as in the preceding examples.

10. *Samuel Rogers in account current with Joseph H. Carter.*

Dr.			*Cr.*		
1848.			1848.		
Apr. 1,	To mdse. at 8 mo.,	$500.00	July 15,	By cash,	$400.00
" 24,	" " cash,	350.00	" 23,	" "	300.00
May 27,	" " at 2 mo.,	75.00	Oct. 15,	" "	150.00
June 15,	" " " 6 mo.,	180.00			

What is the equated time for paying the balance of the above account? *Ans.* Feb. 10, 1849.

11. If the account, example 9, were to be settled Jan. 1, 1849, what sum would pay the balance, reckoning interest at 6 per cent. per annum?

NOTE. Either find the amount of each item up to Jan. 1, 1849, (**110**, Ex. 3, 4, and 5,) or the value of the balance, $175, Jan. 1, 1849; that is, the amount of $175 on interest for 9 days, at 6 per cent. Do it by both methods.

12. What sum would pay the balance of the account in example 10, Jan. 1, 1849, interest at 8 per cent.? (**110** and **113**.)

The following notes refer to the account current and interest account on the next page, and should have followed that account; but for want of room on that page they are placed here.

NOTE 1. As W. F.'s note is not due till 1 month and 14 days after Sept. 1, the interest for that time is not added in the Cr. column, but in the Dr. In making up interest accounts, the time and interest of such items are usually written in red ink. Hence, the entry on the Dr. side, "To interest on Cr. side in red."

2. The interest of the Dr. items being more than that of the Cr., the balance of interest is to be added to the Dr. side of the account.

3. Some merchants state the time in days, and compute the interest by taking one sixth of the number of days as the interest in mills of $1 for the time. This method makes the interest more than the above. For example: the interest of $1000 from May 17, 1847, to Feb. 29, 1848 = 9 mo. 12 d., or 288 days, is $47, if the time be stated in months and days; and $48, if the time be stated in days. But as the interest on both the Dr. and Cr. items is reckoned in the same manner, the *balance* of interest will generally be nearly the same by both methods.

Copy upon paper the accounts on pages 166 and 167, stating the time in days, and find the balance of each account.

13. *Form of an Account Current and Interest Account.*

William Stevens in account current with Stephen Williams.

Interest account to Sept. 1, 1848.

Dr.

		Time. m.	Time. d.	Interest. $	Interest. cts.	Amount. $	Amount. cts.
1848. May 5,	To corn, due June 5th,	2	27	7	25	500	00
" 10,	" butter, due July 20,	1	12		70	100	00
June 17,	" flour, due June 20,	2	12	7	20	600	00
	" Interest on Cr. side in red,	–	–	1	10	–	–
	" Balance of interest acct.,	–	–	–	–	* 4	85
				$16	25	1204	85
1848. Sept. 1,	To balance due this day,					354	85

Cr.

		Time. m.	Time. d.	Interest. $	Interest. cts.	Amount. $	Amount. cts.
1848. Apr. 6,	By bill of mdse., due Apr. 20,	4	12	6	60	300	00
May 20,	" W. F.'s note, due Oct. 15,	1	14	*1	10	150	00
June 20,	" cash,	2	12	4	80	400	00
	" balance of int. to Dr. side,	–	–	4	85	–	–
	" balance of account,	–	–	–	–	354	85
				$16	25	1204	85

Errors and omissions excepted.

Boston, Sept. 1, 1848.

STEPHEN WILLIAMS,
By W. F. Hodges.

* See note 1, page 165.

14. John Smith in account current with H. Brown.

Interest Account to July 1, 1848.

Dr.

1848.		Time. m.	Time. d.	Interest. $	Interest. cts.	Amount. $	Amount. cts.
Jan. 1,	To J. P.'s note, dated Jan. 1, in 4 m.,	1	27	23	75	2,500	00
Mar. 10,	" bill of merchandize, due 15th March,	3	16	22	26	1,260	10
Apr. 18,	" T. L.'s note, dated April 1, in 9 m.,	*6	3	26	20	859	06
May 5,	" paid your order in favor of S. M.,	1	26	9	33	1,000	00
June 20,	" cash per receipt,		11	3	30	1,800	00
	" Dr. int. in *red* on Cr. side,	*	–	11	05	–	–
	" balance of int. to Cr.,	–	–	12	98	–	–
	" balance of account,	–	–	–	–	1,001	72
			$	82	67	8,420	88

Cr.

1848.		Time. m.	Time. d.	Int'st. $	Int'st. cts.	Amount. $	Amount. cts.
Mar. 15,	By ¼ net proceeds brig Edward's cargo, due May 1, 1848,	2	–	47	51	4,750	50
Apr. 10,	" net sales coffee, due June 6, 1848,	0	25	8	96	2,150	50
June 1,	" net sales pepper, due Aug. 15, 1848,	*1	14	11	05	1,506	90
	" Cr. int. on Dr. side in red,	*	–	26	20	–	–
	" balance of int. acct.,	–	–	–	–	12	98
				$ 82	67	8,420	88
1848. July 1,	By bal. due this day, ...					1,001	72

Errors and omissions excepted.

Salem, July 1, 1848.

H. B.

* See note 1, page 165.

QUESTIONS. What is *barter?* What abbreviation do merchants use for *bought?* for *company?* for *merchandise?* for *bill of parcels?* for *received payment?* for *gross?* for *each?* for *at?* How are shillings and pence expressed? the time at which a note falls due on which days of grace are allowed? How is per cent. often expressed? What is an *account current?* For what is the term *Dr.* used? the term *Cr.?* *to?* *by?* What does the word *current* indicate? How often are accounts current usually made up? What is the practice in regard to interest in such accounts? In how many different ways may accounts current be arranged and settled? What is the first? the second? the third? What items are usually written in red ink? What if the interest of the Dr. items is more than that of the Cr.? What if that of the Cr. items is the larger? What is said of stating the time in days in computing the interest?

SECTION XII. — RATIO — PROPORTION.

126. RATIO.

Ratio is the relation which one quantity bears to another of the same kind, and expresses the part that one quantity is of another. Thus, $\frac{3}{5}$ expresses the relation of 3 to 5; it also expresses what part 3 is of 5. The two given numbers are called the *terms* of the ratio. The first term is called the *antecedent*, and the second the *consequent*.

The ratio of one quantity to another is obtained by dividing the antecedent by the consequent. It is expressed either in the form of a common fraction, (**86**,) or the terms are written after each other with the sign ($\div$) expressing division between them. Thus, the ratio of 5 to 7, is written $\frac{5}{7}$, or 5 : 7, which is read, either, *the ratio of* 5 *to* 7, or, *as* 5 *is to* 7.

Obs. In expressing a ratio, the sign ($\div$) is usually written thus, (:), without the horizontal line between the dots.

The question what is the ratio of 5 to 7 is the same as the question, what part of 7 is 5? (**86.**)

If the terms of the ratio are not expressed in the same denomination, they must be reduced to the same denomination.

When the terms of a ratio are not prime to each other, the ratio may be reduced to lower terms, just as common fractions may be reduced to lower terms.

1. Write the ratio of 7 to 3; 8 to 5; 5 to 9; 9 to 16; 87 to 150; 16 to 9.

2. Write and reduce to its lowest terms each of the follow-

ing ratios: 18 to 4; 4 to 18; 15 to 9; 25 to 15; 27 to 45; 105 to 45; 800 to 150.

3. What fraction expresses the ratio of 7 to 8? Of 15 to 24? Of $2\frac{1}{2}$ to $3\frac{1}{2}$? Of $4\frac{3}{4}$ to $6\frac{3}{4}$? Of $\frac{5}{8}$ to $\frac{3}{5}$?

NOTE. Reduce the fractions to a common denominator, and compare the numerators. Thus, the ratio $\frac{15}{40}$ to $\frac{18}{40}$ is the same as that of 15 to 18.

4. Express in a common fraction the ratio of $\frac{4}{9}$ to $\frac{5}{12}$; the ratio of $\frac{2}{3}$ of $\frac{6}{7}$ to $\frac{5}{8}$; of $\frac{4}{5}$ to 7; of $\frac{9}{15}$ to $3\frac{1}{3}$; of $4\frac{1}{5}$ to $7\frac{3}{8}$.

NOTE. $\frac{4}{9}=\frac{16}{36}$; $\frac{5}{12}=\frac{15}{36}$. The ratio of $\frac{16}{36}$ to $\frac{15}{36}$ is the same as the ratio of 16 to 15.

127. PROPORTION.

A proportion consists of two equal ratios. When four numbers are so related to each other, that the first has the same ratio to the second that the third has to the fourth, they constitute a proportion. Thus the numbers 4, 5, 12, 15, form a proportion, because the ratio of 4 to 5 is equal to the ratio of 12 to 15. The proportion may be expressed thus: $4:5=12:15$; or, $4:5::12:15$; or $\frac{4}{5}=\frac{12}{15}$; which is read, 4 is to 5 as 12 is to 15; or, 4 divided by 5 is equal to 12 divided by 15.

The first and fourth terms of a proportion are called the *extremes;* and the second and third, the *means*. When the proportion is expressed in a fractional form, the numerator of the first fraction and the denominator of the second are the extremes, and the denominator of the first and the numerator of the second the means. In every proportion the product of the extremes is equal to the product of the means. In the above proportion $4:5=12:15$, or $\frac{4}{5}=\frac{12}{15}$; the product of the extremes, 4×15, is equal to the product of the means, 5×12.

As the product of the extremes is always equal to the product of the means, we see that if the product of the means be divided by one of the extremes, (**97**,) the quotient will be the other extreme; and if the product of the extremes be divided by one of the means, the quotient will be the other mean.

In questions in simple proportion, there are always three numbers or terms given, to find a fourth term or answer. Two

of the given terms are of the same name or kind, and the other given term is of the same name or kind as the answer.

EXAMPLE. If 3 barrels of flour cost \$18, how many dollars will 8 barrels cost?

In this example the dollars should have the same ratio to each other that the barrels have to each other; viz., the ratio of 3 to 8. The proportion is written $3 : 8 = 18 :$ *Ans.*; and is read, 3 barrels is to 8 barrels as \$18 is to the answer. And, since the product of the means divided by one extreme gives the other extreme, $\frac{8 \times 18}{3} = \frac{144}{3} = 48$; or, by cancelling,

$$\frac{8 \times \overset{6}{\cancel{18}}}{\cancel{3}} = 48,$$ the 4th term, or answer.

Find the unknown terms in the following expressions. Cancel, if possible, before performing the work.

1. $1 : 7 = 9 : —$; $3 : 8 = 48 : —$; $5 : 12 = 8 : —$; $4 : 15 = 12 : —$; $5 : 3 = 8 : —$; $18 : 15 = 27 : —$; $150 : 80 = 95 : —$.

2. $\frac{3}{8} : 4 = \frac{3}{5} : —$; $3\frac{1}{8} : 20 = 7\frac{1}{2} : —$; $15\frac{1}{3} : 18\frac{1}{3} = 25\frac{1}{3} : —$; $\frac{4}{5} : \frac{7}{8} = \frac{9}{16} : —$.

3. $4.7 : 5.03 = 12 : —$; $2.5 : 3.8 = .5 : —$; $.7 : 15 = 1.3 : —$; $\frac{3}{5} : 1.8 = 1.5 : —$.

4. $3 : 7 = — : 15$; $25 : 18 = — : 1.3$; $75 : 1.3 = — : 3.8$.

5. $5.1 : — = 9 : 5\frac{3}{8}$; $3.01 : — = \frac{7}{8} : \frac{4}{9}$; $4\frac{5}{8} : — = 6.5 : 5\frac{3}{16}$.

6. $— : 5 = 18 : 25$; $— : 7.5 = 15\frac{3}{7} : 18$; $— : 1\frac{3}{8} = 5 : 9$.

7. 3 barrels : 8 barrels = \$15 : \$ —; 6 lb. : 11 lb. = 25 cts. : —; 4 lb.* : 5 oz. = 15 cts. : —.

NOTE. When the terms of a ratio are of different denominations, they must be reduced to the same denomination.

(8.) 4 bu. : 9 bu. 3 pk. = \$2.50 : —; 3 yd. 2 qr. : 4 yd. 3 qr. = \$12.50 : \$ —.

(9.) $\frac{3}{8}$ yd. : $\frac{4}{7}$ qr. = \$$\frac{3}{5}$: \$ —; $\frac{5}{8}$ lb. : $3\frac{3}{5}$ lb. = \$$1\frac{5}{12}$: —; $\frac{2}{3}$ cwt. : $\frac{3}{8}$ qr. = \$1.25 : \$ —.

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* Avoirdupois.

128. Since a proportion consists of two equal ratios, and as ratio is the relation of two quantities of the same kind only, the third term must always be of the same kind as the fourth term or answer; and the second must be either greater or less than the first, as the answer or fourth term is to be greater or less than the third term. Hence the following

RULE FOR SIMPLE PROPORTION.

Write the given number which is of the same kind as the required fourth term or answer, for the third term of the proportion. Then consider whether the answer is to be greater or less than the third term; if it is to be greater, write the greater of the two remaining terms for the second, and the other for the first term; but if it is to be less, write the less of the two remaining terms for the second term, and the other for the first.

Divide the product of the second and third terms by the first; the quotient will be the fourth term or answer.

The first and second terms must be of the same denomination; the fourth will be of the same denomination as the third term.

The learner should solve the following questions both by analysis (**47**) and by the rule for proportion.

1. If 9 yards of cloth cost $45, what will 15 yards cost?

$$\overset{yd.}{9} : \overset{yd.}{15} = \overset{\$}{45} : \overset{\$}{-}$$

$$\frac{15 \times \overset{5}{\cancel{45}}}{\underset{1}{\cancel{9}}} = 75.$$

As the answer is to be in dollars, we make $45 the third term; and as 15 yards will cost more than 9 yards, the second term must be larger than the first.

To perform this question by analysis, say, if 9 yards cost $45, 1 yd. will cost $\frac{1}{9}$, and 15 yards $\frac{15}{9}$ of $45 = $75.

2. If 6 men do a piece of work in 20 days, how long will it take 15 men to do it?

NOTE. As 15 men will do it in fewer days than 6 men, the less of these two numbers must be the second term.

3. If 8 acres cost $98.50, what will 360 acres cost?
4. If 54 acres cost $2160, what will 9 acres cost?

5. If 7 men do a piece of work in 25 days, in what time will 5 men do it?

6. If 5 men do a piece of work in 35 days, in what time will 7 men do it?

7. How many men will it take to do in 25 days the work that 5 men will do in 35 days?

8. If 7 pairs of boots cost \$24.50, how many pairs will \$94.50 buy?

9. If 7 pairs of boots cost \$24.50, how much will 27 pairs cost?

10. If $\frac{3}{7}$ of a barrel of flour cost \$2.70, what will 17 barrels cost?

11. If 17 barrels of flour cost \$107.10, how much will \$2.70 buy?

12. How many yards of cloth $\frac{3}{4}$ yd. wide will line a cloak containing 10 yards that are $\frac{7}{8}$ yd. wide?

13. If a cubic foot of water weighs 1000 oz., how many pounds of water will a cistern contain that is 3 ft. wide, 5 ft. long, and 5 ft. high? How many in a cistern $4\frac{1}{2}$ ft. wide, $3\frac{1}{2}$ ft. high, and 6 ft. long?

14. If the interest on a note at 6 per cent. is \$125.15, what would be the interest at 5 per cent.?

15. If the interest at 6 per cent. were \$52.95, at what rate per cent. would it be \$61.77$\frac{1}{2}$?

16. If the rate per year is $7\frac{1}{2}$ per cent., in what time would it be 24 per cent.?

17. If a post $6\frac{1}{2}$ feet high casts a shadow of $7\frac{1}{2}$ feet, on level ground, how high is a steeple which at the same time casts a shadow of 187 ft.? How long a shadow will a pole 60 ft. high cast?

18. Bought 48 yards of broadcloth for £33 12s. What are 27 yards worth at that rate? How many yards will £24 10s. buy?

19. If $\frac{4}{9}$ of a hhd. of molasses cost \$12, what cost $\frac{3}{5}$ of a hhd.?

$$\frac{4}{9} : \frac{3}{5} = 12 : —; \quad \frac{\frac{3}{5} \times \frac{12}{1}}{\frac{4}{9}} = \frac{3}{5} \times \frac{\overset{3}{\cancel{12}}}{1} \times \frac{9}{\cancel{4}} = \frac{81}{5} = 16\tfrac{1}{5}, \text{ Ans.}$$

By Analysis. If $\frac{4}{9}$ cost \$12, $\frac{1}{9}$ would cost $\frac{1}{4}$, and $\frac{9}{9}$ would cost $\frac{9}{4}$ of \$12. $\frac{3}{5}$ of a hhd. would cost $\frac{3}{5}$ of $\frac{9}{4}$ of \$12 = \$16$\frac{1}{5}$, *Ans.*

20. If $\frac{3}{8}$ of a bushel is worth 24 cents, what is $\frac{7}{16}$ of a bushel worth?

21. If $3\frac{3}{4}$ lb. of pork cost 38 cts., what are $15\frac{1}{4}$ lb. worth?

22. If 8 men mow $33\frac{3}{7}$ acres of grass in 18 days, in how many days will they mow $28\frac{3}{4}$ acres?

23. How much carpeting $1\frac{1}{4}$ yd. wide will cover a floor $5\frac{5}{8}$ yards long, and $3\frac{5}{8}$ yards wide?

24. If 8 men can mow a field in 3 days, by working 10 hours per day, how long will it take them if they work only 9 hours per day?

129. COMPOUND RATIO.

A *Compound Ratio* is the ratio of the product of two simple ratios.

The ratio of	8 to 5 is $\frac{8}{5}$
The ratio of	4 to 3 is $\frac{4}{3}$

The ratio compounded of these is 32 to $15 = \frac{32}{15} = \frac{8}{5} \times \frac{4}{3}$.

A compound ratio is reduced to the form of a simple ratio by multiplying the corresponding terms together.

1. Reduce to a simple form each of the following compound ratios.

$$\left.\begin{matrix} 3 : 5 \\ 4 : 7 \end{matrix}\right\}; \quad \left.\begin{matrix} 6 : 5 \\ 3 : 8 \\ 4 : 5 \end{matrix}\right\}; \quad \left.\begin{matrix} 3\frac{1}{2} : 8 \\ 2\frac{1}{4} : 7 \\ 3 : 4\frac{1}{4} \end{matrix}\right\}; \quad \left.\begin{matrix} 4 : 3\frac{1}{4} \\ 4\frac{1}{3} : 3 \\ 8 : 9 \end{matrix}\right\}.$$

130. COMPOUND PROPORTION.

Compound Proportion is the equality of two ratios, one of which is compound and the other simple. Thus, $\left.\begin{matrix} 3 : 5 \\ 4 : 7 \end{matrix}\right\} = 3 : 8\frac{3}{4}$ is a compound proportion. After reducing the compound ratio to a simple one, the proportion becomes a simple proportion.

Reduce the following compound proportions to simple ones, and find the unknown term of the last ratio.

If the antecedents or first terms have factors common to the consequents or second terms, or to the third term, they should be cancelled before multiplying and dividing.

(1.) $\left.\begin{matrix} 3 : 8 \\ 5 : 7 \end{matrix}\right\} = 4 : — ;$ *Ans.* $15 : 56 = 4 : 14\frac{14}{15}$.

$\left.\begin{matrix} 4 : 13 \\ 3 : \ \ 7 \end{matrix}\right\} = 5 : — ;$ $\left.\begin{matrix} 6 : \ \ 5 \\ 15 : 12 \end{matrix}\right\} = 12 : — ;$

(2.) $\left.\begin{matrix} \frac{1}{2} : \frac{2}{5} \\ 5 : 7 \end{matrix}\right\} = 8 \ \ — ;$ $\left.\begin{matrix} 4\frac{1}{2} : 6\frac{1}{2} \\ 5 \ \ : 8\frac{1}{4} \\ 4 \ \ : 5 \end{matrix}\right\} = 3\frac{1}{4} : — ;$

$\left.\begin{matrix} 3 : 8 \\ 2 : 5 \\ 10 . 9 \end{matrix}\right\} = 4 : — ;$ $\left.\begin{matrix} \frac{1}{3} : \frac{3}{5} \\ \frac{3}{8} : \frac{2}{3} \end{matrix}\right\} = \frac{3}{4} : — .$

3. If 8 horses eat 70 bushels of oats in 5 weeks, in what time will 7 horses eat 40 bushels?

40 bushels will last $\frac{40}{70}$ as long as 70 bushels, or $\frac{40}{70}$ of 5 weeks. This is expressed by the proportion $\overset{bu.}{70} : \overset{bu.}{40} = \overset{wks.}{} : \overset{wks.}{5} —$. 40 bushels will last 7 horses $\frac{8}{7}$ as long as it will last 8 horses, or $\frac{8}{7}$ of $\frac{40}{70}$ of 5 weeks. This is expressed by the proportion $\overset{h.}{7} : \overset{h.}{8} = \frac{40}{70}$ of 5 weeks : — weeks.

As the answer does not depend upon either of the ratios alone, but upon both combined, the two ratios 70 : 40, and 7 : 8 may be written together, and then reduced as in the above examples; thus, $\left.\begin{matrix} 70 : 40 \\ 7 : \ \ 8 \end{matrix}\right\} = 5 : — ;$ which being reduced, gives 49 : 32 = 5 weeks : — weeks.

As questions in *simple* proportion have three terms given to find a fourth, so in *compound* proportion five, seven, or some other odd number of terms, are given to find a sixth, or an eighth term, &c.

RULE FOR COMPOUND PROPORTION. *Write that number for the third term which is of the same kind as the answer. Then take the other quantities in pairs, or two of a kind, and arrange them as in simple proportion. Reduce the compound proportion thus formed to the form of a simple proportion, and find the fourth term as in simple proportion.*

4. If 7 men build 84 rods of wall in 12 days, by working 12 hours a day, how many men can build 100 rods in 5 days, working 10 hours a day?

$$\left.\begin{array}{rcl} \overset{1}{\cancel{84}}\text{ rd.} & : & \overset{2}{\cancel{100}}\text{ rd.} \\ \cancel{5}\text{ d.} & : & \cancel{12}\text{ d.} \\ \cancel{10}\text{ h.} & : & 12\text{ h.} \end{array}\right\} = \overset{1}{\cancel{7}}\text{ m.} : \text{—}.$$

$$1 \quad : \quad 24 = 1 : 24 \text{ men.}$$

7 men is put for the 3d term, being of the same kind as the answer. It will take more men to build 100 rods than to build 84; therefore 100 rods should be put for the 2d term. It will take more men to build it in 5 days than in 12 days; therefore 12 days must be the 2d term. It will take more men to build it by working 10 hours a day than by working 12 hours a day; therefore 12 hours must be the 2d term of the ratio.

Solve the following questions both by proportion and by analysis. (47 and 95.)

5. If 14 men build 84 rods of wall in 3 days, how long will it take 20 men to build 300 rods?

6. If 24 horses eat 126 bushels of oats in 36 days, how many bushels will 32 horses eat in 48 days?

7. If 4 men build a wall 10 ft. long, 6 ft. high, 2 ft. thick, in 6 days, how long will it take 12 men to build one 100 ft. long, 8 ft. high, and 3 ft. thick?

8. If \$200 gain \$12 in 12 months, what will \$500 gain in 8 months?

9. If 7 men, working 9 hours a day, dig a ditch 210 ft. long, 3 ft. wide, and 4 ft. deep, in 4 days, in what time will 35 men, working 12 hours a day, dig a ditch 420 feet long, 4 feet wide, and 5 feet deep?

10. If 4 men can build $38\frac{1}{4}$ rods of wall in $3\frac{1}{4}$ days, how long will it take 9 men to build $123\frac{1}{2}\frac{5}{6}$ rods?

11. If 75 men, working 10 hours a day, in 25 days grade 175 rods of rail-road, how many men will it take to grade 448 rods in 40 days, working 12 hours per day?

12. If the carriage of 10 boxes of sugar, weighing each 420 lb., 15 miles, cost \$5, what would the carriage of 15 boxes, each weighing 450 lb., cost for 60 miles?

13. A person undertook to perform a piece of work in 8 days with 15 men, but at the end of 6 days, found $\frac{3}{8}$ of it unfinished; how many more men must he employ to finish it at the set time?

14. If a footman, when the days are 14 hours long, can travel

276 miles in 16 days, in how many days can he travel 852 miles, when the days are but 12 hours long?

15. A garrison of 500 men have provisions for 15 weeks, at the rate of 18 oz. per day to each man; how many men will the same provisions maintain for 10 weeks, allowing each man only 12 oz. per day?

16. If a bar of iron 6 feet long, 3 inches broad, and 2 inches thick, weighs 72 pounds, what will a bar weigh that is 4 feet long, 2 inches wide, and $1\frac{1}{2}$ inch thick?

17. If 248 men in 5 days of 11 hours each can dig a trench 230 yards long, 3 wide, and 2 deep, in how many days of 9 hours long will 24 men dig a trench 420 yards long, 5 wide, and 3 deep?

18. The par value of the pound sterling being $\$4\frac{4}{9}$, how many dollars will pay a debt of £450, the rate of exchange on England being at a premium of $8\frac{1}{2}$ per cent.? at $9\frac{1}{4}$ per cent.?

19. How many pounds sterling are equal to $1000, the premium of exchange being $8\frac{1}{2}$ per cent.? $9\frac{1}{4}$ per cent.?

131. The terms of a proportion may be distinguished into causes and effects. Thus, in the last Art., example 11th, men, days and hours may be regarded as causes, and rods as the effect produced by those causes. So, in example 12th, boxes, pounds and miles may be classed together as causes, and the money expended as the effect. In example 15th, all the terms may be classed as causes; the effect produced upon the men who consumed the provision not being expressed in numbers.

1. If 6 men build a wall 20 feet long, 6 feet high, and 4 feet thick, in 16 days, by working 10 hours per day, in how many days will 24 men build a wall 200 feet long, 8 feet high, and 6 feet thick, by working 12 hours per day?

The statement of the above question by proportion is as follows:

$$\left\{\begin{array}{rcl} \text{24 men} & : & \text{6 men} \\ \text{20 ft.} & : & \text{200 ft.} \\ \text{6 ft.} & : & \text{8 ft.} \\ \text{4 ft.} & : & \text{6 ft.} \\ \text{12 hours} & : & \text{10 hours} \end{array}\right\} : \text{16 days} : \text{— days.}$$

In this example 6 men, 16 days, and 10 hours, are the first set of causes, and 24 men, — days, and 12 hours, the second; 20 feet, 6 feet, and 4 feet, are the first set of effects, and 200 feet, 8 feet, and 6 feet, the second.

Since in every proportion the product of the extreme terms is equal to the product of the mean terms, we see, by the above statement, that the product of the first set of causes multiplied by the second set of effects, which are the numbers constituting the *mean* terms, is equal to the product of the second set of causes multiplied by the first set of effects, which are the numbers constituting the *extreme* terms. That is, the two products of each cause, or set of causes, by the opposite effect, or set of effects, are equal to each other. If, therefore, the terms are arranged so that each set of causes shall be in the same column with the opposite effects, we shall, by making the numbers in the column that contains the blank or unknown quantity factors of a divisor, and those in the other column factors of a dividend, obtain the answer. The terms may be arranged thus:

	3	causes.
$\cancel{6}$ men	$\cancel{24}$ men	
2 $\cancel{16}$ days	— days	
10 hours	$\cancel{12}$ hours	
10 $\cancel{200}$ feet	$\cancel{20}$ feet	effects.
$\cancel{8}$ feet	$\cancel{6}$ feet	
$\cancel{6}$ feet	$\cancel{4}$ feet	

$200 \div 3 = 66\frac{2}{3}$ days.

We first write the first set of causes on the *left* of a vertical line, toward the top, and the corresponding set of effects on the *opposite* side of the line, toward the bottom. We then write the second set of *causes* opposite to the first set, on the *right* of the vertical line, and the second set of effects on the *left* of the vertical line, opposite to the first. The numbers on the left of the line are factors of the dividend, and those on the right, containing the blank, are factors of the divisor. After cancelling equal factors in the divisor and dividend, we multiply the remaining factors and perform the division.

2. If I pay \$40 for the carriage of $5\frac{3}{4}$ cwt. 150 miles, what must I pay for carrying $7\frac{5}{7}$ cwt. 64 miles?

\$40	—
$7\frac{5}{7}$	$5\frac{3}{4}$ cwt.
64	150 miles

\$$\cancel{40}$	—
18 $\cancel{54}$	23
64	$\cancel{150}$ 5
4	7

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\$$22.89\frac{111}{161}$ *Ans.*

The mixed numbers may be changed to improper fractions. Transposing the denominators of fractional quantities to the opposite column, is the same as multiplying each column by the same numbers, which does not alter their relation to each other. (74.)

NOTE. In writing down the terms of the proportion, either cause or either effect may be first written; it will make no difference in the result. It is generally most convenient to write the terms just as they occur in the question, taking care that each cause and its effect be on opposite sides of the *vertical* line.

"All the terms *acting*, *producing*, or *consuming*, are CAUSES; viz., *men*, *horses*, *time*, *capital*, *length*, *breadth*, *thickness*, *or parts of a compound*, &c. EFFECTS are the result or consequence of said causes; viz., *work*, *wages*, *interest*, *superficial and solid contents*, &c. By fixing these distinctions in the memory, the student will soon be able to apply the criterion with ease and certainty." — PLAIN CALCULATOR, BY JOERRES.

3. If 6 oxen in 8 days eat 5 acres of grass, how many acres will serve 12 oxen 96 days?

4. If the interest of \$500 for $3\frac{1}{2}$ years be \$105, what would be the interest of \$1020 for $5\frac{1}{2}$ years, at the same rate per cent.?

5. Six men, by working 8 days, 10 hours a day, can do $\frac{2}{3}$ of a piece of work; in how many days will ten men do $\frac{7}{8}$ of the same work, working 9 hours a day?

6. If a rectangular cistern, 7 feet long, 5 feet wide, and 6 feet deep, hold 13125 pounds of water, how much will a cistern hold that is $10\frac{1}{2}$ feet long, $7\frac{1}{2}$ feet wide, and 9 feet deep?

7. What principal at 6 per cent. will yield \$15.50 interest, in $2\frac{1}{2}$ years?

STATEMENT.

$$\begin{array}{r|l} \$100 & — \\ \text{yr.}\quad 1 & 2\frac{1}{2}\text{ yr.} \\ \hline \$15.50 & \$6. \end{array}$$

8. In what time will the interest of \$500.75 amount to \$35.75, at 6 per cent.?

9. At what per cent. must \$1000 be put on interest, in order to yield \$192.50 in $3\frac{1}{2}$ years?

10. How much flour at $\$6.87\frac{1}{2}$ per barrel, must be given for 150 bushels of corn at $\$0.62\frac{1}{2}$ per bushel?

11. If a staff $5\frac{1}{2}$ feet long cast a shadow $8\frac{1}{4}$ feet long, how high is the steeple which, at the same time, casts a shadow of 175 feet?

NOTE. Let the pupil perform examples in both simple and compound proportion, by this method, till he can perform them with facility.

132. ARBITRATION OF EXCHANGE. CHAIN RULE.

1. If one barrel of flour is worth 4 barrels of apples, and 2 barrels of apples are worth 8 bushels of corn, and 5 bushels of corn are worth 6 bushels of potatoes, and one bushel of potatoes is worth 50 cents, how many barrels of flour will $25 buy? How much are 4 barrels of flour worth?

Questions like the above may be readily solved by the rule of cause and effect, (Art. **131**,) by placing each effect opposite to its cause, and making each effect of the same denomination with the next cause.

NOTE. The *first* numbers in each part of the question are called *antecedents*, and may be regarded as *causes*; the following ones are called *consequents*, and may be regarded as *effects*.

OPERATION.

Causes, or Antecedents.	*Effects, or Consequents.*
	2
1 bbl. flour.	$\cancel{4}$ barrels apples.
$\cancel{2}$ " apples.	8 bush. corn.
5 bush. corn.	6 " potatoes.
1 " potatoes.	\$.$\cancel{50}$ .02.
\$$\cancel{25}$ 1.	— barrels flour.

\$5 ÷ 1.92 = $2\frac{29}{48}$ barrels, the answer.

In the first question, viz., how many barrels of flour will $25 buy, the unknown quantity is an effect or consequent; the blank is therefore in the column of consequents, the numbers in which are factors of the divisor; the antecedents being factors of the dividend.

	2
1 bbl. flour.	$\cancel{4}$ barrels apples.
$\cancel{2}$ " apples.	8 bush. corn.
$\cancel{5}$ bush. corn.	6 " potatoes.
1 " potatoes.	\$.$\cancel{50}$.10.
\$ ——.	4 barrels flour.

\$38.40 Answer.

In the second question, viz., how much are 4 barrels of flour worth, the unknown quantity is a cause or antecedent; the blank is therefore in the column of antecedents, the numbers in which are factors of the divisor; the consequents being factors of the dividend.

2. If 10 barrels of flour can be bought for 54 bushels of wheat, and 9 bushels of wheat for 20 bushels of corn, and 12 bushels of corn for 10 bushels of rye, and 5 bushels of rye for $3.50, how many barrels of flour can be bought for $50? How much are 18 barrels worth?

NOTE. Cancel equal factors before multiplying.

3. If 15 oranges are worth 35 lemons, and 7 lemons are worth 12 apples, and 18 apples are worth 10 pears, and 8 pears are worth 15 peaches, and 3 peaches are worth 2 cents, what are 10 oranges worth? How many oranges will 40 cents buy?

Debts due in foreign countries are often paid through the medium of a number of persons residing in different countries. The method of changing the currency of one country into that of another, through the medium of one or more intervening currencies, is called *Arbitration of Exchange.*

The method of operation is the same as for the above questions.

4. If 1 French crown is equal to 80 pence of Holland, and 40 pence of Holland to 24 pence of England, and 20 pence of England to 35 pence of Hamburg, and 60 pence of Hamburg to 1 florin of Frankfort, how many florins of Frankfort are equal to 100 French crowns?

5. A merchant in New York wishes to pay £1000 in London; how many dollars will pay the amount, if he sends his money to Paris at 5 francs 15 centimes to the dollar, and thence to London at 25 francs 80 centimes for £1?

6. Which is best for the merchant, to buy a bill on London, exchange being at a premium of $9\frac{1}{2}$ per cent., (**122**,) to pay the debt named in the last question, or to remit his money through Paris, as there proposed? How many dollars is the difference?

NOTE. The questions in Art. **128** to **132** should be performed by *Analysis*, as well as by the specific rules given for solving them.

QUESTIONS. What is *ratio*, and what does it express? Give an example. What are the terms of a ratio? What is the first term called? the second? How is the ratio of one quantity to another obtained? How is ratio expressed? Give examples. What must be

done if the terms are not in the same denomination? How may a ratio be reduced to its lowest terms?

What is a *proportion?* When do four numbers form a proportion? In what different ways may a proportion be expressed? Which terms of a proportion are called extremes? Which are called means? What products are always equal to each other? To what is either mean equal? To what is either extreme equal? Why? What are given and what is required in questions in simple proportion? Which terms must be of the same kind? When must the second term be greater than the first? When less? Repeat the rule for simple proportion.

What is a *compound ratio?* How may a compound ratio be reduced to the form of a simple ratio?

What is *compound proportion?* How may a compound proportion be reduced to the form of a simple proportion? In what terms may equal factors be cancelled? What are given and what is required in questions in compound proportion? What is the rule for compound proportion?

Into what may the terms of a proportion be distinguished? Give examples. In statements by cause and effect, what products are always equal? How do we write down the terms in solving questions by this method? Which column of numbers constitutes the divisor? Which the dividend? What may be done if any of the terms are fractions or mixed numbers? Which terms should be written first? What terms are *causes?* What are *effects?*

How are debts due in foreign countries often paid? What is meant by *arbitration of exchange?* How may questions in arbitration of exchange be solved? Of what denomination should each effect be?

133. PARTNERSHIP.

Sometimes two or more persons unite together for the transaction of business. Such a union is termed a *partnership.* The association thus formed is called a *firm*, or *house.* The money or capital employed is called *capital*, or *stock.* The process by which partners divide their gain or loss is sometimes called *fellowship.*

1. A and B form a partnership. A furnishes $300 of the stock, and B $500; they gain $80. How shall the gain be divided between them?

It is evident that they should share the gain in proportion to the stock each furnished; therefore, as A furnished $\frac{3}{8}$ of the stock, he should have $\frac{3}{8}$ of the gain; and as B furnished $\frac{5}{8}$ of the stock, he should have $\frac{5}{8}$ of the gain.

2. Three men hire a pasture for $48. A pastures 4, B 5, and C 3 horses. What part of the whole stock did each furnish? What must each pay?

3. A, B and C formed a partnership. A furnished \$450, B \$500, and C \$600. What part of the whole stock did each furnish? They gained \$248. What part of the gain belongs to each? How many dollars? If they lose \$93, what part of the loss must each sustain? How many dollars?

4. A bankrupt owes \$980; viz., to A \$420, to B \$350, and to C \$210. What part of his effects must each receive? How many dollars should each receive if the bankrupt is worth \$840? How much if he is worth \$280?

RULE BY PROPORTION. *As the whole stock is to each man's stock, so is the whole sum to be divided to each man's share of it.*

NOTE. Perform all the examples in this Art. both by Analysis and Proportion.

5. A, B, C, and D send a vessel to the West Indies. A furnishes \$1600 of the cargo, B \$1800, C \$2550, and D \$1200. They gain \$2288. What per cent. of the money invested do they gain? How many dollars of the gain belong to each?

6. Divide \$850 among 5 men, so that their shares shall be in the proportion of 6, 5, 4, 3, and 2, respectively, (that is, so that the first shall have 6 as often as the second has 5, &c.) What is the share of each?

7. Divide \$75 between 2 persons, so that A shall have $\$\frac{1}{2}$ as often as B has $\$\frac{3}{4}$.

NOTE. A has $\frac{2}{4}$ as often as B has $\frac{3}{4}$. Their shares are therefore as the numbers 2 and 3.

8. Divide \$345 between 3 persons, in the proportions of $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{3}{8}$.

9. Divide \$750 among 4 men, in shares which are in the proportion of $\frac{1}{3}$, $\frac{2}{5}$, $\frac{4}{5}$, and $\frac{5}{9}$.

10. A and B hire a pasture, for which they pay \$20. A's horse was in the pasture $13\frac{1}{2}$ weeks, and B's $15\frac{1}{4}$ weeks. What proportion of the whole should each pay? How many dollars?

11. A bankrupt is worth \$10,000; his debts amount to \$12,500. How much shall a creditor receive to whom he owes \$500?

12. A bankrupt wishes to pay a dividend of \$1000 among four of his creditors. How much can he pay to each, if he owes A \$400, B \$750, C \$575, and D \$825?

134. WHEN THE STOCKS ARE EMPLOYED FOR UNEQUAL TIMES.

1. A and B form a partnership for 8 months. A furnishes $500 at the first. B at first furnishes but $100, but in 2 months he furnishes $600 more. What part of the gain belongs to each?

A's	$500	for 8	months	=	$4000	for 1 month.
B's	100	" 8	"	=	800	" " "
B's	600	" 6	"	=	3600	" " "

It is, therefore, as if A furnished $4000 for 1 month, and B $4400 for 1 month. As A furnished $\frac{4000}{8400} = \frac{10}{21}$ of the stock, he should have $\frac{10}{21}$ of the gain; and as B furnished $\frac{4400}{8400} = \frac{11}{21}$ of the stock, he should have $\frac{11}{21}$ of the gain.

Analyze the following in the same manner. Solve them also by Proportion.

RULE. *Multiply each man's stock by its time; then, as the sum of the products is to each man's product, so is the whole gain or loss to each man's share of it.*

2. A and B hire a pasture for $50. A pastures 3 cows for 8 weeks, and B 5 cows for 7 weeks. What part of the rent should each pay? How many dollars?

3. Three persons in company contract to build a bridge. A employs 15 men for 16 weeks, B 20 men for 21 weeks, and C 30 men for 24 weeks. They gain $1500. How much of the gain should each have?

4. April 1, 1848, A commenced trade, with a capital of $1000. July 1, he admits B as a partner, who furnishes $1500. C is admitted Aug. 1, with a capital of $800. Their gains for the year ending April 1, 1849, amount to $1500. What part of the gain belongs to each?

5. Jan. 1, 1849, A, B, and C form a partnership for one year, each contributing $2000. April 1st, A withdraws $500. May 1st, B withdraws $600, and C adds $800 more. Aug. 1st, C with draws $1000, and A furnishes $900 more. If they gain $2500, how much of it shall each have?

6. A, B, and C contract to grade 5 miles of railroad. A employs 25 men for $3\frac{1}{2}$ months, B 35 men for $2\frac{3}{4}$ months, and C 40 men for $3\frac{1}{4}$ months, They lose by the job $875. How much should each contribute to make good the deficiency?

135. ANALYSIS.

1. A man performed a journey of 162 miles, going twice as far the second day as he did the first, and three times as far the third day as he did the second. How far did he travel each day?

2. Divide the number 152 into three such parts that the second may be three times the first, and the third five times the second.

3. A, B and C entered into partnership. A contributed a certain sum, B $2\frac{1}{2}$ times as much, and C as much as A and B both. How much did each contribute, the whole amount being $4200?

4. A man owns a carriage that cost him 4 times as much as his horse, and both together $600. What did each cost?

5. A man hired an equal number of men and boys, giving each boy 25 cents and each man $1 per day; how many of each does he employ, if their daily wages amount to $11.25?

6. A man hired a certain number of men and 3 times as many boys, agreeing to pay each boy 30 cents a day, and each man $1.25; how many of each were there, if their weekly wages amounted to $103.20?

7. A mechanic hired a certain number of men at $1 per day, $\frac{2}{3}$ as many at 75 cents, and $\frac{1}{2}$ as many at 50 cents. What was the number of each, the daily wages of the whole being $21?

8. A, B, and C, meeting on the road, agreed to dine together. A furnished 5 loaves, B 3 loaves, and C, having no bread, paid 8 pieces of money for his share. How should the money be divided between A and B?

9. A laborer received $1.25 for every day he worked, and forfeited his wages and 35 cents more every day he was idle. He was idle one day in a week, and received $47.20. How long was he employed?

10. A father's age is 5 times his son's, and the sum of their ages is 42 years. How old is each?

11. A farmer bought cows, calves and sheep, of each an equal number, for $156; he gave $30 apiece for cows, $$5\frac{1}{2}$ for calves, and $$3\frac{1}{2}$ for sheep. How many of each did he buy?

12. A farmer bought cows, calves and sheep, for $337.50; he gave $30 apiece for cows, $$5\frac{1}{2}$ for calves, and $$3\frac{1}{2}$ for sheep. There were 3 times as many calves as cows, and

twice as many sheep as calves. How many of each did he buy?

13. Divide the number 336 into four such parts that the second shall be twice the first, the third $2\frac{1}{2}$ times as much as the first and second, and the fourth as much as the other three.

NOTE. The parts are as the numbers 2, 4, 15 and 21. Why?

14. The greater of two numbers is $7\frac{1}{2}$ times the less, and the sum of the numbers is 204. What are the numbers?

15. A spendthrift, after spending $\frac{3}{8}$ of his money and $\frac{3}{4}$ of the remainder, had $500 left. How much had he at first?

16. A cistern has 3 pipes; the first can fill it in $\frac{1}{2}$ an hour, the second in $\frac{1}{3}$ of an hour, and the third in $\frac{1}{4}$ of an hour. How long will it take to fill it if they are all left open together?

17. Three fifths of a certain number exceed $\frac{3}{8}$ of it by 270. What is the number?

18. If it takes a man $3\frac{1}{2}$ days to perform a piece of work, working $9\frac{1}{2}$ hours per day, how long will it take him to perform it if he works $10\frac{3}{4}$ hours per day?

19. If 9 men mow 10 acres of grass in a day, how much will 15 men mow in $3\frac{1}{2}$ days?

20. Bought 978 pounds of sugar, at $8\frac{1}{2}$ cents per lb.; for $\frac{2}{3}$ of which paid in potatoes at 45 cents, and the rest in cash. How much cash and how many potatoes did I pay?

136. MISCELLANEOUS EXERCISES IN SECTION XII.

1. What is the ratio of $8\frac{3}{8}$ to $5\frac{7}{15}$? Of $3\frac{9}{16}$ to $75\frac{1}{24}$?
2. What is the ratio of 8.3 to 9.75? Of 5.47 to 3.3?
3. Express the ratio of .87 to .087; of .00015 to .19, and reduce the ratios to their lowest terms.
4. Express in a simple form the ratio compounded of 8 : 7, and 15 : 4.
5. Express in a simple form the ratio compounded of $8\frac{3}{8} : 7\frac{3}{4}$, and $15\frac{1}{3} : 4\frac{3}{66}$.
6. What is the fourth term of this proportion — $8\frac{1}{2} : 7\frac{1}{2} :: 25 : —$?
7. If 1000 bricks will build a wall 9 inches broad, 26 feet long, 4 feet high, how many will build a wall 18 inches broad, 130 feet long, and 6 feet high?

8. If an iron bar 2 feet long, 3 inches broad, and 1 inch thick, weighs 18 lb., what will be the weight of another bar of iron, which is 7 feet long, 6 inches broad, and $3\frac{1}{2}$ thick?

9. How many men will build a wall 240 yards long, 6 feet high, and 3 feet thick, in 8 days, when 7 men can build another wall 40 yards long, 4 feet high, and 2 feet thick, in 32 days?

10. A person engaged to complete a portion of railway 490 yards long in 38 days, and for that purpose hired 60 men; but at the end of 22 days, he finds no more than 210 yards finished. How many additional men must he employ, to complete the work in the stipulated time?

11. A, B, and C contract to build a railroad for $9150. A employs 20 men 60 days, B 30 men 50 days, and C 60 men 55 days, and he is to receive $150 for superintending the work. How much should each man receive?

12. Three men purchase a vessel. A pays $\frac{3}{10}$ of the cost, B $\frac{4}{9}$, and C the remainder, which was $1150. What was C's share, and how much did A and B pay?

13. If 10 barrels of flour can be bought for 45 bushels of wheat, and 15 bushels of wheat for 25 bushels of corn, and 24 bushels of corn for 20 bushels of rye, and 18 bushels of rye for $12.75, how many barrels of flour can be bought for $100?

14. If 25 lb. at New York are equal to 22 lb. at Nuremburg, and 88 lb. at Nuremburg are equal to 92 lb. at Hamburg, and 46 lb. at Hamburg are equal to 49 lb. at Bourdeaux, how many pounds at New York are equal to 98 lb. at Bourdeaux?

15. Six merchants trade after this manner. A puts in $250 for 6 months, and $300 for 4 months; B puts in $450 for 8 months; C puts in $800 for 5 months, and $500 for 4 months; D puts in $200 for 7 months, and $500 for 5 months; E puts in $1500 for 10 months, $500 for 2 months, and $300 for 5 months. Their whole gain is $1000. What part of it shall each have?

16. A commences trade Jan. 1st, with a capital of $1000. April 1, he admits B as a partner, with a capital of $800. Their profits at the end of the year are $650. What is each person's share of the gain, after paying to A a salary of $350, and deducting interest at 6 per cent. on each person's capital? How much of the whole profits must each receive?

NOTE. A's interest is $60; his salary $350. B's interest is $34. Deducting these sums from $650, there remain $206, to be divided in

the ratio of $\frac{5}{8}$ to $\frac{3}{8}$, (134,) which gives for A's net profit $128.75; for B's, $77.25. A's net profit, $128.75, + his salary, $350, + his interest, $60, = $538.75. B's net profit, $77.25, + his interest, $34, = $111.25.

17. A, B, and C enter into partnership. A at first contributes $500, and in 3 months afterward $250 more; B contributes at first $650, but at the end of 6 months he withdraws $200; C at first puts in $375, and at the end of 7 months $400 more. At the end of the year they find they have gained $875; $275 of which belongs to A for transacting the business. How much of the whole gain must each receive?

18. Four men own a saw-mill. A paid $1000, B $1600, C $1750, and D $2250. What is their yearly income from the investment, if the mill rents for $600, the taxes and other expenses being $150.50? What per cent. do they realize per annum on the money invested?

SECTION XIII. — ALLIGATION.

137. ALLIGATION is of two kinds, MEDIAL and ALTERNATE.

ALLIGATION MEDIAL is the process by which we find the average or mean value of a mixture composed of several different ingredients, when the value and quantity of each are given.

1. A grocer mixes 12 lb. of sugar worth 8 cents per lb., with 15 lb. worth 10 cts. per lb. What is 1 lb. of the mixture worth?

12 lb. at 8 cts. per lb. are worth	$0.96,
15 lb. at 10 cts. " " " "	$1.50,
27 lb. are worth	$2.46; therefore, 1 lb. is

worth $\frac{1}{27}$ of $2.46.

RULE. *Divide* THE TOTAL VALUE *of the quantities by* THE SUM *of the quantities.*

2. If 15 bushels of oats worth 45 cts. per bushel, 10 bushels of rye worth 60 cts., and 12 bushels of barley worth 50 cts. per bushel, be mixed together, what will 1 bushel of the mixture be worth?

3. A goldsmith melted 3 oz. of gold of 18 carats fine, with

5 oz. 20 carats fine, and 4 oz. 21 carats fine, with 2 oz. of alloy. What was the fineness of the mixture?

NOTE. A carat is $\frac{1}{24}$ of any quantity of gold. Gold 18 carats fine is $\frac{18}{24}$ pure gold; the rest is of some baser metal, which is regarded as of no value.

4. What is the average length of 22 pieces of cloth, of which 5 pieces measure $20\frac{1}{4}$ yards each, 8 pieces $20\frac{3}{4}$ yards each, 6 pieces 21 yards each, and 3 pieces $21\frac{1}{4}$ yards each?

5. A composition is made of 18 lb. of tea at 66 cents per lb., with 20 lb. at 75 cents, and 16 lb. at 78 cents per lb. What is the worth of 3 lb. of this mixture?

138. ALLIGATION ALTERNATE is the process by which we find what quantity of each of several ingredients, whose values are given, will compose a mixture of a given rate.

1. In what proportions must I mix barley worth 50 cts. and oats worth 40 cts. per bushel, that the mixture may be worth 47 cents a bushel?

NOTE. I must mix them so as to gain just as much on the oats as I lose on the barley. I gain 7 cts. for every bushel of oats I use, and as I lose but 3 cents on 1 bushel of barley, I must use as many bushels of barley as there are times 3 in 7, or $2\frac{1}{3}$. I must therefore use $2\frac{1}{3}$ bushels of barley to 1 of oats, or 7 of barley to 3 of oats; for the ratio of $2\frac{1}{3}$ to 1 = the ratio of 7 to 3. (**126.**)

Analyze the following examples in the same manner.

Prove your answers to be correct, by Alligation Medial.

2. In what proportions must a grocer mix sugars worth 8 and 12 cts. together, to make a mixture worth 11 cts.? 10 cts.? 9 cts.?

In making a mixture worth 11 cts., he will gain 3 cents on 1 lb. at 8 cents; but he must use 3 lb. at 12 cts., to lose 3 cents. *Ans.* 1 lb. at 8 cents to 3 lb. at 11 cents.

3. In what proportions must wine that costs 80 cents per gallon, and water, be mixed together, to reduce the price of the wine to 75 cts. per gallon? 70 cts.? 50 cts.? 60 cts? 85 cts.?

4. In what proportions must a grocer mix wines at 50 cts.

and 60 cts. per gallon, with water, that the value of the mixture may be 45 cts. per gallon? 40 cts.? 35 cts.? $37\frac{1}{2}$ cts.?

NOTE. To make a mixture at 45 cents per gallon, by using 1 gallon of water, he gains 45 cents; on 1 gallon of wine at 50 cents, he will lose 5 cents; and on 1 gallon at 60 cents, he will lose 15 cents. He may therefore use 1 gallon at 50 cents, and $2\frac{2}{3}$ at 60 cents, to 1 gallon of water; or 1 at 60 and 6 at 50 cents, to 1 gallon of water. The proportions may, therefore, be either 1 at 50, $2\frac{2}{3}$ at 60, and 1 of water; or, 6 at 50 cents, 1 at 60, and 1 of water; or, 2 at 50, $2\frac{1}{3}$ at 60, and 1 of water; or, 3 at 50, 2 at 60, and 1 of water; or, 4 at 50, $1\frac{2}{3}$ at 60, and 1 of water. The proportions may thus be varied indefinitely.

By this method of solving the question, the grocer may use a limited quantity of one or more of the ingredients, or a larger or smaller proportion of either, as may suit his convenience.

5. A grocer has 8 gallons of wine worth 50 cts. per gallon, which he would mix with 3 gallons of wine at 60 cts., and with water, to make a mixture worth 45 cts. per gallon. How much water must he use? How much water, if he use 5 gallons at 50 cts. and 6 at 60 cts.?

6. Mix teas at 30, 36, 40, and 50 cts. per lb., so as to make a mixture worth 42 cents per lb.; 37 cents per lb.; 45 cts. per lb.

7. How much port wine of American manufacture, at \$1.75, temperance wine, at \$1.25 per gallon, and water, may be mixed together, to make a mixture of 500 gallons that may be sold at \$1 per gallon?

NOTE. Find the *proportions* as above, and then find the *quantities*, as in Art. **133**, quest. 6 to 10.

8. A grocer has two kinds of sugar, worth 8 and 11 cents per lb. How much of each must he take to answer an order for 300 pounds at 9 cents per lb.?

9. I have 2 kinds of cloves, one of which cost me 15 cents per lb. and the other 20 cents; I wish to fill an order for 800 pounds at 20 cents. Of how many pounds of each kind shall the mixture be made, that I may gain 20 per cent. on the cost?

10. There is a mixture made of wheat at 4s. per bushel, rye at 3s., barley at 2s., with 12 bushels of oats at 18d. per bushel. How much has been taken of each sort, when the mixture is worth 3s. 6d.?

SECTION XIV. — DUODECIMALS.

139. *Duodecimals* are compound numbers, the denominations of which increase and diminish in a uniform ratio of 12, as in the table.

1 foot =	12 inches or primes,	(′)
1 inch or prime =	12 seconds,	(″)
1 second =	12 thirds,	(‴)
1 third =	12 fourths,	(⁗)

$$1' = \frac{1}{12} \text{ of } 1 \text{ ft.} = \frac{1}{12} \text{ ft.}$$
$$1'' = \frac{1}{12} \text{ of } 1' = \frac{1}{12} \text{ of } \frac{1}{12} \text{ of } 1 \text{ ft.} = \frac{1}{144} \text{ ft.}$$
$$1''' = \frac{1}{12} \text{ of } 1'' = \frac{1}{12} \text{ of } \frac{1}{144} \text{ ft.} = \frac{1}{1728} \text{ ft.}$$
$$1'''' = \frac{1}{12} \text{ of } 1''' = \frac{1}{12} \text{ of } \frac{1}{1728} \text{ ft.} = \frac{1}{20736} \text{ ft.}$$

The marks used to indicate the denominations are called *indices.*

Duodecimals are applied to square and cubic measure. The dimensions are usually taken in feet and inches, the feet being integers, the inches 12ths of a foot, or primes.

If the length of any square surface in feet be multiplied by the width in feet, the product will be square feet; and if the surface in square feet be multiplied by the depth in feet, the product will be cubic feet. If the length in feet be multiplied by the width in inches, the product will be 12ths of a foot or primes; and if 12ths be multiplied by 12ths, the product will be 144ths. Or, as $1 \times \frac{1}{12} = \frac{1}{12}$, so $1 \times 1' = 1'$; and as $\frac{1}{12} \times \frac{1}{12} = \frac{1}{144}$, so $1' \times 1' = 1''$; as $1 \times \frac{1}{144} = \frac{1}{144}$, so $1 \times 1'' = 1''$; $\frac{1}{12} \times \frac{1}{144} = \frac{1}{1728}$, so $1' \times 1'' = 1'''$. Hence we derive the following rule for finding the denomination of any product in duodecimals.

The product of any two denominations must have as many indices as there are indices in both factors.

NOTE. Feet, being integers, have no index.

1. What is the product of feet multiplied by feet? feet by primes? primes by primes? feet by seconds? primes by seconds? seconds by thirds? seconds by fourths? thirds by seconds? by thirds? by fourths?

Required the cubical contents of a block of marble 7 ft. 8 in. long, 6 ft. 4 in. wide, and 3 ft. 5 in. high?

To get the area of one end of the block, we multiply its width by its height, (**56, 70.**) Beginning with the feet of the multiplier, $4' \times 3 = 12' = 1$ ft. to be carried to the product of feet by feet. 6 ft. $\times$ 3 ft. = 18 ft., and 1ft. are 19 ft. We next multiply by the $5'$; $4' \times 5' = 20'' = 1'\ 8''$. We write the $8''$ to the right, and carry the $1'$ to the next product. $6 \times 5' = 30'$, and $1'$ are $31' = 2$ ft. $7'$. Adding the two partial products gives the square surface of 1 end of the block. Multiplying this by the length in the same manner, we obtain the cubical contents.

6	4'		
3	5'		
19	0'		
2	7'	8''	
21	7'	8'	
7	8'		
151	5'	8''	
14	5'	1''	4'''
165	10'	9''	4'''

RULE FOR MULTIPLYING DUODECIMALS.

1. *Write the corresponding denominations of the factors under each other.*

2. *Multiply each denomination in the multiplicand, beginning with the lowest, by each denomination in the multiplier, and write each partial product under the corresponding denominations of the factors.*

3. *Add the partial products together.*

1. Multiply 3 ft. 5' by 4 ft. 8'; 6 8' by 5'; 15 4' by 9'; 12 4' 3'' by 5' 6''; 13 6' by 4; 18 0' 6'' by 1 5'.

2. How much is 3 4' 2'' $\times$ 3 1'? 6 5' 2'' $\times$ 3 5' $\times$ 3' 10''?

3. How many square feet in a board 15 ft. 4 in. long, and 8 in. wide? 1 ft. 5 in. wide? 25 ft. 8 in. long, and 2 ft. 3 in. wide?

4. How many square feet on the floor of a room 16 ft. 5 in. long, and 14 ft. 8 in. wide?

5. How many square feet in 3 doors, each 6 ft. 2 in. high, and 3 ft. 10 in. wide?

6. How many square feet of surface are there in the ceiling, floor, and the four sides of a room, which is 15 ft. 6 in. long, 14 ft. 5 in. wide, and 9 ft. 4 in. high?

7. How many square feet are there on the surface of a chest, the outside dimensions of which are 8 ft. 3 in. long, 3 ft. 5 in. wide, and 3 ft. 7 in. deep?

8. How many bushels of corn will the above box hold, if the boards are one inch thick, the bushel being 2150.4 cubic inches?

9. How many square yards upon a floor that is 25 ft. 3 in. long, and 21 ft. 4 in. broad?

10. How many cubic feet in a pile of wood 12 ft. 4 in. long, 10 ft. 3 in. high, and 3 ft. 10 in. wide? How many cords?

SECTION XV. — POWERS AND ROOTS.

140. INVOLUTION.

1. **What is the product of 5 multiplied by itself? *Ans.* $5 \times 5 = 25$.**
2. **What is the product of 5 used three times as a factor? *Ans.* $5 \times 5 \times 5 = 125$.**
3. **What is the product of 6 used twice as a factor? 3 times? 4 times? 5 times?**

The multiplying of a number by itself is called *involution*. The product obtained by involution is called a *power*. The number multiplied by itself to obtain the power is called the *first power*, or *root*. It is called the root, because all the powers spring from it. If the root or first power is used twice as a factor, the product is called the second power; if it is used three times, it is called the third power, &c.

The second power of a number is often called the square of the number; because the area of a square is equal to the 2d power of one of its sides. The third power is often called the cube of the number; because the solid content of a cube is equal to the 3d power of the length of one of its sides.

The power of a number is generally indicated by a small figure, called an index or exponent, placed at the right of the number, a little above it.

Thus, $4^2 = 4 \times 4 =$ the square or 2d power of 4; $4^3 = 4 \times 4 \times 4 =$ the cube or 3d power of 4; $4^4 = 4 \times 4 \times 4 \times 4 =$ the 4th power of 4.

To find any required power of a number, either integral or fractional.

RULE. *Multiply the number by itself, until it is used as many times as a factor as the name of the power indicates.*

4. What is the square or 2d power of 1? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? Of $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{5}$? $\frac{3}{5}$? $\frac{4}{7}$? $1\frac{1}{2}$? $2\frac{1}{2}$? $3\frac{1}{2}$? $4\frac{1}{3}$? $2\frac{2}{3}$? .1? .2? .3? .4? .5? .6? .7? .8? .11?
5. What is the cube or third power of 1? 2? 3? 4? 5? 6? Of .1? .2? .3? .4? .5? 6?

6. Find the value of the following expressions: 2^3; 5^3; 35^2.
(7.) 54^2; $(4.2)^2$; $(8.05)^3$. (8.) $(5\frac{4}{5})^4$; $(3\frac{3}{4})^3$; $(8.51)^3$.

141. EVOLUTION.

Evolution is the reverse of Involution. Involution teaches to find a product of equal factors; but Evolution is the process of resolving a product into its equal factors. Each of these factors is called a root of the number. If the number be resolved into two equal factors, each factor is called the square root, or 2d root, of the number. If the number be resolved into three equal factors, each factor is called the cube root, or 3d root.

Thus, 5 is the square root of 25; 3, the cube root of 27; 2, the 4th root of 16; because $5 \times 5 = 25$; $3 \times 3 \times 3 = 27$; $2 \times 2 \times 2 \times 2 = 16$; — the name of the root expressing the number of equal factors into which the given number is resolved.

The square root is indicated by this sign $\sqrt{\ }$ placed before the number; thus, $\sqrt{9}$ means the square root of 9. The other roots are indicated by the same sign, with the index of the root placed over it. The cube root is expressed thus, $\sqrt[3]{27}$; the 4th root, $\sqrt[4]{16}$. Or, the roots may be indicated by a fractional exponent; as, $(9)^{\frac{1}{2}}$; $(27)^{\frac{1}{3}}$; $(16)^{\frac{1}{4}}$, which are read, the square root of 9; the cube root of 27; the 4th root of 16, &c.

If several numbers are included in the power, with the sign plus or minus between them, the root is expressed thus:

$$\sqrt{30 + 15 - 9} = 6; \quad \sqrt[3]{40 + 3 - 16} = 3.$$

$$\text{Or, } (30 + 15 - 9)^{\frac{1}{2}} = 6; \quad (40 + 3 - 16)^{\frac{1}{3}} = 3.$$

Express in both ways the square root of 20; the cube root of 40; the 4th root of $\frac{15}{16}$; the 5th root of 60.

A *perfect power* is a number whose root can be exactly extracted; as, $\sqrt{16}$, $\sqrt[4]{16}$, $\sqrt[3]{27}$, $\sqrt{\frac{25}{4}}$; and its root is called a *rational* number.

An *imperfect power* is a number whose root cannot be exactly extracted; as, $\sqrt{8}$, $\sqrt[3]{9}$, $\sqrt[4]{25}$; and the root of such a power is called a *surd*, or *irrational number*.

A number may be a perfect power of one degree, and an imperfect power of another; thus, 25 is a perfect 2d power, but an imperfect 3d or 4th power.

142. EXTRACTION OF THE SQUARE ROOT.

The Extraction of the Square Root of a number is the process of finding one of its two equal factors; or, of finding a

number which, being multiplied into itself, will produce the given number. The following exhibits the 2d powers or squares of the first twelve integral numbers:

Roots,	1	2	3	4	5	6	7	8	9	10	11	12.
Squares,	1	4	9	16	25	36	49	64	81	100	121	144.

The square of any number cannot have more than twice as many figures as its root, and but one less than twice as many.

This is true when the root consists of one figure; for 1^2 contains one figure, and 9^2 contains but two figures. It is true also when there are two figures in the root; for 10^2 contains three figures; 99^2 contains but four figures. Is it true when there are three figures in the root? Prove it. Prove it to be true when there are four figures in the root.

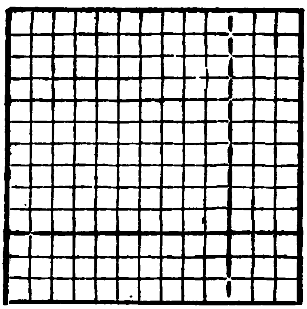
If, then, the square number consists of one or two figures, there will be one figure in the square root; if it contains three or four figures, the root will contain two, &c.

1. How many figures will there be in the root, if there are 5 in the square? How many if the square contains 6 figures? 7 figures? 8? 9? 10? 11? 12? Why?

2. What is the square of .2? .25? .105? 3.001? How many decimals will the root contain, if the square contains 4 decimals? Why? 6 decimals? 8 decimals? Why? 10 decimals?

The number of figures in the root may be known by placing a dot over every second figure in the square, *beginning with units*.

3. What must be added to the square of 10, to make the square of 13, or of $10+3$?



If to two adjacent sides of a square of ten feet, represented as in the margin, we add three rows of 10 squares each, we see that 3^2, or 9 more squares of 1 foot each, are wanted to complete the square of 13. Therefore, if to the square of 10 we add twice 10×3 and 3^2, we shall obtain the square of $10+3$; and the square of 13, thus expressed, will be, *The square of* 10, *plus twice the product of* 10 *into* 3, *plus the square of* 3; *or*, 10^2+ *twice* $10\times3+3^2$.

4. Express in like manner the square of each of the following numbers, illustrating each by a diagram of your own constructing: $(10+1)^2$; $(10+2)^2$; $(10+4)^2$; $(10+5)^2$; $(3+2)^2$; $(5+3)^2$.

From these examples and illustrations, we see that *the square of the sum of any two numbers is equal to the square of the first, plus twice the product of the first into the second, plus the square of the second.*

5. Find by this method the square of 4+3; 5+8; 1+½; 2½=2+½; 5½; 6½; 12½; 20½; 30½.

143. Let us now extract the square root of 1444.

```
       8
  1444(3~~9~~
   9
 8
6~~9~~) 544
    ~~621~~
     544
```

1. As the square contains 4 figures, the root must contain 2 figures, tens and units; and the whole square must consist of *the square of the tens, and twice the product of the tens into the units, and the square of the units.*

2. As the square of tens is hundreds, the square of the tens of the root will be found in the hundreds of the square. The greatest square in 14 is 9; and its root, 3, is the tens of the root. Subtracting the square of the tens from the whole square, the remainder contains twice the tens of the root into the units, and the square of the units; or the product of *twice the tens, plus the units*, multiplied by the units. (Why?) (**97.**) Therefore, dividing the remainder by the former factor, viz., twice the tens plus the units, would exactly give the latter factor, viz., the units. (Why?) But, as the units' figure is not yet found, we take twice the tens, or 6, as a *trial* divisor. Dividing the 54 tens by this trial divisor, gives 9 for the units of the root. Adding this to the trial divisor for a *complete* divisor, and multiplying the latter by the units, we get too large a product. Consequently, our units' figure was too large, as was to have been expected, since our trial divisor was too small. We, therefore, erase this units' figure and the product; and, trying a smaller units' figure, proceed as before. Hence the following

RULE FOR EXTRACTING THE SQUARE ROOT.

1. *Separate the given number into periods of two figures each, by placing a dot over every second figure, beginning at units;* thus, $8\dot{4}1\dot{6}5.0\dot{4}1\dot{6}8\dot{0}$. The number of dots will show how many figures, whether integers or decimals, the root will consist of.

2. *Find by trial the greatest square number in the left hand period,* (which may consist of one or two figures,) *and place its root on the right hand of the given number, as you do a quotient in division. Subtract the square of the root thus found from the first period, and to the remainder bring down the second period for a dividend.*

3. *Double the root already found, and place it on the left of this dividend, for a trial divisor. See how many times it is contained in the tens of the dividend, and annex the result both to the root already found,*

and also to the trial divisor, for a complete divisor. If the trial divisor is not contained in the tens of the dividend, annex a naught to the root, and to the trial divisor, for the next trial divisor, and bring down the next period for a dividend.

4. ***Multiply the complete divisor by the last figure in the root, subtract the product from the dividend, and to the remainder annex the next period for a new dividend.***

If the product should exceed the dividend, diminish the last figure of the root, and of the complete divisor.

5. *Repeat the same process, viz.: Double all the figures in the root for a new trial divisor, and, dividing by it, find as before the next figure in the root; and continue the operation till all the periods are brought down.*

To extract the root of a common fraction, reduce it to its lowest terms, and extract the root of the numerator and of the denominator, for the numerator and denominator of the root, *when both terms of the fraction are perfect squares;* if they are not, reduce the fraction to a decimal, and extract the root by the above rule.

If the number of decimals is odd, a naught must be annexed; and if there is a remainder after bringing down all the periods, decimal naughts forming new periods may be annexed.

PROOF. *Square the root; the result should equal the given square.*

1. Extract the square root of $78689\frac{7}{8}$.

As the decimal .875 consists of an odd number of figures, a naught is annexed to make the number even. The second trial divisor, 56, not being contained in the tens of the dividend, a naught is annexed to the root and to the trial divisor, and another period brought down, as directed in the rule.

```
               7̇86̇89̇.87̇50̇(280.517
               4
         48 ) 38,6
              384
       5605 ) 2898,7
               28025
      56101 ) 9625,0
               56101
     561027 ) 401490,0
               3927189
                 87711
```

2. Extract the square root of 1225; 2401; 7569.

3. Extract the square root of 15625; 531441; 1048576.

4. Extract the square root of 20421361; of 36529936; of 72726784.

5. Find $\sqrt{12.25}$; $\sqrt{24.01}$; $\sqrt{.7569}$.

6. Find $\sqrt{5314.41}$; $\sqrt{53.1441}$; $\sqrt{1.5625}$; $\sqrt{104.8576}$.

7. Extract the square root of 2042.1361; of 204.21361.

8. Extract the square root of $\frac{9}{16}$; of $\frac{324}{576}$; of $\frac{1225}{7569}$.

9. What is the square root of $12\frac{1}{4}$? (Change it to an improper fraction.) Of $456\frac{1}{4}$? of $14\frac{11}{25}$?

10. What is $\sqrt{3\frac{1}{15}}$? $\sqrt{25\frac{3}{85}}$? $\sqrt{24.301}$? $\sqrt{45164.8106}$?

NOTE. Carry the root in Nos. 10 to 15, to six decimal places.

11. What is the square root of 30.4167081? of $3\frac{5}{24}$? of $504\frac{3}{7}$?

12. What is the square root of 410.680716? of .00081645? of $\frac{1}{857}$?

13. Extract the square root of $\frac{25}{18475}$; of $\frac{2}{3}$; of $8\frac{2}{87185}$.

14. Find $\sqrt{\frac{3}{8} + \frac{5}{6} + 4\frac{5}{12}}$.

NOTE. Add the quantities before extracting the root.

15. Find $\sqrt{3\frac{5}{8} + \frac{4}{9} \text{ of } 2\frac{7}{15}}$; $\sqrt{3\frac{7}{8} \div \frac{8}{15}}$.

For the application of the square root, see Art. 160 to 165, which the pupil may learn before proceeding to the cube root, if his teacher should think it best.

QUESTIONS. What is *involution?* a *power?* the *first power?* the *root?* Why is it called the *root?* What is the *second power?* the *third?* What is the *square* of a number? Why so called? The *cube* of a number? Why? How is a power indicated? Give examples. Repeat the rule for finding any required power of a number.

What is *evolution?* What is the difference between *involution* and *evolution?* What is the *root* of a number? the *square* root? the *cube* root? How is the square root indicated? the cube root? the 4th root? In what other way? Give examples. What is a *perfect* power? a *rational* number? an *imperfect* power? a *surd?* Show that a number may be a *perfect* power of one degree, and an *imperfect* power of another.

What is the extraction of the *square root?* Repeat the first 12 integral numbers, and their squares. How many figures does a square number contain? how many decimals? Prove it to be true. How may the number of figures in the root be known? To what is the square of the sum of two numbers equal? Illustrate this. Repeat the rule for extracting the square root, and show the reason for the different parts of the process. How do you extract the root of a common fraction? When is it to be reduced to a decimal?

144. EXTRACTION OF THE CUBE ROOT.

The *Extraction of the Cube Root* of a number is the process of finding one of its three equal factors; or, of finding a number which, being multiplied into itself, and then into that product, will produce the given number. The following numbers in the upper line represent roots, and those in the lower line their third powers, or cubes.

Roots.	1	2	3	4	5	6	7	8	9	10	11	12.
Cubes.	1	8	27	64	125	216	343	512	729	1000	1331	1728.

The cube of any number cannot have more than three times its number of figures, and never but two less than three times as many.

This is true when the root has but one figure; for $1^3=1$; $2^3=8$; $3^3=27$; $5^3=125$; $9^3=729$. It is also true when the root has two figures; for $10^3=1000$; $50^3=12500$; $99^3=970299$. Show that it is true when the root consists of three figures; four figures.

There will be three times as many decimals in the cube as in the root.

1. What is the cube of .1? .03? .005? .0007?

Hence the number of figures in the root, both of integers and decimals, may be known by placing a dot over every third figure in the cube, beginning at units.

2. How many figures will there be in the root, if there are 11 in the cube? 12? 13? 14? 15?

3. Find the cube of $28=20+8$.

$$
\begin{array}{lll}
\text{Multiply} & 20+8 & \\
\text{by} & 20+8 & \\
\hline
 & 20^2+20\times8= & \text{prod. of } \overline{20+8}\times20. \\
 & \qquad 20\times8+8^2= & \text{“ “ } \overline{20+8}\times8. \\
\hline
\text{Multiply} & 20^2+2\times20\times8+8^2= & \text{“ “ } \overline{20+8}\times\overline{20+8}. \\
 & 20+8 & \\
\hline
 & \multicolumn{2}{l}{20^3+2\times20^2\times8+20\times8^2=(20^2+2\times20\times8+8^2)\times20} \\
 & \multicolumn{2}{l}{\qquad 20^2\times8+2\times20\times8^2+8^3=(20^2+2\times20\times8+8^2)\times8} \\
\hline
 & \multicolumn{2}{l}{20^3+3\times20^2\times8+3\times20\times8^2+8^3=(20+8)\times(20+8)\times(20+8).}
\end{array}
$$

By cubing 28, as in the above example, we see that the cube of the sum of two numbers, consisting of units and tens, is *the cube of the tens*, + 3 *times the square of the tens multiplied by the units*, + 3 *times the tens multiplied by the square of the units*, + *the cube of the units*; or, *The cube of the tens; plus the product of* 3 *times the square of the*

tens, increased by 3 times the tens multiplied by the units, and the square of the units, all multiplied by the units (**97**); *plus the cube of the units;* that is, $20^3+(3\times 20^2\times 8)+(3\times 20\times 8^2)+8^3=20^3+(\overline{3\times 20^2}+\overline{3\times 20\times 8}+8^2)\times 8+8^3$.

145. We will now extract the cube root of 21952.

		8
t. d.	~~1200~~	$2\dot{1}95\dot{2}$(2~~9~~
	~~540~~	8
	~~81~~	13952
c. d.	~~1821~~	~~16389~~
t. d.	1200	13952
	480	13952
	64	
c. d.	1744	

1. The root will contain two figures, tens and units. (Why?) 2. The cube of the tens will be found in the thousands. (Why?) The greatest cube in 21 is 8, and its root, 2, will be the tens of the root. 3. Subtracting the cube of the tens from the given cube, the remainder contains three *times the square of the tens, plus three times the tens multiplied by the units, plus the square of the units*, all multiplied by the units. (Why?) (**97.**) Therefore, dividing the remainder by the former factor, viz., 3 times the square of the tens, &c., would exactly give the latter factor, viz., units. But as the units' figure is not yet found, we will take three times the square of the tens, or 12 hundreds, as a *trial* divisor. If we divide the remainder by this trial divisor, the quotient would be more than 11. We know the units' figure cannot be more than 9; therefore the quotient is too large, as we might expect it would be, since our divisor is but a *partial* divisor. A *complete* divisor is to be formed by adding to the trial divisor 3 times the tens multiplied by the units, and the square of the units. 3 times 2 tens multiplied by 9, or, (which is the same,) 30 times 2 multiplied by 9, is 540, and 9^2 is 81. Adding these to the trial divisor for a complete divisor, and multiplying by 9, we get too large a product. Our quotient figure is therefore still too large. We must erase it, and go over this part of the work again. Taking 8 as a quotient figure, we make the complete divisor 1744, which being multiplied by 8, gives the product 13952. Hence the

RULE FOR EXTRACTING THE CUBE ROOT.

1. *Separate the given number into periods of three figures each, by placing a dot over every third figure, beginning at units;* thus, $3\dot{1}48\dot{6}.10\dot{0}84\dot{0}$. (What will the dots show? Why?)

2. *Find by trial the greatest cube number in the left hand period,* (which may consist of one, two, or three figures,) *and place its root on the right. Subtract the cube of this root from the first period, and to the remainder bring down the 2d period for a dividend.*

3. *Take 300 times the square of the root already found for a trial divisor. Divide the dividend by this trial divisor, and place the result in the root.* (If the trial divisor is not contained in the dividend, annex

one naught to the root, and two naughts to the trial divisor for the next trial divisor, and bring down the next period for a dividend.)

4. *Take 30 times the root previously found, multiplied by the figure last placed in the root, and the square of this last figure, and add them to the trial divisor, for a complete divisor. Multiply the complete divisor by the last figure* in the root; subtract the product from the dividend; bring down the next period, and proceed as before. See 3.

The remarks concerning fractions and decimals, annexed to the rule for the extraction of the square root, are equally applicable to the cube root, by substituting the word cube for square.

The number of decimals in the cube must be a multiple of 3; as 3, 6, 9, &c.

1. Extract the cube root of $81563\frac{1}{2}$.

	$\dot{8}1\dot{5}6\dot{3}.50\dot{0}(43.3$
$4^3 =$	64
$4^2 \times 300 = 4800$	17563
$3 \times 4 \times 30 = 360$	
$3^2 = 9$	
First divisor, 5169	15507
$(43)^2 \times 300 = 554700$	2056500
$43 \times 3 \times 30 = 3870$	
$3^2 = 9$	
Second divisor, 558579	1675737
	380763

2. Extract the cube root of 79507; 132651; 704969.

(3.) 1124864; 28652616; 40001688.

(4.) 68417929; 480048687; 527514112.

5.* How much is $\sqrt[3]{809}$? $\sqrt[3]{300}$? $\sqrt[3]{4.10846}$?

(6.) $\sqrt[3]{341}$? $\sqrt[3]{54.0061071}$?

7. How much is $\sqrt[3]{\frac{8}{27}}$? $\sqrt[3]{\frac{343}{729}}$? $\sqrt[3]{\frac{6859}{10648}}$?

(8.) $\sqrt{12\frac{19}{27}}$? $\sqrt{52\frac{47}{64}}$? $\sqrt{4\frac{4737}{4913}}$?

9. How much is $\sqrt[3]{\frac{3}{8}}$? $\sqrt[3]{\frac{1}{18}}$? $\sqrt[3]{\frac{1}{1200}}$?

(10.) $\sqrt[3]{15\frac{3}{14}}$? $\sqrt[3]{18\frac{7}{28}}$?

For the application of the cube root, see Art. **178** and **179**.

QUESTIONS. What is the *extraction of the cube root?* Repeat the first 12 integral numbers, with their cubes. How many figures are

* Extract the root to the nearest ten thousandth.

there in the cube, compared with the number in the root? How many decimals? Give examples to show the truth of this. To what is the cube of a number, consisting of units and tens, equal? Show this by an example. How may the number of figures in the root be ascertained? Why? Repeat the rule for the extraction of the cube root, and give the reason for each step in the process.

SECTION XVI. — ARITHMETICAL SYMBOLS.

146. All the arithmetical symbols or signs usually employed have been introduced in different parts of this work. As the different processes in arithmetic can be so clearly and concisely indicated by means of them, it is important that the pupil should become so familiar with them, as to be able to use them properly, and to understand them when correctly used by others.

The expression $3+4\times5-2$ is read 3 plus 4 times 5 minus 2; and is the same as $3+20-2$, or $3+18=21$.

$\overline{3+4}\times5-2$, or $(3+4)\times5-2$, is the same as $7\times5-2=35-2=33$.

The expression $5\times4\times8\div2+2$ is $20\times4+2$; or, $160\div2+2=80+2=82$. $5\times4\times8\div\overline{2+2}$ is $160\div4=40$.

The expression $6\times15\div3\times2$ is $90\div6=15$; $6\times15\div3\times2-4$ is $90\div6-4=15-4=11$; $6\times15\div\overline{3\times2-4}$ is $90\div2=45$.

The expression $3+8\div4-2$ is $3+2-2=3$; but $\overline{3+8}\div4-2$ is $11\div4-2=2\frac{3}{4}-2=\frac{3}{4}$; and $(3+8)\div(4-2)$ is $11\div2=5\frac{1}{2}$.

The expression $(5-3+4)\times(5-1+2)\div(4+7)$ is $6\times6\div11=\frac{36}{11}=3\frac{3}{11}$.

The expression $(\frac{1}{2}+\frac{3}{4})\times\frac{5}{6}\div(\frac{10}{9}-\frac{1}{3})$ is $\frac{5}{4}\times\frac{5}{6}\div\frac{7}{9}=\frac{5}{4}\times\frac{5}{6}\times\frac{9}{7}=\frac{75}{56}=1\frac{19}{56}$.

The expression $\sqrt{}(25-16)\times\sqrt[3]{}(18+9)$ is $\sqrt{9}\times\sqrt[3]{27}=3\times3=9$. So $\sqrt{\overline{29+20}}-\sqrt[3]{64}\times\frac{1}{8}$ is $\sqrt{49}-\sqrt[3]{64}\times\frac{1}{8}=7-4\times\frac{1}{8}=7-\frac{1}{2}=6\frac{1}{2}$; but $\sqrt{\overline{29+20}}-\sqrt[3]{\overline{64\times\frac{1}{8}}}$ is $\sqrt{49}-\sqrt[3]{8}=7-2=5$.

The expression $[(\frac{5}{8} - \frac{5}{12} \div \frac{1}{4} + \frac{3}{8}) \times \frac{4}{5}] \div \frac{2}{3}$ is $[(\frac{5}{24} \div \frac{5}{8}) \times \frac{4}{5}] \div \frac{2}{3} = \frac{1}{3} \times \frac{4}{5} \div \frac{2}{3} = \frac{2}{5}$.

Find the value of the following expressions.

(1.) $25 + 13 - 1 + 7$. (2.) $25 + 13 - (1 + 7)$.

(3.) $18 \times 5 - 3 \times 4$. (4.) $18 \times \overline{5 - 3} \times 4$.

(5.) $\overline{45 \div 9} - 4 + 3 \times 8$.

(6.) $45 \div \overline{9 - 4} + 3 \times 8$. (7.) $45 \div \overline{9 - 4 + 3 \times 8}$.

(8.) $(75 - 5) \times 8 + 4 \div 2 + \overline{6 \times 8} - 3$.

(9.) $75 - (5 \times 8 + 4) \div 2 + 6 \times 8 - 3$.

(10.) $\overline{75 - 5} \times (\overline{8 + 4} \div 2) + 6 \times \overline{8 - 3}$.

(11.) $75 \div \overline{15 - 10} + 8 \times 4 - \overline{15 \div 3} + 2$.

(12.) $75 \div \overline{15 - 10} + (8 \times 4) - 15 \div \overline{3 + 2}$.

(13.) $\frac{3}{8} + \frac{5}{12} - \frac{13}{24}$. (14.) $(\frac{3}{4} \times \frac{5}{9} + \frac{5}{8}) \times (\frac{5}{6} - \frac{3}{8})$.

(15.) $\frac{3}{4} \times (\frac{5}{9} + \frac{5}{8}) \times \frac{5}{6} - \frac{3}{8}$.

(16.) $2\frac{1}{2} \div (1 - \overline{\frac{4}{13} + \frac{1}{2} + \frac{1}{17}})$.

(17.) $42 \div [(2 \times 1.50) - .50]$.

(18.) $(2\frac{5}{8} + 9\frac{1}{3}) \div \frac{1}{2} \times \frac{3}{17}$.

(19.) $16\frac{3}{8} \div \frac{4}{5} - (\frac{4}{5} \times 8\frac{2}{3} - 2\frac{7}{8})$.

(20.) $\sqrt{25} + 24 - (\sqrt[3]{64} + \sqrt[3]{125})$.

(21.) $\sqrt{(25 + 24)} - \sqrt[3]{64 + 125}$.

(22.) $\frac{1}{2} \times \frac{3}{4} \div \frac{3}{8} - \frac{5}{14}$. (23.) $\sqrt{36 - 16 + \frac{1}{4}}$.

SECTION XVII. — PROGRESSION, OR SERIES.

147. ARITHMETICAL PROGRESSION.

Arithmetical Progression, or *Series by Difference*, is a series of numbers which uniformly increase or decrease, so that the difference between any two adjacent numbers is the same throughout the series. This difference is called the *common difference*. When the *common difference* is *added* to each term to make the succeeding one, it is called an *ascending series;* as, 1, 3, 5, 7, &c. When it is subtracted, it is called a *descending series;* as, 7, 5, 3, 1.

The numbers that compose the series are called the *terms* of the series. The first and last terms are called the *extremes*, and the others, the *means*.

In every arithmetical series there are 5 things to be considered, viz.:

1. The first term.
2. The last term.
3. The common difference.
4. The number of terms.
5. The sum of all the terms; any three of which being given, the rest may be found.

1. If in a series the first term is 2, and the common difference 3, what is the 2d term of the series? the 3d? the 5th? the 10th term?

From this example we see that, to find the second term, we add the common difference *once* to the first term; to get the third term, we add the common difference *twice* to the first term, &c. Hence the following rules under CASE I. and II.

CASE I. The first term, common difference, and number of terms being given, to find the last term.

RULE. *Multiply the common difference by one less than the number of terms, and add the product to the first term.*

2. If in a series the first term is 3, and the common difference 5, what is the 4th term? the 8th? 15th? 20th?

3. A stone falls 16.1 feet during the first second of its descent in free space, 48.3 ft. the 2d second, 80.5 ft. the 3d, &c.; how far would it fall the 8th second?

4. A man lends $100 at 6 per cent. simple interest; what will it amount to in 1 year? in 2 years? in 5 years? in 8 years? The principal and the amounts for the successive years form an arithmetical series; what is the first term? the common difference? the last term?

CASE II. The two extremes and the number of terms being given, to find the common difference.

RULE. *Subtract the less extreme from the greater, and divide the remainder by 1 less than the number of terms; the quotient will be the common difference.*

5. The extremes of an arithmetical series being 6 and 76, and the number of terms 15, what is the common difference?

SOLUTION. $\frac{76-6}{14}=5$, the answer.

6. A man has 5 sons; the youngest is 6 years old, and the eldest 18; their ages increase in arithmetical progression; what is the common difference of their ages?

CASE III. To find the sum of all the terms of an arithmetical series.

7. What is the sum of all the terms of the series 2, 5, 8, 11, 14, 17?

If we write the series twice, reversing the order of the terms as in the margin, and add the corresponding terms, we see that *twice* the sum of all the terms of the series is equal to 19 multiplied by 6; or, *to the product of the sum of the extremes multiplied by the number of terms.* Hence the following

2,	5,	8,	11,	14,	17
17,	14,	11,	8,	5,	2
19,	19,	19,	19,	19,	19

RULE. *Find the last term as in Case I., and then multiply the sum of the extremes by half the number of terms.* Or,

Multiply the sum of the extremes by the number of terms, and take ½ the product.

8. How many times does the hammer of a clock strike in 12 hours?

SOLUTION. $(1+12)\times 6=78$. Or, $\frac{(1+12)\times 12}{2}=78$.

9. How many times would it strike in 24 hours, if the hours were numbered from 1 to 24?

10. How far will a stone fall in 3 seconds? in 6 seconds? in 8 seconds?

NOTE. The last term is the distance it will fall in the 3d, the 6th, or the 8th second. See question 3.

11. If a young man, at the age of 21, "a temperate drinker," spends 5 cents per day, for his favorite beverage, for 300 days in the year, how much does it cost him in one year?

12. If for each succeeding year it costs him $3 more than

the preceding, (a very moderate estimate,) how much will he have spent at the age of 40, if he should escape the drunkard's grave till that age?

NOTE. The last term is $3 \times (19-1) + 15$.

13. Thirteen wheels revolve in arithmetical progression; the first makes 3 and the last 51 revolutions per second. How many revolutions does each wheel exceed the former, and how many do they all together make in one second?

NOTE. Find the common difference, by Case II.

14. Eleven wheels revolve in arithmetical progression. The first makes 3 revolutions, and the last 33, per second. Required the sum of the series.

148. ANNUITIES AT SIMPLE INTEREST.

A sum of money due annually, quarterly, or at any other regular periods, is called an *annuity*. The periodical sums are sometimes called *instalments*.

The *present worth* of an annuity is that sum which being put at interest for the time, will be sufficient to pay it.

The *amount* of an annuity is the interest of all the instalments added to their sum.

An annuity at simple interest is an example of arithmetical progression; in which the first term is the periodical sum, the common difference is the interest on the periodical sum for the time that elapses between two successive instalments, and the number of terms the number of instalments.

1. A man hired a house for \$200 per annum, agreeing to pay the rent quarterly; but for 2 years the rent has remained unpaid. How much does he now owe, reckoning simple interest at 8 per cent. per annum on each quarterly payment?

NOTE 1. The quarter's rent just due is \$50. That due 3 months ago is \$50, and the interest on it for 3 months; that due 6 months ago is \$50, and 6 months' interest. If, then, we call \$50 the first term in the series, what will be the second term? The common difference? The number of terms? The last term? The sum of all the terms?

2. A man bought a house-lot for \$500, agreeing to pay for

it in 5 equal yearly instalments, without interest. What sum, at the end of 5 years, will pay the whole amount, reckoning interest on each instalment from the time it becomes due?

NOTE. The payment last due is $100 without interest; the last payment but one is $100 and one year's interest; the last but two is $100 and two years' interest, &c. If, then, we call $100 the first term, what will be the common difference? The number of terms? The last term? The sum of all the terms?

From these examples, To find the amount of an annuity at simple interest, we derive the following

RULE. *Find the last term of the series as in Case I., Art.* **147**, *and then the sum of the series as in Case III.*

3. What is the amount of an annuity of $150, that has been *in arrears*, that is, that has not been paid, for 10 years, reckoning interest at 6 per cent. per annum?

4. What will an annuity of $375 amount to in 5 years, reckoning simple interest at 8 per cent.?

5. If a pension, payable quarterly, of $100 per annum, remains unpaid for 3 years, what will be due at the end of that time, reckoning simple interest at the rate of 5 per cent. per annum?

149. GEOMETRICAL PROGRESSION.

Geometrical Progression, or *Series by Quotient*, is a series of numbers which increase or decrease so that the quotient of any term divided by the preceding one shall give the same quotient throughout the series. This quotient is called the *Common Ratio.*

When the ratio is more than one, the series is called an *Ascending Series;* when the ratio is less than one, the series is called a *Descending Series.*

Thus, in the ascending series 3, 12, 48, 192, &c., the ratio is 4. In the descending series 72, 42, 8, $2\frac{2}{3}$, $\frac{8}{9}$, the ratio is $\frac{1}{3}$.

In Geometrical Progression, 5 things are to be considered, viz.:

1. The first term.
2. The last term.
3. The common ratio.
4. The number of terms.
5. The sum of all the terms.

CASE I. 1. If the first term of a geometrical progression is 5, and the ratio 3, what is the 5th term of the series? $5 \times 3 \times 3 \times 3 \times 3 = 5 \times 3^4 = 405$. Hence

The first term, common ratio, and number of terms being given, to find the last term.

RULE. *Raise the ratio to a power whose exponent is one less than the number of terms, and multiply the power by the first term; the product will be the last term.*

2. A man bought 10 yards of cloth, for which he agreed to pay 1 cent for the first yard, 3 cents for the 2d, 9 for the 3d, &c. What did he pay for the last yard?

3. What will \$20 amount to, in 8 years, at 6 per cent., compound interest?

NOTE. The first term is the principal, the ratio 1.06, the number of terms 9.

4. What will \$1050 amount to, in 10 years, at 5 per cent., compound interest?

5. What is the 8th term of the series 20, 60, 180, &c.?

CASE II. The ratio and the two extremes being given, to find the sum of the series.

6. What is the sum of the series 5, 15, 45, 135, 405?

The series 15, 45, 135, 405, 1215, is obtained by multiplying the given series by the ratio, 3. If, therefore, we subtract the first series from the 2d, the remainder will be twice the first series. From the order in which the terms are arranged, it will be seen that this remainder is obtained by subtracting the first term in the given series from three times the last term. Therefore, $1215 - 5 = 1210$ is twice the sum of the given series; and $\frac{1210}{2} = 605$, is the sum of the given series.

Hence, to find the sum of a series by quotient, we have the following

RULE. *Find the last term as in the preceding article; multiply the last term by the ratio, and divide the difference between the product and the first term by the difference between the ratio and 1.*

7. What is the sum of 10 terms of the series 1, 4, 16, 64, &c.?

8. A man bought a house, agreeing to pay for it in 12 monthly payments. The first payment was to be \$1, the 2d \$3, the 3d \$9, &c. How much was the last payment? How much did they all amount to?

150. Annuities at Compound Interest.

1. What will an annuity of \$1 amount to in 5 years, at 6 per cent., compound interest?

This question forms a geometrical series of 5 terms, in which \$1 is the first term, the amount of \$1 for 1 year, or \$1.06, the ratio, and the amount of \$1 for 4 years the last term. Therefore, $1 \times (1.06)^4 =$ the last term; and $\frac{1 \times (1.06)^4 \times 1.06) - 1}{1.06 - 1}$ $= \frac{(1 \times 1.06^5) - 1}{1.06 - 1} =$ the amount.

Hence, to find the amount of an annuity at compound interest, we have this

Rule. *Find the sum of the series, as in the last article.**

2. What will an annuity of \$50 amount to in 10 years, at 5 per cent., compound interest?

Note. First find the amount of an annuity of \$1, as in the last example. $\frac{(1 \times 1.05^{10}) - 1}{1.05 - 1} \times 50 =$ ——.

3. James Phillips was born Aug. 1, 1824. Aug. 1, 1834, his father deposited for James, in the savings bank, \$10, and did the same on each returning birth-day, till his son was 21 years old. What did all the deposites amount to at that time, at 6 per cent.? at 5 per cent.?

4. "What would be the difference, at the end of 10 years, between two young men, A and B? A spends \$100 a year, in theatres, amusements, &c., and B invests the same sum in business, in such a way that the principal and interest of one year yield him 15 per cent. for the next year?" — Wayland.

5. What is the difference between the amount of an annual deposite of \$100 for 10 years, bearing an annual compound interest at 5 per cent., and a semi-annual deposite of \$50 for

* See table III. page 262.

10 years, bearing a semi-annual compound interest, at $2\frac{1}{2}$ per cent.?

MISCELLANEOUS EXAMPLES IN ANNUITIES.

II. 6. A young man has bought a farm for \$3500. He pays \$2000 cash, and gives his note on interest at 6 per cent. for the remainder, the payment of which is secured by a mortgage of the whole farm. How much must he save yearly, in order to pay his interest money annually, and clear his farm in 10 years?

NOTE. The amount of all his yearly savings, at compound interest, must be equal to the amount of the *whole note* for 10 years at compound interest. Therefore, divide the amount of the note, \$1500, at compound interest for 10 years, by the amount of an annual payment of \$1 for 10 years (Quest. 1), the quotient will be the answer.

7. How much must he save yearly, to clear his farm in 8 years? in 5 years?

8. A man wishes to put at compound interest such a sum of money as will afford him annually \$100 for 10 years, at the end of which time the principal and interest shall be exhausted. What sum must be put at interest, the rate being 6 per cent.?

NOTE. Divide the amount of an annuity of \$100 for 10 years by the amount of \$1 at compound interest for 10 years.

9. How much must he put at interest at 5 per cent., to yield an annuity of \$200 for 20 years?

10. What is the present worth of an annuity of \$150 to continue 10 years, allowing compound interest at 6 per cent.?

NOTE. This question is the same as the preceding. Why?

When an annuity is not to commence till some specified time has elapsed, or till the occurrence of some future event, it is called *an annuity in reversion.*

11. A person leaves an estate, the annual rent of which is \$400, to his widow, during her life, and the reversion of the same to his son for 10 years after her death. What is the present value of each legacy, allowing compound interest at 5 per cent., supposing the widow to live 15 years after the death of her husband?

NOTE. The present worth of the annuity to continue 25 years will be the present value of both their legacies. Its present worth for 15 years is the present value of the widow's, and this subtracted from the value of both will give the value of the son's. Express the rule for this in your own language.

12. What would be the present worth of each of the above legacies, if the widow should survive her husband 10 years? 5 years?

151. PERMUTATION.

Permutation is the method of finding in how many ways any number of things may be arranged. To do this, we have the following

RULE. *Multiply continually together all the terms of the natural series, from* 1 *up to the given number; the product will be the answer required.*

1. For how many days can 5 persons be placed in a different position around a table at dinner? $1 \times 2 \times 3 \times 4 \times 5 = 120$. For how many days can 10 persons? 12?

2. How many changes can be made of the letters in the word Charlestown? New York? Manchester?

QUESTIONS. What is *arithmetical progression?* What is the *common difference?* What is an *ascending series?* a *descending series?* Give examples. What are the *terms* of the series? the *extremes?* the *means?* What 5 things are to be considered in an arithmetical series? Give the rule when the first term, common difference and number of terms are given, to find the last term;—the two extremes and the number of terms being given, to find the common difference. What is the rule for finding the sum of all the terms? Why? Demonstrate the rule.

What is an *annuity?* an *instalment?* the *present worth* of an annuity? the *amount* of an annuity? Show that an annuity at simple interest is an example of arithmetical progression. (See quest. 1, note.) What is the *first term?* the *common difference?* the *number of terms?* What is meant by an annuity in *arrears?*

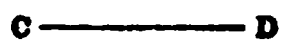
What is *geometrical progression?* What is the *common ratio?* What is an *ascending series?* a *descending series?* Give examples. What are the 5 things to be considered in a geometrical series? What is the rule when the first term, ratio, and number of terms are given, to find the last term? Give an example. The first term and ratio given, to find the sum of all the terms? Demonstrate the rule. Show that an annuity at compound interest is an example of a geometrical series. What is the *first term?* the *ratio?* the *last term?* the *sum of all the terms?* How may the present worth of an annuity be found? What is an annuity in reversion? How do you find the present value of an annuity in reversion?

SECTION XVIII.—SURFACES.

152. GEOMETRICAL DEFINITIONS.

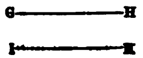
1. A POINT is position only, without dimensions.

2. A LINE has one dimension only, length.

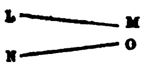


3. A STRAIGHT LINE is extension in one direction only; it is the shortest distance between two points.

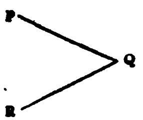
4. A CURVE LINE constantly changes its direction.



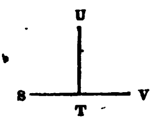
5. PARALLEL LINES are equally distant in every point, and never meet, though ever so far extended.



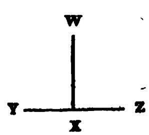
6. OBLIQUE OR INCLINED LINES change their distance from each other, and would meet if sufficiently extended.



7. AN ANGLE is the opening between two lines which meet in a point. The point of meeting is called the VERTEX of the angle. Thus, the opening between the lines P Q and Q R is an ANGLE, and the angular point at Q is the VERTEX of the angle. Angles are denoted by three letters, the middle letter denoting the angular point; as, P Q R, or R Q P, denotes the angle at Q.



8. RIGHT ANGLES are angles which are made by two lines meeting so as to form equal angles. S T U and V T U are right angles.

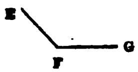


9. PERPENDICULAR LINES are lines which meet at equal angles.

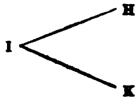
The lines W X and Y Z are perpendicular to each other.

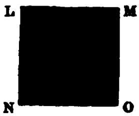
10. HORIZONTAL LINES are lines parallel to the plane of the horizon.

11. VERTICAL LINES are lines perpendicular to the plane of the horizon.



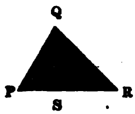
12. AN OBLIQUE ANGLE is either greater or less than a right angle. When greater, it is called an OBTUSE angle, as E F G; when less, it is called an ACUTE angle, as H I K.



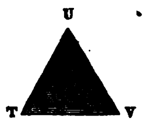


13. A SURFACE is the outside of anything. It has two dimensions, length and breadth.

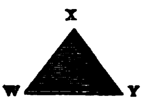
14. A PLANE SURFACE does not change its direction; that is, it is perfectly flat or level.



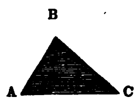
15. A TRIANGLE is a figure bounded by three sides. Its *altitude* or *height* is the perpendicular distance between one of the angles and the opposite side. The side to which the perpendicular is drawn is called the *base*. Thus, P Q R is a triangle; Q S is the *altitude*, and P R the *base*.



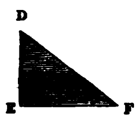
16. AN EQUILATERAL TRIANGLE is a figure that has its three sides equal.



17. AN ISOSCELES TRIANGLE is one that has two of its sides equal.



18. A SCALENE TRIANGLE is one that has its three sides unequal.



19. A RIGHT-ANGLED TRIANGLE is one that has one right angle, as at E. The *hypothenuse* is the side opposite the right angle; as, the side D F.

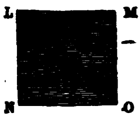
20. An obtuse-angled triangle has one obtuse angle.

21. An acute-angled triangle has all its angles acute.

22. A quadrilateral or quadrangle is a figure bounded by 4 straight lines; as, G H I K.

23. A parallelogram is a quadrilateral that has its opposite sides parallel.

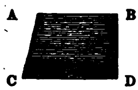
24. A rectangle is a right-angled parallelogram; as, G H I K.



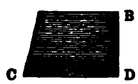
25. A square is an equilateral rectangle; as, L M N O. The line L O is a diagonal.

26. A rhomboid is an oblique-angled parallelogram; as, P Q R S.

27. A rhombus is an equilateral rhomboid; as, T U V W.

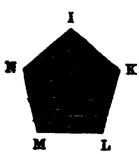


28. A trapezoid is a quadrilateral which has only one pair of its opposite sides parallel; as, A B C D.



29. A Trapezium is a quadrilateral, neither pair of whose opposite sides are parallel.

30. Plane figures bounded by more than four straight lines are called *polygons*. A polygon of 5 sides is called a pentagon; of 6 sides, a hexagon; of 7, a heptagon; of 8, an octagon.

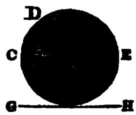


31. A regular polygon has all its sides and all its angles equal. If they are not all equal, the polygon is irregular. The figure I K L M N is a regular polygon.

The equilateral triangle and square are *regular* figures.

32. The PERIMETER of a figure is the sum of all its sides.

33. A figure all the sides of which are straight lines is called a *rectilinear figure*.

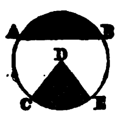


34. A CIRCLE is a plane surface, enclosed by a curve line called the circumference, every part of which is equally distant from the centre.

35. The DIAMETER of a circle is a straight line passing through its centre and terminated by the circumference; as, C E.

36. The RADIUS of a circle is a straight line extending from the centre to the circumference of the circle; as, F C, or F E. The point F is the centre.

37. A TANGENT is a straight line which touches the circumference only in one point, but which, when extended, does not cut it; as, G H.

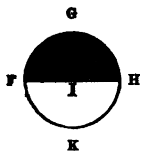


38. An ARC of a circle is any part of the circumference; as, A B or C E.

39. A CHORD is a straight line joining the extremities of an arc; as, A B.

40. A SEGMENT is any part of a circle bounded by an arc and its chord; as the surface included between the chord A B and the arc A B.

41. A SECTOR is any part of a circle bounded by an arc and two radii drawn to its extremities; as, the surface C D E.

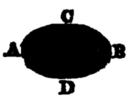


42. A SEMICIRCUMFERENCE is one half the circumference; as, the line F G H.

43. A QUADRANT is one quarter of the circumference; as, the line F G or G H.

44. A SEMICIRCLE is a half of a circle bounded by a diameter and a semicircle; as, the surface F H K.

45. The CIRCUMFERENCE of every circle is supposed to be divided into 360 equal parts, called degrees; each degree into 60 equal parts, called minutes; and each minute into 60 equal parts, called seconds. Hence, a semicircle contains 180 degrees, and a quadrant, 90 degrees.



46. AN ELLIPSE is an oval figure, having two diameters or axes. The longer axis is called the *transverse*, and the shorter the *conjugate* axis, or diameter.

153. MENSURATION OF SURFACES. (Art. 56.)

The area of a SQUARE *is equal to the square of one of its sides; consequently, the side of a square is equal to the square root of its area.*

1. What is the area of a square field, each side of which is 40 rods? 15 rods? $20\frac{1}{2}$ rods? 18 rods $3\frac{1}{2}$ yards? 5 rd. 3 yd. 2 ft.?

2. What is the side of a square field whose area is 900 sq. rods? 15 acres? 25 acres?

3. The area of a circle is 361 sq. ft. How long is a square of equal area?

4. How many sq. ft. in the floor of a square room whose side is 15 ft. 8 in.?

154. *The area of a rectangle is found by multiplying its longer side by the shorter.*

5. How many acres in a rectangular lot of land which is 20 rods long and 12 rods wide? What is one side of a square of equal area?

6. How many yards of plastering in the ceiling of a room which is 65 feet long and 38 feet wide?

7. How many feet of boards will it take to cover the four sides of a barn which is 57 feet long, 38 feet wide, and 18 feet high?

155. *The area of any parallelogram is found by multiplying the base by the altitude.*

156. *The area of a trapezoid is found by multiplying the sum of its parallel sides by half the perpendicular distance between them.*

8. How many square feet in a board that is 15 ft. long, one end of which is 15 and the other 10 inches wide? If one end is 5 and the other 15 inches wide? If one end is 25 and the other 23 inches wide?

157. A triangle is one half of a parallelogram of the same base and altitude. Therefore,

RULE 1. *The area of a triangle is equal to the product of half the base multiplied by the altitude.*

9. What is the area of a triangular field, one side of which is 15 rods, and the perpendicular to the corner opposite this side is 12 rods?

NOTE. Any triangular field or other surface may be measured by this method. Or,

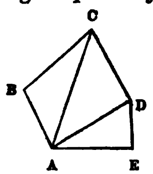
RULE 2. *Measure the three sides of the field; add them together, and from half their sum subtract each side separately; then multiply the half sum and the three remainders together; the square root of this product will be the area.*

10. The three sides of a triangular field measure 15, 20 and 25 rods respectively. What is the content of the field?

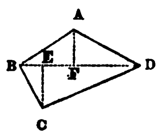
Solution. $\frac{15+20+25}{2}=30$. $30-15=15$; $30-20=10$; $30-25=5$. $\sqrt{30\times15\times10\times5}=$ the area.

158. QUADRILATERALS AND POLYGONS.

The area of any surface that is bounded by straight lines may be found by dividing it into triangles, and measuring each triangle separately.



11. Let A B C D E represent a pasture. The side A B measures 20 rods, B C 30, C D 25, D E 15, and E A 25 rods; the diagonal A C 40, and A D 30 rods. What is the area of the field?



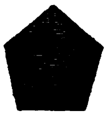
12. Required the area of the quadrilateral A B C D, in which the diagonal B D is 133, and the perpendiculars A F 37, and C E 44 yards.

13. If in the above quadrilateral the side A B measures 72, C B 46, C D 125, D A 80, and the diagonal B D 133 feet, what is the area? What would be the cost of such an area at 10 cents per square foot?

159. Regular Polygons.

Irregular polygons may be measured as in Art. **158**, but to find the area of *regular* polygons,

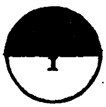
Multiply half the perimeter by the perpendicular let fall from the centre upon one of its sides.



14. What is the area of a regular pentagon, each side of which is 250 feet, and the perpendicular from the centre 172.05 feet?

15. What is the area of a regular octagon, each side of which is 20 yards, and the perpendicular from the centre 24.14 yards?

160. The Circle.



The *circumference* of a circle is about $3\frac{1}{7}$, or, more nearly, 3.1416 times its *diameter*.

To find the area of a circle,

Rule 1. *Multiply half the diameter by half the circumference.* Or,

2. *Multiply the square of the diameter by* .7854. Or,
3. *Multiply the square of the circumference by* .0795775.

16. What is the circumference of a circle whose diameter is 15 inches? 25 in.? 23 ft.? 12 ft.?
17. What is the diameter of a circle whose circumference is 25 ft.? 37 ft.?
18. What is the circumference of a circle whose radius is 30 ft.?
19. What is the area of a circle the diameter of which is 15 inches? 25 in.? 23 ft.? 12 ft.?
20. Find the area of a circle whose circumference is 54 inches.

161. The area being given to find the diameter.

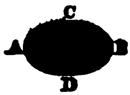
Since the area $=$ *(diameter)*$^2 \times .7854$, *the diameter* $= \sqrt{\textit{area} \div .7854}$.

RULE. *Divide the area by .7854, and take the square root of the quotient.*

21. What is the diameter of a circle the area of which is 1 acre? $\frac{1}{4}$ of an acre? 4 acres?

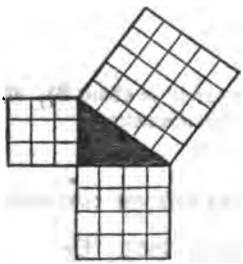
162. To find the area of an ellipse,

Multiply the product of the two diameters by .7854.



22. What is the area of an ellipse whose diameters are 20 and 24 feet?

23. What is the area of an ellipse the axes of which are 30 and 40 yards?



163. In every right-angled triangle, the square described on the hypothenuse is equal to the sum of the squares described upon the base and perpendicular. If E F G be a right-angled triangle, and right-angled at F, then will the square H, described on the hypothenuse E G, be equal to the sum of the squares I and K, described on the base F G and perpendicular E F. By counting the small squares, we find the number of small squares in the square H to be equal to the number of small squares in the squares I and K.

Hence, the square of the hypothenuse of a right-angled triangle is equal to the sum of the squares of both the other sides; and, therefore, the hypothenuse is equal to the square root of the sum of the squares of the other sides; and either side is equal to the square root of the difference between the square of the other side and the square of the hypothenuse.

24. The base of a right-angled triangle is 20 feet, and its height 30 feet. How long is the hypothenuse?

25. What is the distance between the opposite corners of a room 15 feet long and 12 feet wide?

26. The hypothenuse of a right-angled triangle being 75 ft., and the base 25 ft., what is its height?

27. How long is the diagonal of a square whose side is 8 feet?

28. If the diagonal of a square is 54 ft., what is one side of the square?

164. SIMILAR SURFACES.

1. What is the area of a square, one side of which is 1 inch? 2 inches? 3 inches? 4 inches? 5 inches?

2. If the area of a square is 1 inch, what will be the area of a square the side of which is twice as long? 3 times as long? 4 times as long? 5 times as long?

3. What is the area of a triangle whose base and altitude are 2 feet and 3 feet respectively? 4 and 6 ft.? 6 and 9? 8 and 12? 10 and 15? 20 and 30?

4. If the area of a triangle is 3 feet, how many times as large will be the area of one the sides of which are twice as long? 3 times as long? 4 times? 5 times?

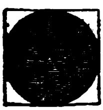
5. What is the area of a rectangle whose length and breadth are 3 and 4 respectively? 6 and 8? 9 and 12? 15 and 20?

6. If the area of a rectangle is 12, how many times as large will be the area of a rectangle whose base and altitude are twice as long? 3 times? 4 times? 5 times?

NOTE 1. *Similar rectangles or triangles, or other rectilinear figures*, are those whose corresponding dimensions are proportioned; thus, in No. 5 above, the length and breadth of each rectangle are in the ratio of 3 to 4. In No. 3 the bases and altitudes of the triangles are in the ratio of 3 to 4. They are therefore similar. *All squares are similar to each other; so are all circles.* Why?

NOTE 2. If a line is drawn in a triangle, parallel to one of the sides, and meeting the other two sides, it divides those two sides proportionally; the small triangle cut off is, therefore, similar to the whole undivided triangle.

NOTE 3. *The areas of similar surfaces are in proportion to the squares of their like dimensions.* Illustrate this truth by the above examples.



7. Draw upon your slate a square, one side of which shall be one inch. Inscribe in this square a circle; that is, draw in it a circle whose circumference shall touch each side of the square. What is the diameter of the circle? What is the area of the square? *The area of the circle is .7854 of an inch.* (160, Rule 2.)

8. Draw a square whose side is 2 inches, and inscribe in it a circle as before. What is the diameter of the circle? What is the area of the square? What is the area of the circle?

9. Draw another square whose side is 3 inches, and ask and answer the same questions as before.

Give, if you can, a reason for the rule for finding the areas of circles; viz., "Multiply the square of the diameter by .7854."

10. Draw two circles about the same centre, one with a radius of 2 inches, and another with a radius of 3 inches. What is the ratio of their areas?

From the above examples and illustrations we may derive the following rules for the use of those who need them.

I. To find the area of any surface which is similar to a given surface.

Rule. ***As the square of either of the sides or dimensions of the surface whose area is known, is to the square of the corresponding side or dimension of the other surface, so is the area of the first surface to that of the second.***

II. To find the side, diameter, or circumference of any surface which is similar to a given surface whose dimensions are known.

Rule. *As the area of the figure whose side is known, is to the area of the other, so is the square of any dimension of the former to the square of the corresponding dimension of the latter. Extracting the square root of this fourth term will give the answer.*

11. If a pipe 1 inch in diameter discharge 2 gallons of water per minute, how many gallons will be discharged by a pipe 2 inches in diameter? $2\frac{1}{2}$ inches? 3 inches? $3\frac{1}{4}$ inches?

12. If a $1\frac{1}{2}$ inch tube discharge 20 gallons in 18 minutes, how large a tube will be needed to discharge 80 gallons in the same time? 180 gallons? 320 gallons?

13. There is a right-angled triangle 12 feet in perpendicular height. How far from the base must a line be drawn parallel to the base, to cut off $\frac{5}{12}$ of the whole triangle? (Note 2.)

14. How would a line parallel to the base, and 6 feet from it, divide the triangle?

15. If a rope 6 inches in circumference consists of 450 threads or strands, required the number of such threads to make a 14 inch cable.

16. If 30 feet in length of a cable 10 inches in circumference weighs 117 pounds, how much will 30 feet of another cable of the same stock weigh, that is 15 inches in circumference? What must be the circumference of a rope which would weigh $\frac{1}{4}$ as much as the first cable?

17. There is a triangle containing 85 square rods, and one

of its sides measures 15 rods. What is the area of a similar triangle, the corresponding side of which measures 10 rods?

18. If a perpendicular pole 10 feet long casts a shadow 7½ feet long, what is the height of a steeple that casts a shadow of 140 feet at the same time?

19. A triangular board is 18 inches wide at the base, and 12 feet long. At what distance from the base must a line be drawn to cut off half of it? ⅔ of it? ⅓ of it?

165. MISCELLANEOUS EXERCISES IN SURFACES.

1. What is the side of a square floor containing 1521 square feet?

NOTE 1. To find the side of a square equal in area to any given superficies,

Extract the square root of the given area.

2. The side of a square is 15 feet. How long is the diagonal?

3. The diagonal of a square is 20 feet. What is one side of the square?

4. A rectangle is 25 feet long and 20 feet wide. What is its area? How long is its diagonal?

5. Two ships have sailed from the same port; one north 50 miles, the other east 60 miles. How far apart are they?

6. Four boys attend the same school. Charles lives 150 rods north from the school-house, James 100 rods east, William 75 rods south, and Henry 200 rods west from the school-house. How far, in a direct line, do Charles and James live apart? James and William? William and Henry? Henry and Charles? Charles and William? James and Henry?

7. A carpenter, wishing to test the correctness of his "square," makes a mark on one of its arms 6 feet, and also another 8 feet, from the point where the two arms meet. How far apart are these marks, if the arms are exactly perpendicular?

8. What is the area of a circular plot of land, the diameter of which is 18 rods? 36 rods? 9 rods? 3 rods?

9. What is the diameter of a circular field, the area of which is 16 acres? 64 acres? 4 acres? 1 acre?

10. A rectangular court is 100 feet long and 20 feet wide. How much further does he travel who goes from one corner to the opposite one, following the direction of the fence, than he who crosses it diagonally?

11. A rectangular field is 15 rods long and 12 wide. What is it worth, at $180 per acre? How much will it cost to fence it, at $2.50 per rod? How far from either corner is the centre of the field?

12. If a leaden pipe, $1\frac{1}{2}$ inch in diameter, will fill a cistern in two hours, what will be the diameter of another pipe, to fill the same cistern in one hour?

13. If 20 feet of iron railing weigh 1120 lb. when the bars are $1\frac{1}{4}$ inch square, what will 30 feet weigh, if the bars are $\frac{7}{8}$ inch square? What will 50 feet weigh, and what will it come to at $6\frac{1}{4}$ cents per pound?

14. If a round pillar 7 inches in diameter has four cubic feet of stone in it, what must be the diameter of a pillar of equal length, to contain ten times as much stone?

15. A gentleman has a fish-pond in the form of a triangle, containing 480 poles; he wants another, one half as large, in the form of a square. Required the side. (Note 1.)

16. There is a rectangular field 220 yards long and 22 yards broad. What length of fence will enclose the same area in a square?

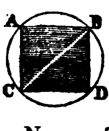
17. A farm consists of 4 fields; the first, 2 A. 3 R. 14 sq. rd.; the second, 3 A. 1 R.; the third, 1 A. 0 R. 18 sq. rd.; the fourth, 4 A. 3 R. 24 sq. rd. What shall be the side of a square field equal in area to all four?

18. The length of a line, stretched from the top of a steeple to a station 250 feet from its bottom, was found to measure 330 feet. What was the height of the steeple?

19. The breadth of a building is 32 feet, and the height of the angle for the roof, that is, of the ridge above the beams, is 9 feet. Required the length of the rafter.

20. What is the perpendicular of an equilateral triangle, each side of which is 144 yards?

21. A carriage wheel is to be $5\frac{1}{2}$ feet in diameter; what will be its circumference? How long must each felloe of the wheel be, if there are to be 6 felloes? How long, if there are to be 5 felloes?

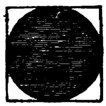


22. If the diameter of a circle is 1 inch, how long will be the *diagonal* of an inscribed square? What will be the *side* of the square? What will be the side of an inscribed square, if the diameter is 42 inches?

NOTE. The square root of one half of the square of the diagonal will be one side of the square. Why?

23. The radius of a circle is 34 inches. What is the area of an inscribed square?

24. Required the difference in area between a circle whose diameter is 24, and a square inscribed in the same circle.



25. What is the difference in area between a square whose equal side is 24, and the largest circle that can be inscribed in the same square?

QUESTIONS. What is a point? a line? a straight line? a curve line? What are parallel lines? oblique or inclined lines? What is an angle? the vertex of an angle? What are right angles? perpendicular lines? horizontal lines? vertical lines? What is an oblique angle? an obtuse angle? an acute angle? a surface? a plane surface? a triangle? the height or altitude of a triangle? the base of a triangle? Give examples of each. What is an equilateral triangle? an isosceles triangle? a scalene triangle? a right-angled triangle? the hypothenuse? an obtuse-angled triangle? an acute-angled triangle? a quadrilateral or quadrangle? a diagonal? a parallelogram? a rectangle? a square? a rhomboid? a rhombus? a trapezium? a polygon? What is a polygon of 5 sides called? of 6 sides? of 7 sides? What is a heptagon? an octagon? a regular polygon? an irregular polygon? What is the perimeter of a figure? What is a rectilinear figure? a circle? the diameter of a circle? the radius? a tangent? an arc of a circle? a chord? a segment? a sector? a semicircumference? a semicircle? How is the circumference of every circle supposed to be divided? How many degrees in a semicircle? in a quadrant? What is an ellipse? What is the longer axis called? the shorter?

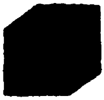
To what is the area of a square equal? the side of a square? What is the rule for finding the area of a rectangle? of any parallelogram? of a trapezoid? of a triangle? Another rule? How may the area of any surface which is bounded by straight lines be found? What is the rule for finding the area of a regular polygon?

To what is the circumference of a circle equal? How may the area be found? How may the diameter be found, when the area is given? What is the rule for finding the area of an ellipse?

To what is the hypothenuse of a right-angled triangle equal? To what is the hypothenuse equal? To what is either side equal? What are *similar* rectilinear figures? What figures are always similar? How does a line drawn parallel to one side of a triangle and meeting the other two sides, divide those sides? To what are the areas of similar figures proportional? Give a reason for the rule for finding the area of circles, viz.: "Multiply the square of the diameter by .7854." What is the rule for finding the area of a surface that is similar to a given surface? What is the rule for finding the dimensions of a surface that is similar to a given surface whose dimensions are known?

SECTION XIX. — SOLIDS.

166. GEOMETRICAL DEFINITIONS.

1 

1. A SOLID is a figure having length, breadth and thickness.

2 

2. A PRISM is a solid whose ends are equal polygons, and whose sides are parallelograms.

3. A prism whose sides are all squares is called a *cube*.* If its ends are triangles, it is called a *triangular prism*. If the ends are squares, it is called a *square prism*;* if pentagons, a *pentagonal prism*, &c.

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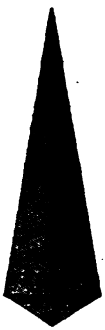
4. A CYLINDER is a round column whose ends are equal circles.

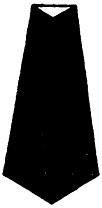
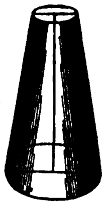
C 

D 

5. A PYRAMID is a solid having a triangle, a square, or a polygon for its base; its sides being triangles whose vertices meet in a point at the top, called the vertex of the *pyramid*.

6. A CONE is a solid having a circular base, and tapering uniformly to a point at the top.

* A solid contained by six quadrilateral planes, every opposite two of which are equal and parallel, is called a *parallelopipedon*.

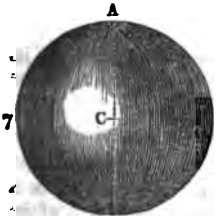
E  F 

7. A SEGMENT of a solid is the part cut off from the top by a plane parallel to its base.

8. A FRUSTUM is the part left at the bottom after the segment has been cut off.

9. The *axis* of a solid is a line drawn from the middle of one end to the middle of the opposite end.

A



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10. A SPHERE, OR GLOBE, is a solid bounded by a curve surface, all the points of which are equally distant from a point within called the *centre*.

11. A *hemisphere* is half a sphere.

12. The *axis* or *diameter* of a sphere is a line passing through the centre and terminating at the surface, as the line A B.

13. The *height* of a solid is the perpendicular distance between its top or vertex and its base.

14. The *slant height* of a *pyramid* is a line drawn from the vertex to the middle of one of the sides at the base.

15. The *slant height* of a *cone* is the shortest line that can be drawn from the vertex to the circumference of the base.

16. A *spheroid* is a solid generated by the revolution of an ellipse about one of its diameters. If the ellipse revolves about its longer diameter, the solid is called *prolate* or *oblong* spheroid; if about its shorter diameter, an *oblate* or *flattened* spheroid.

167. MENSURATION OF SOLIDS.

TO FIND THE AREA OF THE SURFACE OF A CUBE.

RULE. *Multiply the square of the length of one side by 6; the product will be the area.* (Why?)

1. The side of a cube is 15 inches; what is the area of its surface?

2. What is the area of the surface of a cube, the side of which is 10 inches? 1 ft. 6 in.? 5 ft. 8 in.?

168. TO FIND THE SOLID CONTENTS OF A CUBE.

RULE. *Cube the side given.*

3. How many solid feet in a cubical block of marble, the length of which is 15 inches? 1 ft. 4 in.? 2 ft. 5 in.?

169. TO FIND THE SURFACE OF A PRISM, PARALLELOPIPEDON, OR CYLINDER.

RULE. *Multiply the perimeter or circumference of the base by the height, and to this product add the area of the two ends; the sum will be the area.*

5. What is the surface of a triangular prism, whose length is 12 feet, and each of its equal sides 4 feet? $5\frac{1}{4}$ ft.? (**157**, Rule II.)

6. What is the surface of a square pyramid, or parallelopipedon, the height of which is 15 feet, and each side of its base $3\frac{1}{2}$ feet?

7. What is the *convex* surface of a cylinder 15 inches long, and its base 15 inches in circumference? 1 ft. 9 in. long, and 1 ft. in diameter?

8. What is the *whole* surface of a cylinder, whose length is 25 feet, and the diameter of its base 4 feet?

$(3.1416 \times 4 \times 25) + (4^2 \times .7854 \times 2) =$ the answer.

9. Required the inside and the outside surface of a box, measuring $4\frac{1}{2}$ feet long, $2\frac{1}{2}$ feet wide, and 3 feet deep on the outside, the boards of which it is made being 1 inch thick.

170. TO FIND THE SOLID CONTENTS OF A PRISM OR CYLINDER.

RULE. *Multiply the area of the base by the altitude; the product will be the solidity.*

10. What is the solidity of a triangular prism, whose length is 12 feet, and each of its equal sides 4 feet? 5½ ft.?

11. What are the solid contents of a square pyramid, the height of which is 15 feet, and each side of its base 3½ feet?

12. What is the solidity of a cylinder 15 inches long, and its diameter 6 inches? 1 ft. 9 in. long, and 1 foot in diameter?

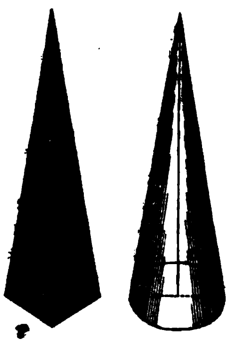
13. How many solid inches in a grindstone 28 inches in diameter, and 5 inches thick?

14. How many solid feet in a round cistern 5½ feet in diameter, and 8 feet deep?

15. If a cubic foot contains 7½ gallons, how many gallons will the above cistern hold?

171. To find the surface of a pyramid or cone.

Rule. *Multiply the perimeter, or the circumference of the base, by one half the slant height.*



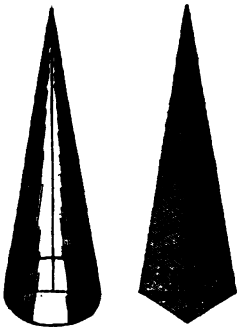
16. What is the surface of a pyramid, the perimeter of its base being 15 feet, and its slant height 25 feet? perimeter 12, and the slant height 20 inches?

17. What is the convex surface of a cone whose slant height is 20 inches, and the circumference of its base 15 inches?

18. What will be the expense of painting a conical spire of which the height is 118 feet, and the circumference of the base 64 feet, at 8d. per square yard?

172. To find the solid contents of a pyramid or cone.

Rule. *Multiply the area of the base by one third of the altitude; the product will be its solidity.*



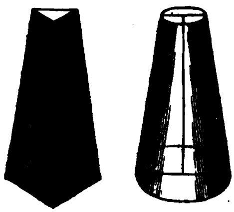
19. What are the solid contents of a square pyramid, of which the height is 42 inches, and one side of the base 14 inches? if the height be 12 ft. 3 in., and a side of the base 1 ft. 4 in.?

20. What is the solidity of a cone whose height is 15 ft., and the diameter of the base 2 ft. 4 inches?

21. How many solid ft. in a square stick of timber that tapers towards one end to a point, the length being 20 feet and a side of the base 25 inches?

173. To find the surface of a frustum of a pyramid or cone.

RULE. *Add the perimeters or circumferences of the two ends together, and multiply half the sum by the slant height, for the upright or curve surface, to which add the areas of the two ends, to get the whole surface.*



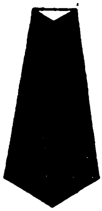
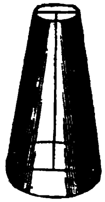
22. Required the surface of a frustum of a square pyramid, the sides of the ends being 40 and 26 inches, and the slant height 10 feet.

23. Required the surface of the frustum of a cone, the diameters of the bases being 43 and 23 inches, and the slant height 9 feet.

24. What will be the cost of dressing the upright surface of a square stone pillar, of which each side of the base is $3\frac{1}{2}$ feet, and of the top 2 feet, the pillar being 15 feet high, at $12\frac{1}{2}$ cents per square foot?

174. To find the solid content of the frustum of a pyramid or cone.

RULE. *Find the areas of the two ends, and take the square root of their product. To this add the two areas; the sum, multiplied by one third of the perpendicular height, will give the solid content.*

25. What is the content of a mast 57 ft. high, and the girths at its ends 63 and 38 inches?

26. What are the contents of a square stick of timber 25 feet long, the sides of the ends being 18 and 13 inches?

27. What is the weight of a square stone pillar, 12 feet high, each side of whose base is 4 feet, and of the top 3 feet, allowing $12\frac{1}{2}$ cubic feet to weigh a ton, 2000 pounds?

175. TO FIND THE SURFACE AND SOLID CONTENT OF A WEDGE.

RULE FOR THE SURFACE. *Find the areas of the rectangle, the two parallelograms or trapezoids, and the two triangles of which its surface consists, and add them together.*

RULE FOR THE SOLID CONTENT. *To twice the length of the base add the length of the edge, and multiply the sum by the breadth of the base, and by one sixth of the perpendicular from the edge upon the base; the product will be the content.*

28. Required the superficial and the solid contents of a wedge of which the sides of the base or "head" are 36 and 9 inches, the edge 44 inches, and the perpendicular height 22 inches.

$\sqrt{(4\frac{1}{2})^2 + (22)^2} = 22.456$, slant height of the sides. $36 \times 9 =$ the area of the rectangle; $22 \times 9 =$ the two triangles, and $(36 + 44) \times 22.456 =$ the two trapezoids. Hence, $324 + 198 + 1796.48 = 2318.48$ sq. in. in the whole surface.

(3 ft. + 3 ft. + 3 ft. 8 in.) $\times$ 9 in. $\times 22 \times \frac{1}{6} =$ 2 c. ft. 2 in. 7″, solid content.

29. How many solid feet in a wedge, of which the height is 25 inches, the edge 28 inches, and the sides of the base 34 and 10 inches?

176. TO FIND THE SURFACE OF A SPHERE, OR OF ANY SEGMENT OR ZONE OF IT.

RULE. *Multiply the circumference of the sphere by the axis, or by the part of it corresponding to the zone or segment required; the product will be the surface.*

30. What is the surface of a globe whose axis is 15 feet?

31. If the diameter of the earth is 8000 miles, how many square miles of surface does it contain?

II. 32. The part of the earth's axis corresponding to each of the frigid zones is 327.19 miles, and to each temperate zone 2053.467 miles, and to the torrid zone 3150.677 miles. What is the surface of each zone the earth's diameter being 7912 miles.

177. TO FIND THE SOLID CONTENT OF A SPHERE.

RULE. *Multiply the cube of the axis by .5236.*

33. What is the solidity of a sphere, of which the diameter is 16 inches?

34. What is the solidity of the moon, supposing her to be a perfect sphere, the axis being 2180 miles?

178. TO FIND THE SIDE OF A CUBE, EQUAL IN CONTENT TO ANY GIVEN SOLID.

RULE. *The cube root of the cubical content given is the side of a cube of equal solidity.*

35. The diameter of a globe is 3 feet; what is the side of a cube of equal solidity?

36. A chest is 4 ft. 7 in. long, 2 ft. 3 in. broad, and 1 ft. 9 in. deep. Required the side of a cube of equal solidity.

37. What must be the side of a cubical cistern that shall hold 5000 gallons of water? (59.)

38. A cylindrical vessel is 15 inches in diameter, and 18 inches deep; what would be the side of a cubical vessel of equal capacity?

39. The side of a cubical vessel is 12; what must be the side of a cubical vessel that will contain 3 times as much?

40. The side of a cubical vessel is 24; required the side of another vessel that will contain only $\frac{3}{4}$ as much as the first.

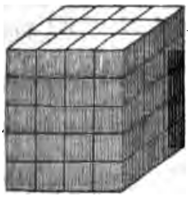
41. What must be the side of a cubical bin which will contain 1000 bushels of corn? (58.)

179. SIMILAR SOLIDS.

42. There is a cubical box, each of whose inside dimensions is 1 inch. What is the content of the cube? What is the content of the largest globe that can be included within the box? *Ans.*, .5236 of a cubic inch.

43. Suppose each side of the box to be 2 inches; what would be its capacity? How many times as large as the other? How times as large would be the globe that it would include?

44. Suppose each side of the box to be 3 inches. Ask and answer the same questions.



45. What is the content of a block of marble, if its length, width, and thickness are 5, 4, and 3 feet respectively? What, if 10, 8, and 6 feet? What, if 15, 12, and 9 feet? What, if $2\frac{1}{2}$, 2, and $1\frac{1}{2}$ feet?

NOTE 1. *Similar solids are those whose corresponding sides or dimensions are proportional.* Thus the different solids mentioned in example 45 are similar.

NOTE 2. *The contents of similar solids are in proportion to the cubes of their corresponding dimensions;* that is, *The content of one solid is to the content of another similar solid, as the cube of any dimension of the former is to the cube of the like dimension of the latter.*

46. Can you illustrate this truth by the above and similar examples?

47. Can you show from the above examples why the solidity of a sphere is obtained by multiplying the cube of the diameter by .5236?

NOTE 3. *If a plane pass through a cone or pyramid parallel to its base, it divides the lines it meets proportionally; the small cone or pyramid cut off by it is, therefore, similar to the whole cone or pyramid.*

48. If a ball 20 inches in diameter weighs $555\frac{5}{9}$ pounds, what is the diameter of one of the same metal that weighs 15 pounds?

49. If a vessel, one of whose sides is 2 feet, will contain 37.63 gallons, what will another similar vessel hold, whose corresponding side is 15 feet?

50. If a tree whose diameter is 2 feet at the base contains 3 cords of wood, how much wood will there be in a tree of the same shape, the diameter of which is 3 feet?

51. If an ox whose girt is 7 feet weighs 1000 pounds, how much will an ox of the same form weigh, whose girt is 6 ft. 6 in.? What should be the girt of an ox of the same form, which weighs 1500 pounds?

52. A square pyramid of wood, 12 feet long, and each side of whose base is 18 inches, is to be balanced upon a pivot passing through it. How far from the base must the pivot be placed to balance it?

53. A cone is 15 feet high; how many feet of its top must be taken off to remove one half of it? one third of it? two thirds?

54. What part of the cone will a plane 5 feet from the base cut off?

55. If a stack of hay 12 feet high weighs 4 tons, how much will a similar stack weigh, that is 15 feet high?

56. If a common brick 8 inches long weighs 4 pounds, how much will a brick of similar shape weigh, that is 12 inches long?

57. If a man 6 feet high weighs 200 pounds, what will a giant of similar form and of equal solidity weigh, that is 8½ feet high?

58. A cubic foot of lead weighs 11352 ounces; how much will a leaden ball 2 inches in diameter weigh? 4 inches? 6 inches? 8 inches? 12 inches?

59. How large a ball of lead will weigh 709¼ ounces?

NOTE. The ball will contain $\frac{709\frac{1}{4}}{11352} = \frac{1}{16}$ of a cubic foot. $\sqrt[3]{\frac{1}{16} \div .5236}$ = answer. Or, $.5236 : \frac{1}{16} :: 1^3 :$ the cube of the answer; that is, by Note 2, the solid content of a ball 1 foot in diameter, is to the solid content of the required ball, as the cube of the diameter of the first ball, is to the cube of the diameter of the second.

180. MENSURATION OF BOARDS AND TIMBER.

The unit of measure for boards, plank, joists, beams, &c., is the square foot; they are usually surveyed by board measure, the board being estimated at one inch thick. Thus, a board

10 ft. long, and 1½ feet wide, contains 15 square feet, if it is 1 inch thick; if it is 1½ inch thick, it contains 22½ square feet; if 2 inches thick, 30 square feet. Round timber is sometimes measured by the ton, and sometimes by board measure.

RULE FOR MEASURING BOARDS, PLANK, JOIST, BEAMS, &C.

Multiply the length in feet by the width in inches, and this product by the depth, or thickness, in inches, and divide the last product by 12; *the quotient will be the number of square feet.*

1. How many square feet in a board 23 feet long, 17 inches wide, and 1 inch thick? ⅞ in. thick? ¾ in. thick? 1¼ in. thick?

2. How many square feet in a joist 30 feet long, 6 in. wide, and 3 in. thick? 5½ in. wide, and 2½ in. thick?

To find the solid contents of any rectangular stick of timber that does not taper, see Art. **170**; if it does taper, see Art. **174**.

181. To find the side of the largest square stick of timber that can be hewn or sawn from a round log, whose diameter is given.

RULE. *Multiply the diameter of the smaller end by* .7071. (Art. **165**, quest. 22.)

SECTION XX.—PROBLEMS IN MENSURATION.

From the principles and illustrations detailed in the two preceding sections, we deduce the following useful and practical problems.

182. To find the solid contents of the walls of a rectangular cistern or building, of any given dimensions.

RULE. *From the* OUTSIDE *perimeter of the walls*, (p. 214, def. 32,) *subtract four times the thickness of the walls; the remainder will be the length of the walls. Then multiply this length by the height, and this product by the thickness.*

183. To find the content of the gable ends.

RULE. *Multiply the breadth of the house by the perpendicular height of the ridge above the eaves; the product will be the area of both gable ends.* For the solid contents, *Multiply this area by the thickness.*

20*

184. To find the solid contents of the walls of a *cylindrical* shaped structure; as round cisterns; wells, &c.

RULE. *To the inner diameter of the cylinder, add the thickness of the wall; the sum will be the mean of the inside and outside diameters. Multiply this sum by* $3\frac{1}{7}$, *or by* 3.1416; *the product will be the mean circumference.* For the solid content, *Multiply this mean circumference by the height, and this product by the thickness.*

185. To find the solid contents of the bottom, or foundation-work, of a cylindrical cistern.

RULE *Multiply the diameter from outside to outside by* $3\frac{1}{7}$ *for the circumference: then multiply half the diameter by half the circumference, and this product by the thickness, for the cubical content.*

NOTE. To find the capacity of a cistern, the top and bottom of which are not equal, see Art. **174.**

186. To find the number of bricks it will take to build any wall, or other work, the solid contents of which are known.

A common brick is 8 inches long, 4 inches wide, and 2 inches thick; its solid content is therefore 64 cubic inches, or $\frac{1}{27}$ of a cubic foot. Hence the

RULE. *Multiply the number of cubic feet by* 27; *the product will be the answer.*

187. To find how many gallons a rectangular cistern will contain.

RULE. *From the inside dimensions find the cubical contents in feet,* (**170**,) *and multiply the content thus found by* $7\frac{1}{2}$. *This will be the number nearly.* Or, *Find the content in cubic inches, and divide by* 231.

188. To find how many gallons a cylindrical cistern will contain.

RULE. *Multiply the inside diameter by itself, and this product by the height, the dimensions being taken in feet; then multiply the last product by* $5\frac{7}{8}$.

189. To find the number of quarts a cylindrical vessel will hold.

RULE. *Take the dimensions in inches; multiply the square of the diameter by twice the height, and divide the product by* 147.

190. To find the number of bushels a rectangular bin will hold.

RULE. *Take the dimensions in feet; multiply the length, width, and height together; then multiply this product by* 45, *and divide by* 56.
Or, *Take the dimensions in inches; multiply the length, width, and height together; multiply this product by* 5, *and divide by* 10752.

191. To find the inside dimensions of any box, cistern, &c., of a given capacity, the dimensions of which are to have a given proportion to each other.

RULE. *Divide the capacity of the required box, &c., by the capacity of one whose dimensions are expressed by the numbers of the given proportion; and multiply each of these numbers by the cube root of the quotient; the several products will be the dimensions required.*

1. Required the dimensions of a rectangular cistern which shall hold 3000 gallons, and whose length, width, and depth shall be as the numbers 5, 3, and 4.

SOLUTION. — $5 \times 3 \times 4 \times 7\frac{1}{2} = 450$ gallons, the capacity of a cistern whose dimensions are 5, 3 and 4 feet. $\sqrt[3]{3000 \div 450} = \sqrt[3]{6.\dot{6}} = 1.882$. Then $1.882 \times 5 = 9.41$ ft. = length; $1.882 \times 3 = 5.646$ ft. = the width; and $1.882 \times 4 = 7.528$ the height.

2. Required the inside dimensions of a round cistern to contain 10000 gallons, the diameter of which shall be to the height as 2 to 3.

SOLUTION. — $2 \times 2 \times 3 \times 5\frac{7}{8} = 70\frac{1}{2}$ gallons, the capacity of a cistern whose dimensions are 2 and 3 feet. $\sqrt[3]{10000 \div 70\frac{1}{2}} = \sqrt[3]{141.844} = 5.215$. $5.215 \times 2 = 10.43$ ft. = the diameter; $5.215 \times 3 = 15.645$ feet = the height.

NOTE. Of *rectangular* forms, the *cube* gives the greatest capacity from a given amount of materials, *if the top is to be covered;* if not, a *square* base is the best, with an indefinite height.

In *cylindrical* forms, the greatest capacity from a given amount of materials is obtained by making the diameter and height equal, *if the top is to be covered;* if not, a given amount of materials will enclose the greatest space when the diameter is one half the height.

192. To find the dimensions of any surface which shall enclose a given area, the dimensions of which are to have a given proportion to each other.

RULE. *Divide the area of the required surface by the area of a surface whose dimensions are expressed by the numbers of the given proportion, and multiply each of these numbers by the square root of the quotient; the several products will be the dimensions required.*

1. Required the dimensions of a rectangular field to contain 40 acres, and which shall be 4 times as long as it is wide.

$1 \times 4 = 4$ sq. rods $=$ area of a surface whose dimensions are 1 and 4 rods. $\sqrt{6400 \div 4} = \sqrt{1600} = 40$. $1 \times 40 = 40$ rods $=$ the width; $4 \times 40 = 160$ rods $=$ the length.

193. EXERCISES IN THE FOREGOING PROBLEMS. (**182** to **192.**)

1. How many bricks will it take to build the walls of a house 35 feet long, 25 feet wide, and 20 feet high, the perpendicular height of the ridge above the eaves being 10 feet, the walls 1 foot thick, and the gable ends 8 inches thick?

Solution. $120 - 4 = 116$, the length of the walls. $116 \times 20 \times 1 = 2320$ cu. ft. in the walls. $25 \times 10 \times \frac{2}{3} = 166\frac{2}{3}$ cu. ft. in the gable ends. $2320 + 166\frac{2}{3} = 2486\frac{2}{3}$ cu. ft., the solid content of the walls, including the gable ends. $2486\frac{2}{3} \times 27 = 67{,}140$, the number of bricks required.

2. How many bricks will it take to build a rectangular cistern, whose *inside* dimensions are to be 8 feet long, 5 feet wide, and 7 feet deep, the foundation and walls being 8 inches thick, and the top being left uncovered?

3. How many solid feet of masonry are there in a cylindrical cistern, the inside diameter of which is 5 feet, its depth 8 feet, and the walls 10 inches, and the foundation 8 inches thick, the top being left uncovered?

4. How many solid feet are there in the walls of a cellar, the inside dimensions of which are 38 feet long, 23 feet wide, and 5 feet high, the walls being 15 inches thick?

5. How many bricks are required to build a cylindrical cistern to contain 20,000 gallons, the diameter of which is to be two thirds of the depth, the walls and foundation being 8 inches, and the top 6 inches thick?

6. What must be the inside dimensions of a bin to hold

1060 bushels of wheat, the length, width, and height being as the numbers 4, 3, and 2?

7. How many feet of boards will it take to make the walls of the above bin, the boards being $1\frac{1}{2}$ inches thick, allowing nothing for waste; and what will they cost, at $18 per thousand feet, board measure? (**180.**)

QUESTIONS. What is a solid? a prism? a cube? a triangular prism? a square prism? a parallelopipedon? a pentagonal prism? a cylinder? a pyramid? the vertex of a pyramid? a cone? a segment of a solid? a frustum? the axis of a solid? a sphere or globe? a hemisphere? the axis of a sphere? the height of a solid? the slant height of a pyramid? of a cone? a spheroid? a prolate spheroid? an oblate spheroid?

What is the rule for finding the area of the surface of a cube? for finding the solid content of a cube? for finding the surface of a prism, parallelopipedon, or cylinder? for finding the solid contents of a prism or cylinder? the rule for finding the surface of a pyramid or cone? the solid content of a pyramid or cone? the rule for finding the surface of a frustum of a pyramid or cone? to find the solid content of the frustum of a pyramid or cone? to find the surface of a wedge? to find the solid content of a wedge?

What is the rule to find the surface of a sphere, or of any segment or zone of it? to find the solidity of a sphere? to find the side of a cube, the content being given? What are similar solids? To what are the contents of similar solids proportional? How does a plane parallel to the base of a pyramid divide the lines it meets?

What is the unit measure for boards, plank, &c.? Give examples. What is the rule for measuring boards, plank, &c.? What is the rule for finding the side of the largest square stick that can be hewn from a round log whose diameter is given? Draw a circle and inscribe a square within it, and see if you can demonstrate the correctness of this rule. (**165**, quest. 22.)

What is the rule to find the solid contents of the walls of a rectangular building? to find the content of the gable ends? to find the solid contents of the walls of a cylindrical shaped structure? to find the solid contents of the foundation of a cylindrical cistern? to find the capacity of a cistern, the top and bottom of which are not equal? to find what number of bricks it will take to build any work, the solid contents of which are known? to find how many gallons a *rectangular* cistern will contain? a *cylindrical* cistern? to find the number of quarts a cylindrical vessel will hold? to find the number of bushels a rectangular bin will hold? to find the inside dimensions of any box, cistern, &c., the dimensions of which are to have a given proportion to each other? Of rectangular forms, what gives the greatest capacity from a given amount of materials? What is said of cylindrical forms? What is the rule to find the dimensions of any surface which shall enclose a given area?

SECTION XXI.—THE MECHANIC POWERS.

194. *Machines* are certain contrivances employed for the purpose of changing the direction of moving powers, of enabling them to produce any required velocity, or to overcome any required force.

All machines, however complicated, are formed by combining a few simpler machines, commonly called the "Mechanic Powers." They are, the Lever, the Wheel and Axle, the Pulley, the Inclined Plane, the Wedge, and the Screw.

In any machine, the force or original prime mover is called the POWER.

The resistance to be overcome by the power, through the intervention of the machine, is called the WEIGHT.

The great law of equilibrium in the Mechanic Powers, and which applies to all machines, whether simple or complex, is the following:

"*The power multiplied by the space through which it moves in a vertical direction, is equal to the weight multiplied by the space through which it moves in a vertical direction.*" Or,

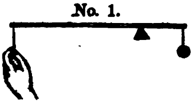
The power is to the weight as the distance through which the weight moves is to the distance through which the power moves. Hence, (Art. **97**, or **127**,) any three of these terms being given, the fourth may easily be found.

In the practical application of the Mechanic Powers, a certain allowance must be made for friction. In some, this is much greater than in others. No account will be taken of friction in the following exercises, unless it is particularly mentioned.

195. THE LEVER.

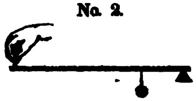
The LEVER is a solid, unyielding rod or bar, working upon a fixed point, called its *fulcrum* or *prop*. In theory, it is an inflexible and imponderable line, supported upon one point, upon which it can turn. It is of three kinds.

No. 1.



In the first kind, the fulcrum is between the power and the weight, as in No. 1.

No. 2.



In the second, the weight is between the fulcrum and the power, as in No. 2.

No. 3.

In the third, the power is between the weight and the fulcrum, as in No. 3.

There are other species of lever, such as the bent lever, the curvilinear lever, &c. The mode of action and the theory of all are the same.

The crowbar, pincers, tongs, nut-crackers, chaff-cutters, hand-barrows, a door turning on its hinges, steelyards and other weighing machines, are examples of levers. The common claw-hammer, when used for drawing nails, is an example of the bent lever.

In the use of the lever, the relative distances through which the power and weight move are in exact proportion to their distance from the fulcrum. Hence, by the law above stated,

The power and weight will be at rest when the product of the power multiplied by its distance from the fulcrum is equal to the product of the weight multiplied by its distance from the fulcrum.

NOTE 1. The pupil, before attempting to solve any question in Mechanic Powers, should draw upon his slate a figure in which the relative position, &c., of the weight and power should be indicated. He will, by this means, get a clearer idea of the nature of the question, and at the same time be cultivating a talent which is too apt to be neglected.

1. If a lever 50 inches long have its fulcrum 4 inches from the weight, what power will be needed to balance a weight of 644 pounds?

SOLUTION. — $644 \times 4 = 2576$, is the product of the weight multiplied by its distance from the fulcrum; it is, therefore, the product of the *power* multiplied by *its* distance from the fulcrum; and, therefore, (**97**,) $\frac{644 \times 4}{46} = 56$ pounds, the answer.

2. If a man weighing 150 pounds rest upon one end of a lever 12 feet long, what weight will he balance at the other end, the fulcrum being $1\frac{1}{2}$ feet from the weight?

$$\frac{150 \times 10\frac{1}{2}}{1\frac{1}{2}} = 1050 \text{ lb.}$$

3. A lever 15 feet long rests on a fulcrum $1\frac{1}{4}$ feet from the end; how large must the power be to balance a weight of 2000 pounds? How large a weight will 2 men balance, one weighing 150 lb., and the other 175 lb., by resting upon the end of the longer arm?

4. If the weight be 2500 lb., and the power 150 lb., where must the fulcrum be placed, under a lever 16 feet long, so as to have the power and weight balance each other?

NOTE 2. The fulcrum must be placed as much nearer the weight than the power, as the weight is heavier than the power. The lever

has to sustain $2500 + 150 = 2650$ lb. The weight must, therefore, be $\frac{150}{2650}$ of 16 ft. from the fulcrum, and the power $\frac{2500}{2650}$ of 16 ft. from the fulcrum.

5. A weight of 30 lb. and a power of $3\frac{1}{4}$ lb. are to be so adjusted to a lever $3\frac{1}{2}$ feet long as to balance each other. Where shall the prop be placed?

6. Three men are to carry a stick of timber, 15 feet long, and of uniform size from end to end, one by lifting at one end of the stick, and the other two by a bar placed under the stick. Where should the bar be placed, that they may each carry an equal part of the weight?

NOTE 3. The bar that is to sustain two thirds of the weight must be placed twice as near centre of gravity, or the middle of the stick, as that which is to sustain only one third; and the same for any other proportion.

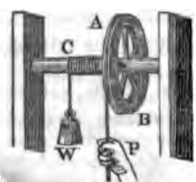
7. In a lever of the second kind, 10 feet long, what power will be necessary to balance a weight of 500 lb., the weight being suspended 2 feet from the fulcrum? 8 feet from the fulcrum? 5 ft.? $3\frac{1}{2}$ ft.?

8. In a lever of the second kind, 5 feet long, how large a weight $1\frac{1}{2}$ feet from the fulcrum will a power of 18 pounds balance? Where shall a weight of 100 lb. be placed, to balance a power of 18 lb.?

9. In a lever of the third kind, suppose the weight to be 100 lb., 10 feet from the fulcrum, what power 5 feet from the fulcrum will balance it? Where must a power of 300 lb. be placed to balance it?

196. THE WHEEL AND AXLE.

The WHEEL AND AXLE is a modification of the lever; it is, indeed, a continuously acting lever.



It consists of a cylinder, C, revolving upon an axis, and having a wheel, A B, of larger diameter, immovably affixed to it. The power is applied to the circumference of the wheel, and the weight to that of the axle.

The radius of the wheel is to be regarded as the longer arm of a lever, and that of the axle as the shorter arm. *Equilibrium, therefore, takes place when the product of the power, multiplied by the radius of the wheel, equals the product of the weight multiplied by the radius of the axle.*

NOTE. The *diameter* or the *circumference* may be substituted for the radius in the above formula.

1. The wheel for raising goods in a grain store is 4½ feet in diameter; the axle 6 inches in diameter. What power applied to the rope passing over the wheel, will it take to balance a bag of wheat weighing 180 lb., attached to a rope passing over the axle?

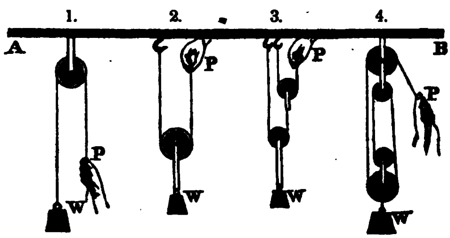
2. If the radius of a wheel is 6 feet, and that of the axle 5½ inches, what weight upon the axle would be balanced by a power of 145 lb. upon the wheel?

3. The radius of a wheel being 4½ feet, what must be the radius of the axle, in order that a power of 25 lb. may balance a weight of 375 lb.?

4. The arm of a windlass used in raising water from a well is 15 inches long, measuring from the centre of motion; the axle on which the rope is wound is 6 inches in *diameter*. What power at the crank will it take to balance a bucket of water weighing 50 lb.?

197. THE PULLEY.

The PULLEY is a wheel round the rim of which a groove is cut in which a cord can work, and the centre of which moves on a pivot in a block. The wheel is sometimes called a *sheave*.



Pulleys are either *movable* or *fixed*.

By a fixed pulley we mean one that only revolves on its axis, without changing its place. It gives no mechanical advantage, its use being only to change the direction of the weight. Thus, in No. 1, the power and weight pass over equal spaces in equal times; their force must, therefore, be equal.* In No. 2, the pulley is movable, and the

* The pulley, in No. 1, may be regarded as a lever of the first kind, with equal arms, the pivot being the fulcrum, and the radius of the wheel the length of each arm.

power moves through twice the distance that the weight does. The advantage gained is, therefore, as 2 to 1.* In Nos. 3 and 4, the power moves through 4 times the distance of the weight; the advantage gained is, therefore, as 4 to 1.

When several movable pulleys act as in No. 3, *the power and weight balance each other when the power is to the weight as* 1 *is to that power of* 2 *which equals the number of movable pulleys.*

Thus, if, as in figure 3, there are two movable pulleys, the power is to the weight as 1 to 2^2; that is, as 1 to 4. If *three* movable pulleys were arranged in this manner, the proportion would be as 1 to 2^3, or as 1 to 8.

When several movable and fixed pulleys are employed as in No. 4, *equilibrium is produced when the power equals the weight divided by twice the number of movable pulleys; or, when the weight equals the power multiplied by twice the number of movable pulleys.*

1. If a power of 160 pounds be applied to a rope connecting a system of 4 movable pulleys, arranged as in No. 3, what weight will the power balance? $1 . 2^4 = 160 : 2560$ pounds. What power would be required to balance a weight of 640 lb.?

2. What weight will be balanced by a power of 15 pounds attached to a cord that passes over 3 movable pulleys arranged as in No. 4? What power will it take to balance a weight of 1500 pounds?

198. THE INCLINED PLANE.

AN INCLINED PLANE is an unyielding plane surface, inclined at an acute angle to the horizon. When a body is placed on such a plane, a part of the weight is supported by the plane, and the remainder, which urges the weight down the plane, is the part to be supported by the power.

When the power acts in the direction of the *plane*, *equilibrium ensues when the power multiplied by the length of the* PLANE *is equal to the weight multiplied by the height of the plane; that is, when the power is to the weight, as the perpendicular of the plane is to its length.*

If the power acts in the direction of the *base*, *equilibrium is obtained when the product of the power multiplied by the length of the* BASE *equals the product of the weight multiplied by the height of the plane; or, when the power is to the weight, as the perpendicular of the plane is to the length of the base.*

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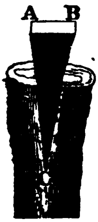
* The pulley in No. 2 is a lever of the 2d kind upon a movable fulcrum, in which the diameter of the wheel is the longer arm, and the radius of the wheel the shorter arm.

1. An inclined plane is 10 feet long, and its perpendicular height 2 feet. What weight will a power of 10 pounds balance, if it act parallel to the plane? What weight will it balance, if it act parallel to the base? (**163.**)

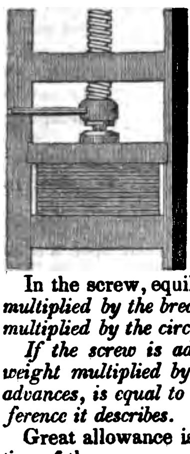
2. A hill rises 8 feet in 100. How much more power must a horse exert to draw a load of 1000 pounds up the hill than on level ground, the friction in the two cases being equal?

199. The Wedge.



The Wedge may be regarded as two inclined planes, laid base to base. It is not usually employed by the agency of continued pressure, but by that of percussion, as the blows of a hammer or mallet upon its back. Its power increases as the thickness of its back, compared with the length of its sides, is diminished. When acted on by continued pressure, the condition of equilibrium is, that the product of the power multiplied by the length of the wedge be equal to the product of the weight multiplied by half the thickness of the head.

200. The Screw.



The Screw is a cylinder with a spiral line cut round it. This spiral line is called the *worm* or *thread* of the screw. The distance from one thread to another is called *the breadth* of the worm.

In most cases, the screw requires a corresponding cavity, called a *nut*, in which it may work. Sometimes the screw moves in the nut, and sometimes the nut moves on the screw. The power is applied to the end of a lever attached to the movable part; the weight or resistance is applied to the end of the screw.

In the screw, equilibrium is obtained, *when the product of the weight multiplied by the breadth of the worm, equals the product of the power multiplied by the circumference of the circle described by it.* Or,

If the screw is advancing through a nut, when the product of the weight multiplied by the distance through which the head of the screw advances, is equal to the product of the power multiplied by the circumference it describes.

Great allowance is to be made for friction in the practical application of the screw.

1. If the lever of a screw is 8 feet long, and the distance between the threads $1\frac{1}{4}$ inches, what power will balance a weight of 4000 pounds, making no allowance for friction?

What weight would be balanced by a power of 15 pounds applied to the extremity of the lever?

2. How broad must the worm of a screw be, so that, with a lever 6 feet long, 10 pounds will balance 8000 pounds? How long must the lever be, if the worm is ½ inch broad?

3. If the breadth of the worm of a screw is one inch, what weight would be balanced by a power of 100 pounds acting at the extremity of a lever 6 feet long?

4. How large a weight will the power in the last question balance, if the weight rests upon an inclined plane 15 feet long and 4 feet high?

5. If in the last question the power acts upon the lever by means of a system of pulleys, with 3 movable pulleys arranged as in the diagram No. 3, what weight will be balanced by it? What weight, if the pulleys are arranged as in No. 4?

QUESTIONS. What is a MACHINE? How are machines formed? What are the MECHANIC POWERS? What is the power? the weight? What is the great law of equilibrium in the Mechanic Powers? Show from Art. **97** and **147** that any three of these terms being given, the fourth may be found. What is said of friction?

What is the LEVER? How many kinds? What is the first kind? the second? the third? What other species of lever are there? Give examples of the lever. In what proportion are the relative distances through which the power and weight move? When will the power and weight be at rest? What should the pupil do before attempting to solve a question in the Mechanic Powers?

What is the WHEEL AND AXLE? Of what does it consist? How are the radius of the wheel and axle to be regarded? What is the law of equilibrium in the wheel and axle?

What is the PULLEY? How many kinds? What is a fixed pulley? What is its use? Show its relation to the lever. Draw upon a slate or blackboard figures of the different kinds of pulley, and explain them. What is the advantage gained in the use of the pulley when arranged as in No. 1? in No. 2? in No. 3? in No. 4? State the law of equilibrium in each.

What is an INCLINED PLANE? How is a body placed on such a plane supported? What is the law of equilibrium when the power acts in the direction of the *plane?* What is it when the power acts in the direction of the *base?*

What is the WEDGE? How is it usually employed? How does its power increase? What is the law of equilibrium in the wedge when it is acted on by continued pressure?

What is the SCREW? What is the worm, or thread? the breadth of the worm? the nut? Where are the power and the weight applied? When is equilibrium obtained in the use of the screw?

SECTION XXII. — MISCELLANEOUS EXERCISES,

MENTAL AND ORAL.

201. 1. Add 305 + 418; 876 + 245; 354 + 768; 5010 + 3108.

2. Add 15.55 + 184.83; 20.04 + 35.28; 1085.45 + 357.85; 857.108 + 318.516.

3. How much is 387 — 218? 416 — 84? 685 — 95? 5101 — 764?

4. How much is 27.3 — 18.3? 254.23 — 81.07? 1001.86 — 318.59?

5. What is the complement of 45? 87? 29? 38? 56? 73? 78?

6. What is the complement of 146? 387? 594? 871? 176? 549?

7. How much are 8¼ times 37? 9¾ times 29? 8⅔ times 177?

8. How much are 15 times 375? Say ten times 375 = 3750; five times 375 will be one half as much = 1875, which added, &c.

9. Multiply 354 by 12½; by 15; by 20; by 25; by 33⅓; by 50; by 75; by 125.

10. Divide 4025 by 12½; by 20; by 400; by 25; by 33⅓; by 50; by 125.

11. What is 5 per cent. of $75? 5½ per cent. of $340? 6 per cent. of $875?

12. What is the interest of $150 for 2 years and 6 months, at 6 per cent.? at 7 per cent.?

13. What is the interest of $275 for 1 year 8 mo. 24 da., at 6 per cent.? at 6½ per cent.?

14. Compute the interest on $850 for 3 mo. 18 da., at 1 per cent. per month.

15. What is the time from Dec. 15, 1840, to Apr. 20, 1841? to Apr. 10, 1841? to May 1, 1842? to July 11, 1841? (**69.**)

16. What is the time from May 23, 1849, to Sept. 18, 1849? to Oct. 15? to Nov. 27? to Jan. 1, 1850? to April 21, 1852?

17. How many square feet in the four walls of a room 18 feet long, 15 feet wide, and 9½ feet high? How many square yards?

18. How many square feet of hewn stone in the top, one side and two ends of a door-step, the length of which is 5½ feet, the width 2½ feet, and the depth 8 inches? What will the hammering of the stone come to, at 50 cts. per sq. ft.? at 25 cts.? at 33⅓ cts.?

19. A block of marble 3 feet wide and 2½ feet deep contains 75 cubic feet. How long is it? If it were 3 feet wide and 12½ feet long, how deep must it be to contain 75 cubic feet? How deep to contain 112½ cubic feet?

20. What will the digging of a cellar cost, at 75 cents per cubic yard, the dimensions being 36 feet long, 20 feet wide, and 7½ feet deep?

21. In a right-angled triangle, the base is 12 feet, and the perpendicular is 16 feet. How long is the hypothenuse?

22. A rectangular lot of land, containing 7¼ acres, is 40 rods long.

How wide is it? How long is a diagonal line from one corner to the opposite? What will it cost to build a fence around the lot, at $\$1\frac{1}{4}$ per rod?

23. A rectangular garden is 100 feet long and 81 feet wide. What must be the dimensions of a similarly shaped garden, to contain 4 times as much surface? 9 times as much? 25 times as much? $\frac{1}{16}$ as much?

24. What must be the dimensions of a lot of the same shape as the last, to contain 2025 square feet? 32400 square feet?

25. A brick of common size is 8 inches long, 4 inches wide, and 2 inches thick. If it take 10000 such bricks to build a wall, how many blocks of stone of the same form, each side of which is twice as long, will be required to build the same wall? How many, if the dimensions are 5 times as great?

26. A pyramid of stone is $2\frac{1}{2}$ feet square at the base, and 6 feet high. What is the solid content? What is the solidity of a pyramid, each of whose dimensions is 3 times as large? What part of the pyramid will be cut off by a plane two feet from the base, and parallel to it?

27. A can perform a piece of work in half a day, B can do it in $\frac{1}{3}$ of a day, and C in $\frac{1}{5}$ of a day. What part of a day will it take them all to do it?

28. A, B, and C can build a wall in 3 days, B can do it alone in 8 days, and C in 10 days. In what time can A build it?

29. A man having spent $\frac{1}{3}$ and $\frac{1}{4}$ of his money, and $\frac{1}{3}$ of the remainder, has $100 left. How much had he at first?

30. Three men trade in company. A puts in $200 for 4 months, B $300 for 6 months, and C $500 for $2\frac{4}{5}$ months. Their gain is $362. What is each partner's share of the gain?

31. Three men trade in company. A puts in $500 for 9 months, B $700 for 5 months, and C 500 bushels of corn for 10 months. They gain $450, of which C's share is $150. What was C's corn valued at per bushel?

32. A's age is to B's as 5 to 6; and their ages united amount to 55 years. How old is each?

33. John's age is to William's as 3 to 4; and three times John's age added to four times William's is just 100 years. What is the age of each?

34. A lever is 15 feet long. Where shall the fulcrum be placed so that a weight of 10 pounds on one end may balance 15 pounds on the other?

35. Three men are to carry a stick of timber, of uniform size, 25 feet long. Where shall the bar be placed by which two of them shall carry two thirds of it, the other lifting at the end?

36. An inclined plane is 12 feet long, and its perpendicular elevation 4 feet. If a hogshead of molasses weighing 1200 pounds be placed upon the plane, and a rope, one end of which is fastened at the top of the plane, be passed round the hogshead, what power at the other end of the rope will be required to balance it?

FOR THE SLATE.

202. 1. Write in figures: Six hundred millions, fifty thousand and thirty-four. Three hundred and fifteen thousand and three. Ninety billions, ninety thousand and ninety. Three hundred and seven billions, six millions and fifty. Seventy-five trillions, four billions and four hundred. Eight hundred trillions, seventy-five millions and one.

2. Add the above numbers together.

3. Write in figures: 23 mil., 875 thous. and 4; 307 bil., 8 mil., 21 thous. and 70; 5 mil., 3 thous. and 74; 97 bil., 8 thous. and 1; 16 tril., 105 mil. and 400; 875 tril., 15 bil. and 25 thous.

4. Add the numbers in No. 3 together.

5. Write in figures: Forty-five *hundredths;* 30,—and 6 *hundredths;* 1000,—and 73 *thousandths;* 7816,—and 5101 *ten thousandths;* 418,—and 5075 *millionths;* 10,—and 15 *hundred thousandths.*

6. Write in figures: Three hundred and fifty-four *ten thousandths.* Eighty-four thousand and fifteen *millionths.* Twenty-five,—and three hundred and thirty-four *ten thousandths.* Fifty-seven,—and twenty-five *thousandths.* Three hundred thousand,—and eight hundred and nine *hundred thousandths.*

7. Add the numbers in No. 5 and 6 together.

8. Express in figures: 30186,—and 50164 *millionths.* 8001,—and 40008 *ten millionths.* 90006,—and 800004 *billionths.* 890106,—and 50087 *hundred millionths.* 51008009,—and 80701080 *hundred billionths.*

9. Find the sum of the numbers in No. 8.

10. Find the sum, the difference, and the product of these two numbers; viz., three thousand and ten,—and ninety-seven *thousandths;* and forty-four thousand and twenty,—and five thousand and seven *hundred thousandths.*

11. Find the sum, the difference, and the product of $15\frac{3}{16}$ and $207\frac{5}{48}$.

12. Divide three hundred thousand and twenty-four by one hundred and six ten thousandths.

13. Divide $87\frac{5}{8}$ by $65\frac{7}{48}$; $805\frac{3}{8}$ by .3 of $\frac{5}{8}$ of $45\frac{1}{8}$.

14. Reduce 34180476 inches to miles; 4418679 square feet to acres; 156485160 cubic inches to cubic yards.

15. What cost 450 yards of broadcloth, at £1 2s. $4\frac{1}{2}$d. per yard?

NOTE. $450 = 5 \times 9 \times 10$. (28.)

16. What cost 74 cwt. of cutlery, at £3 4s. $8\frac{1}{4}$d. per cwt.?

NOTE. $74 = 9 \times 8 + 2$.

17. Multiply 3 A. 3 R. 27 sq. rd. 18 sq. yd. 7 sq. ft. 58 sq. in. by 3; by 12; 18; 48.

18. Divide 5 A. 2 R. 15 sq. rd. 19 sq. yd. 1 sq. ft. 100 sq. in. by 5; by 10; 15; 25.

19. Find the least common multiple of 24, 32, 36, 48 and 60; of 25, 35, 45 and 75.

20. What is the greatest common measure of 144, 256, 872, and 996?

21. Divide the product of 25, 16, 12, 32, 15 and 48 by the product of 5, 40, 8, 24, and 144.

22. Add $3\frac{5}{8}$, $4\frac{8}{9}$, $3\frac{5}{18}$, $15\frac{16}{27}$, and $104\frac{13}{54}$.

23. What must be added to $5\frac{9}{11}$ to make $27\frac{3}{8}$? to $137\frac{24}{25}$ to make $180\frac{18}{55}$?

24. Add £$\frac{3}{7}$, .32s., $\frac{4}{9}$d., and $\frac{1}{4}$ qr. together.

25. Add $\frac{3}{14}$ mile, $\frac{4}{9}$ fur., $\frac{5}{12}$ rd., $\frac{8}{15}$ yd., $\frac{3}{4}$ ft., and $\frac{4}{5}$ inch.

26. Express in lower denominations £$\frac{3}{7}$; £.45; .3s.; $\frac{4}{9}$s.; $\frac{5}{14}$ da.; $\frac{15}{22}$ gal.

27. The sum and difference of two numbers are $154\frac{3}{7}$ and $103\frac{5}{8}$. What are the numbers?

28. A person has paid on a debt of \$10,000, 15 per cent. of it, 17 per cent. of the remainder, and 20 per cent. of the last remainder. How much remains unpaid?

29. What is the interest of \$145.75 from Oct. 7, 1845, to May 2, 1848, at $6\frac{1}{2}$ per cent.?

30. What is the amount of \$751.16 from Jan. 1, 1848, to Apr. 3, 1850, at $5\frac{3}{4}$ per cent.?

31. *Salem, July* 25, 1846.

For value received, I promise to pay Hezekiah Sanborne, or order, five hundred dollars, on demand, with interest at 6 per cent.

\$500 JOSEPH STEVENSON.

Endorsements.

Jan. 1, 1847, received one hundred dollars.
Apr. 15, 1847, received seventy-five dollars.
Nov. 17, 1847, received ten dollars.
Jan. 15, 1848, received one hundred and fifty dollars.

Sept. 1, 1848, received one hundred and eighty-five and $\frac{14}{100}$ dollars.

What was due on the note Jan. 1, 1849, reckoning interest by the "legal rule"? How much by the "common rule"?

How much would be due if semi-annual compound interest be reckoned on the note and on each endorsement to the time of settlement?

32. There is a room 25 feet long, 20 feet wide, and 10 feet high. How many square feet of surface are there in it?

33. How far is it from one lower corner to the opposite upper corner of the above room?

34. What would be the superficial and the solid contents of the largest globe that could be placed in such a room?

35. Sound travels at the rate of 1120 feet per second. How long would it take for sound to travel from Boston to Washington, the distance being 450 miles?

36. John Smith's garden is $145\frac{1}{2}$ feet long and $65\frac{1}{4}$ feet wide. How many feet of surface does it contain? There is a circular pond in the middle, 15 feet in diameter. How much land is there, exclusive of the pond?

37. The wall around the pond is 1 foot thick and 4 feet deep. How many solid feet of masonry does it contain? How many superficial feet are there in the face of the wall?

38. There is a walk laid out around the pond, $3\frac{1}{2}$ feet wide, including the wall of the pond. How much surface is occupied by the walk?

39. How long a line will just reach from one corner of the garden to the centre of the pond?

40. The garden is to be enclosed by a brick wall 6 feet high and 1 foot thick. How many feet of surface will it cover? How many bricks 8 inches long, 4 inches wide, and 2 inches thick, will it take to build it?

41. It is to be surmounted with a sandstone entablature, 6 inches thick, and projecting 2 inches beyond each face of the wall. How many solid feet of sandstone will it take?

42. What would be the cost of excavating the New York and Erie Canal, supposing it to be 360 miles long, 40 feet wide at the surface, 32 feet wide at the bottom, and averaging 6 feet in depth, at $16\frac{2}{3}$ cents per cubic yard?

43. A bookseller purchased 150 gross of steel pens, for two thirds of which he paid $62\frac{1}{2}$ cents per gross, and for the remainder $56\frac{1}{4}$ cents. If he should keep 2 gross for his own use,

and sell the rest at $66\frac{2}{3}$ cents per gross, what per cent. profit would he make by the transaction?

44. A trader, having increased his capital stock by $\frac{1}{5}$ of itself, lost $\frac{1}{4}$ of what he then had; he afterwards gained a sum equal to $\frac{1}{3}$ of the remainder, when he was worth $12,960. What was his capital?

45. A merchant purchased goods to the amount of $2000, which he sold at a loss of $12\frac{1}{2}$ per cent.; he then invested the proceeds in other goods, which he sold at a profit of 20 per cent. What per cent. profit did he make on the two transactions?

46. A can mow an acre of grass in $10\frac{1}{4}$ hours; B, in $8\frac{1}{2}$ hours; and C, in $7\frac{3}{4}$ hours. How long will it take them, all working together, to mow 1 acre?

47. A merchant sold flour at $5.40 per barrel, and by doing so he lost 10 per cent.; whereas, when he purchased the flour, he expected to realize a profit of 15 per cent. At what price per barrel did he expect to sell it?

48. A carpenter is to make a box 5 ft. 8 in. long, 4 ft. 5 in. wide, and $3\frac{1}{2}$ ft. deep, measuring on the outside; it is to be made of plank 2 inches thick. How many feet of plank, *board measure*, will it take to make it, allowing nothing for waste? What will be the cost of the plank, at $25 per thousand feet? How many cubic feet will the box hold?

49. What will the painting of a conical spire cost, the perpendicular height of which is 40 feet, and the diameter at the base 12 feet, at 20 cents per square yard?

50. The earth turns round once, or 360°, in 24 hours. How many degrees does it turn in 1 hour? How long is it in turning 1 degree? What part of a degree does it turn in 1 minute?

NOTE. If the difference in longitude between two places is 15 degrees, the difference in time is 1 hour; that is, if it is 12 o'clock at the place furthest westward, it will be 1 o'clock at the other, &c.

51. When it is 12 o'clock at Salem, what is the time at 45° east of Salem? at 45° west of Salem? at 50° east? at 55° west? at London, which is 70° 54′ east of Salem?

52. When it is 9 o'clock A. M. at Boston, what time is it at London? at New Orleans? at San Francisco? at Constantinople? The longitude of Boston being 71° 4′ W.; of New Orleans, 90° W.; San Francisco, 122° 14′ W., and Constantinople 28° 59′ E. from London.

53. A person, on arriving at Washington, finds his watch,

which was correct when he started from home, 20 minutes too slow. Did he come from the east or the west? How far?

54. A captain at sea finds that when the sun is on the meridian, his chronometer, which keeps London time correctly, points to 15 min. past 3, P. M. What is his longitude from London?

55. The earth's equatorial diameter is 7925.648 miles. What is the circumference of the equator? How many miles are the inhabitants on the equator carried per hour by the earth's revolution?

56. If the diameter of the planet Jupiter is 11.25 times that of the earth, what is the ratio of their surface? of their solid contents?

57. If a globe of metal 8 inches in diameter weighs 100 lb., what will a ball of the same metal 4 inches in diameter weigh? 16 inches in diameter? What must be the diameter of a ball of the same metal, to weigh $337\frac{1}{2}$ lb.?

58. A garden, 50 feet long and 25 feet wide, is to have a ditch, 3 feet wide, dug around it, on the outside. To what depth must the ditch be dug, that the earth thrown up may raise the surface of the garden 9 inches?

59. How many yards of carpeting that is 1.25 yd. wide will it take to cover the floor of a room 15.5 ft. long and 12.8 feet wide? What will the carpeting cost, at $\$1\frac{1}{8}$ per yard? How much will the painting of the walls of the room cost, at 20 cents per square yard, the room being $9\frac{1}{2}$ feet high?

60. How many bricks will it take to build a house 35 feet long and 22 feet wide, the walls to be 20 feet high and 16 inches thick, the perpendicular height of the ridge being 12 feet above the eaves, and the thickness of the gable ends 12 inches. There are 3 doors, 8 feet high and 3 feet 10 inches wide; 17 windows in the two lower stories, 2 feet 10 inches wide and 6 feet 3 inches long, and 2 windows in the attic, 2 feet 6 inches wide and 5 feet 4 inches long; and the bricks are 8 inches long, 4 inches wide, and 2 inches thick.

61. What will an annual saving of $150 amount to in 20 years, reckoning compound interest at 6 per cent.?

62. How much must a young man, who is 21 years of age, save annually, that his savings, when he is 50 years old, may amount to $10,000, his savings being invested at 6 per cent. compound interest?

NOTE. Find how much an annual saving of $1 will amount to, and divide this amount into $10,000. Why?

63. A cylindrical cistern is 10 feet deep. What must be its diameter, that it may contain 5000 gallons of water?

64. A merchant owes as follows: \$500, due Jan. 15, 1849; \$350, due Feb. 3; \$290, due April 13; \$140, due June 17; \$960, due July 21; \$620, due Aug. 9; \$430, due Oct. 18; \$870, due Nov. 1; and \$710, due Dec. 11. What is the equated time for the payment of the whole? What sum would pay the whole Apr. 1, 1849? Jan. 1, 1850, money being worth 10 per cent.?

65. The Winchester bushel is a cylinder 8 inches deep, and $18\frac{1}{2}$ inches in diameter. How many cubic inches does it contain? What must be the side of a cubical box which shall contain the same quantity?

66. A and B can together do a piece of work in 3 days, B and C can together do it in 4 days, and A and C can do it in 5 days. How long would it take them all working together to do it?

NOTE. They can together do $\frac{47}{60}$ of it in 2 days; consequently they can do $\frac{47}{120}$ of it in one day.

67. A and B set out to travel round a certain island which is 60 miles in circumference. A travels 12 miles a day, and B 15 miles a day. How many days must B travel to overtake A? How many miles?

NOTE. A may be regarded as having 60 miles the start of B.

68. If the island were 100 miles in circumference, and A travel 12 miles, B 15 miles, and C 18 miles per day, in how many days will they be together again? How many miles will each have travelled?

69. Divide \$1000 between A, B, and C, giving A \$120 more than B, and C \$50 more than A.

70. Divide \$1000 between A, B, and C, so that B may have \$75 more than A, and \$65 less than C.

71. If the velocity of a cannon ball is 1200 feet per second, what time would it require to move 240,000 miles, the distance of the moon from the earth? What time to move 95 millions of miles, the distance of the sun from the earth, the year being 365.25 days? How long to move 3700 millions of miles, the distance of the planet Neptune from the sun?

For other Miscellaneous Exercises, see Art. 124 and 126.

BOOK-KEEPING.

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203. Book-keeping is the method of recording business transactions. In making such a record, the names of the articles, their prices, and the names of the persons traded with, are to be written down in a regular manner.

Book-keeping is divided into two kinds, called Single Entry and Double Entry. The method by Single Entry is the most simple, and is the method adopted by most persons whose business is small, such as farmers, mechanics, retailers, &c. Two books are necessary for keeping accounts by this method, viz., the Day-book, in which every business transaction is originally entered, and particularly described; and the Ledger, in which are collected in a condensed form all the scattered accounts of the Day-book, so that all the transactions with any one individual are brought together in one account.

In the Day-book each transaction is recorded, by writing, first the name of the person or customer traded with, followed by the term *Dr.*, if he becomes debtor by the transaction, or *Cr.*, if he becomes creditor by it.

In case of law-suit or dispute, the Day-book, or the book in which the original charges are made, is the only one produced in evidence. It is, therefore, important that a plain and accurate description of every transaction in business should be recorded as early as possible, before any of the particulars are forgotten. Such a course will prevent many losses and disputes which, without such care, will be almost certain to occur.

The person who receives property of any kind is debtor, and the person who parts with property is creditor, for the amount.

Thus, if I sell William Perkins 5 bushels of corn, at 60 cents per bushel, he becomes my debtor for the amount of the corn; and if he works for me 10 days, at 1.50 per day, he becomes my creditor for the amount. So, if I pay money to him or for him, he becomes my debtor; and if I receive money of him, he becomes my creditor.

A Cash-book, in which are recorded all payments and receipts of cash, is very useful, and to those whose sales are extensive, or who frequently receive and pay money, it is an almost indispensable auxiliary to the Day-book and Ledger.

The following will illustrate.

204. THE DAY-BOOK.

Charles W. Choate, April 1, 1849. Page 1

Ledger folio.	*Salem, April* 1, 1849.		$	cts
	William Perkins, ... Dr.			
*√1	To 5 bush. corn, @ 60 cts., ...	3 00		
	" 2 " potatoes, @ 75 cts., ...	1 50		
	" 4 lb. butter, @ 20 cts., ...	80	5	30
	3*d*.			
	Henry Stevens, ... Dr.			
√2	To 3 barrels apples, @ $2, ...	6 00		
	" 4 days' work, by myself, boy and team, (2 yoke of oxen and plough,) . . .	22 00	28	00
	5*th*.			
	Henry Stevens, ... Cr.			
√2	By 1 bbl. flour, ...	6 75		
	" sundry other articles from store, as per bill,† .	5 75	12	50
	6*th*.			
	Joseph Thacher, ... Dr.			
√2	To 2450 lb. hay, at 60 cts. per hund., ...		14	70
	9*th*.			
	Frederick Anderson, ... Cr.			
√2	By shingles and lumber, as per bill, ...		30	25
	14*th*.			
	William Perkins, ... *Cr.*			
√1	By 4 days' work shingling barn, himself and apprentice, — at $1.75 per day for himself, and $1 for apprentice, ...		11	00
	15*th*.			
	Frederick Anderson, ... Dr.			
√2	To 3 bush. potatoes, @ 75 cts., ...	2 25		
	" 5 " rye, @ 83 cts., ...	4 15		
	40 lb. salt pork, @ 10 cts., ...	4 00	10	35
	18*th*.			
	William Perkins, ... Dr.			
√1	To cash per receipt, ...		10	00
	"			
	Joseph Thatcher, ... Dr.			
√2	To 2 cords wood, @ $7, ...	14 00		
	" 1 bush. barley, ...	75		
	" 2 " oats, ...	1 00		
	" 2 days' by myself and team, ...	7 50	23	25
	20*th*.			
	Henry Stevens, ... Cr.			
√2	By sundry articles from the store, as per bill, ...		10	50

* When an account is posted, some mark is placed in the margin, to indicate that it has been posted.

† In recording accounts in which the person traded with is Dr., *each item should be stated particularly.* In Cr. accounts this is not always necessary.

Salem, April 23, 1849. p. 2

		$	cts.
	Joseph Thatcher, ... Dr.		
✓2	To 3 calves, weighing 75, 90, and 95 lb., @ 5 cts., . .	13	00
	—— 28*th*. ——		
	William Perkins, ... Cr.		
✓1	By 1 cow, 4 years old, and calf, ...	30	00
	—— 30*th*. ——		
	Henry Stevens, ... Cr.		
✓2	By Cash in full of his account, ...	5	00
	—— " ——		
	Joseph Thatcher, ... Cr.		
✓2	By sundry charges the past month, as per bill, . . .	8	75
	—— *May* 1. ——		
	Frederick Anderson, ... Dr.		
✓2	To Cash to balance his account, ...	19	90

205. Remarks on the Ledger and Posting.

In the Ledger, the debits and credits of each individual are sometimes placed on two pages of the Ledger, facing each other, and sometimes on opposite sides of the same page.

Either a whole page, or a part of a page, may be appropriated to each account. The name of the person with whom the account is kept should be written in large letters at the top of the account.

Transferring an account from the Day-book to the Ledger is called *posting* it. There should be a column on the left of each page, for the date, and three columns on the right, — one for the page of the Day-book, and the other two for dollars and cents.

When several articles are entered at the same place in the Day-book, they need not be specified in the Ledger, but be posted under the general name of "Sundries."

The difference between the Dr. and Cr. side of an account is called the *balance*. In transferring an account to a new page in the Ledger, or to a new Ledger, this balance is debited in the old account and credited in the new, as in William Perkins' account, below, or credited in the old account and debited in the new, as in Joseph Thatcher's account, on the next page.

206. Ledger A.

William Perkins. page 1

1849.					1849.				
Apr. 1.	To sundries, . .	1	5	30	Apr. 14.	By labor, ...	1	11	00
" 18.	" Cash, ...	1	10	00	" 28.	" cow and calf,	2	30	00
May 1.	" bal. trans. to new acc't,	1	25	70					
			41	00				41	00
					1849. May 1.	By bal. trans. from old acc't, . . .	1	25	70

HENRY STEVENS. p. 2

1849.					1849.				
Apr. 3.	To sundries, . . .	1	28	00	Apr. 5.	By sundries, . . .	1	12	50
					" 20.	" "	1	10	50
					" 30.	" Cash, ...	2	5	00
			28	00				28	00

JOSEPH THATCHER.

1849.					1849.				
Apr. 6.	To hay, 2450 lb., .	1	14	70	Apr.30	By sundries, . . .	2	8	75
" 18.	" sundries, . . .	1	23	25	May 1	" bal. trans. to to new acc't,	2	29	20
			37	95				37	95
1849. May 1.	To bal.trans.fr'm old acc't, .	2	29	20					

FREDERICK ANDERSON.

1849.					1849.				
Apr.15	To sundries, . . .	1	10	35	Apr. 9.	By shingles, &c.,	1	30	25
May 1.	" Cash, ...	2	19	90					
			30	25				30	25

REMARK. When a page in the Ledger is filled, either the *balance* may be transferred to a new page, as remarked on the preceding page, or the Dr. and Cr. columns may be added up, and the *amount* of each transferred to the corresponding column of a new page.

Persons whose business is small find it more convenient to copy every charge in full into the Ledger, than to abridge them as in the above examples, for the reason that it is so much easier to draw off a person's account from the Ledger when the charges are posted in this way, than to turn to the Day-book for every separate item in the account.

207. CASH ACCOUNT.

The *Cash-book*, or Cash Account, has been spoken of as an important auxiliary to the Day-book and Ledger. In this book, all transactions in which cash is either received or paid away, are entered; receipts and money on hand when the account is commenced, being written in the Dr. side of the account, and payments in the Cr.; so that, by adding the debtor and creditor sides of the account, the difference will always show, if there has been no mistake or omission, the amount of cash on hand, and the correctness of the account may be tested by actually counting the money.

The following will illustrate the Cash Account. It may be ruled as the Ledger is ruled, with the exception of the columns for the page of the Day-book, which, in single entry, are not needed.

CASH.

1849.				1849.			
Apr. 3.	Cash on hand, . . .	25	23	Apr. 3.	By cow bought, . .	25	00
" 4.	To vegetables sold, .	5	15	" 5.	" stores, ...	3	18
" 7.	" " "	3	27	" 12.	" labor paid for, .	8	17
" "	" apples sold, . . .	10	00	" 18.	" William Perkins,	10	00
" 12.	" butter and cheese,	15	45	" 20.	" expenses to Boston,	3	50
" 17.	" corn, 15 bushels,	11	25	May 1.	" Fred'k Anderson,	19	90
" 30.	" Henry Stevens, .	5	00	" "	" bal. to new acc't,	15	65
		75	35			75	35
May 1.	To bal., Cash on hand,	15	65				

208. It is sometimes desirable to keep an account of the expenses and income of some particular interest, such as a house that is let, a lot of land, the dairy, &c. Farmers' sons, and *daughters too*, would find it a pleasant and useful exercise to keep such an account with some particular field or garden, a crop of corn, potatoes, &c., and in this way be able at any time to compare the whole expense of labor, &c., laid out upon it, with the income. Such accounts may be kept by making particular and daily entries of every item, or by making one entry at the close of each week.

The following may serve as a model.

THE SOUTH FIELD, 3½ ACRES. *Dr.* *Cr.*

1849.					
Jan. 1.	To value, at $80 per acre, ...	280	00		
Apr. 14.	" 25 cords compost, @ $5, ...	125	00		
" 21.	" Labor on do., ...	5	00		
" 28.	" Breaking up, ...	15	00		
May 4.	" Harrowing and furrowing, ...	5	00		
" 11.	" Planting, ...	15	00		
" "	" Potatoes and corn, ...	6	00		
June 9.	" Cultivating and weeding, ...	5	00		
" 23.	" Cultivating, ...	1	50		
" 30.	" Hoeing, ...	3	25		
July 21.	" " &c., ...	2	75		
Sept. 15.	By early potatoes sold, ...	. . .	.	11	50
Oct. 27.	To harvesting, &c. &c., ...	20	00		
" "	By 120 bushels corn, @ 65 cts., ...	. . .	.	78	00
" "	" 75 " potatoes, @ 60 cts., ...	. . .	.	45	00
" "	" Fodder, ...	. . .	.	20	00
1850.					
Jan. 1.	To interest on value, @ 5 per cent., ...	14	00		
" "	By balance, present value $98 per acre, . .			343	00
		497	50	497	50
1850.					
Jan. 1.	To value, ...	343	00		

209. The following is the form of keeping accounts usually adopted by farmers, mechanics, and others whose business is limited. By this method the Ledger form only is used, the Day-book being dispensed with.

Solomon Giddings. Dr. Solomon Giddings. Cr.

1849.		$	cts.	1849.		$	cts.
Apr. 13	To 3 bush. Potatoes, at 87 cts.	2	61	Apr. 7.	By sundries from store, as per bill, . . .	5	15
" 14.	" 2 days' labor, at $1.25, .	2	50	" 13.	" 1 bbl. Flour, ...	6	25
" 18.	" 2317 lb. Hay, at 65 cts. per 100 lb., ...	15	06	" 17.	" 2 pr. Shoes, at $1.50,	3	00
" 21.	" 10 bush. Corn, at 70 cts.,	7	00	" 25.	" 8 gal. Molasses, at 25 cts., ...	2	00
" 25.	" 1 day's labor, ploughing; myself, boy, 1 yoke of oxen, and horse, . . .	4	00	" "	" Cash in full, ...	14	77
		31	17			31	17

210. Exercises for the Pupil.

Make a Day-book and Ledger of your own, and show how James Sullivan, of Salem, should prepare his books and keep his accounts, both by single entry, (**204**,) and by the form Art. **209**, beginning Apr. 1, 1849,—if he sells to Stephen Lummus, Apr. 3, 2 bush. of potatoes, at 75 cts. a bushel, 10 pounds of cheese, at 8 cents a pound, and 2 bushels of rye, at 75 cents a bushel; sells to James Roberts, Apr. 5, 15 bushels of oats, at 40 cents, and 12 bushels of corn, at 70 cents, and buys of him 1 lb. of tea, at 50 cts., 10 lb. of coffee, at 12 cts., 2 lb. of cocoa, at 15 cts., and 10 gallons of molasses, at 27 cts.; works 3 days, himself and team, Apr. 10, 11 and 12, for Simeon Patterson, at $2.25 per day; receives, Apr. 12, of James Roberts, $5 in cash; sells William Stevens 2 calves, at $3 each; employs Joseph Smith, Apr. 16 and 17, to repair his barn, at $1.50 per day; buys of Stephen Lummus, Apr. 16, 500 feet of boards, at $18 per thousand feet, and one half a thousand of shingles, at $2.25 per thousand; and of James Roberts, 20 lb. nails, at 7 cts., 10 lb. sugar, at 8 cts.; and of William Stevens, 6 lb. of veal, at 8 cts.; sells Stephen Lummus, Apr. 21, 1350 lb. of hay, at 60 cts. per hundred; Apr. 23, has employed Richard Williams one month, at $12 per month; pays Richard Williams, Apr. 24, cash, $5; buys of William Stevens, Apr. 26, 1 cow, for $30; sells Stephen Lummus, Apr. 29, 20 lb. salt pork, at 10 cts. per lb.; to Joseph Smith, 3 bushels of potatoes, at 75 cents per bushel, and 3 lb. of butter, at 21 cents per lb.; pays William Stevens, Apr. 30, cash, $20; employs Simeon Patterson to repair his plough, $1.25, and to shoe his oxen, $1.50; May 1, settles with William Stevens, and with Joseph Smith, receiving or paying cash for the balance due.

BUSINESS FORMS.

211. BILLS.

No. 1. *Common Form.*

MR. STEPHEN PALMER,

To WILLIAM JOHNSON, *Dr.*

1849.		
May 5.	For 3 bu. Corn, at 75 cts. per bu.,	$2.25
" 10 to 13.	" 3 days' Labor, at $1.50 per day,	4.50
June 3.	" 18 lb. Cheese, at 10 cts. per lb.,	1.80
		$8.55

June 15, 1849, *Received Payment.*

WILLIAM JOHNSON.

No. 2. (See **206.**)

MR. WILLIAM PERKINS,

To A. B., *Dr.*

			$	*cts.*
1849.				
Apr. 1.	For 5 bush. Corn, @ 60 cts.,	$3.00		
" "	" 2 " Potatoes, @ 75 cts.,	1.50		
" "	" 4 lb. Butter, @ 20 cts.,	80		
			5	30
" 18.	" Cash per receipt,		10	00
			15	30
	Cr.			
Apr. 14.	By Labor,	$11.00		
" 28.	" Cow and Calf,	30.00		
			41	00
May 1.	Balance due him,		25	70

The pupil should make out bills from the accounts kept with each of the persons named in the preceding pages.

Due Bills.

Salem, *May* 15, 1849. Due to James Sullivan, on settlement, ten dollars. REUBEN CONVERSE.

NOTES.

1. *Note on demand.*

Boston, *Nov.* 25, 1848. For value received, I promise to pay to John Peterson, or order, thirty-five dollars, on demand, with interest.

$35. JOSEPH TOLMAN.

2. *Note payable after a certain time.*

Charlestown, *Feb.* 15, 1849.

For value received, I promise to pay Alpheus Pratt, or order, one hundred and fifty dollars and twenty-seven cents, in two months, with interest after.

JOSEPH GOLDTHWAITE.

$150.27.

3. *Note by two persons.* (See page 144.)

4. *Note to be discounted at a Bank.* (See page 144.)

5. *Note payable to bearer.*

Beverly, *June* 27, 1848.

For value received, I promise to pay Ephraim Sawyer, or bearer, fifty dollars, on demand. STEPHEN SHERMAN.

REMARKS ON NOTES.

1. A note of any of the above forms, except No. 5, is negotiable, that is, may be bought or sold, and the amount collected of the promissor or signer, by any person who may hold it; *provided, that it is endorsed by the person to whom it is made payable.*

2. The person who endorses the note binds himself, by doing so, to pay the note if the person who holds it cannot collect it of the signer; unless when endorsing it he write the words, "without recourse to me," or some other expression of similar import. In this case, he is free from any such obligation.

3. A note payable to a certain person or bearer, as No. 5, can be collected of the signer, by any one who holds the note, without being endorsed.

4. If interest is not mentioned in the note, a note on demand draws interest as soon as it is demanded; and a note payable at a certain time draws interest after that time.

5. If several persons sign a note promising jointly and severally to pay the amount, as in No. 3, the amount can be collected of either of them.

6. If a note is payable within a certain time, in a particular article, as flour or iron, for example, the holder is not obliged to receive the article after the expiration of that time, but can demand the money.

ORDERS AND RECEIPTS.

An Order is a written request to pay a specified sum of money, or amount of merchandise, to the person in whose favor the order is drawn, on the presentation of the order.

Order for Money.

Salem, *May* 4, 1849.

MR. NATHANIEL SHEPARD. Please to pay George W. Saunders, or order, ten dollars, value received, and charge the same to the account of Your obedient servant,

DAVID PORTER.

$10.

Order for Goods.

Salem, April 1, 1849.

MR. JAMES BAKER. Please to deliver to Samuel Baker, goods from store to the amount of fifty dollars, and charge the same to the account of Your obedient servant,

JOSEPH BURBANK.

$50.

Receipt for Money on Account.

Received of Moses Curtis fifteen dollars, eighty-seven cents, on account.

MARY BUFFUM.

$20.

Receipt in full of all accounts.

New York, May 8, 1849.

Received of Thomas A. Brooks thirty-five dollars, in full of all accounts. SIMON FAITHFUL.

USE OF THE FOLLOWING TABLES.

1. To find the amount for which a note must be written, payable at a future time, that shall have a known present value.

RULE. *Multiply the number in Table III. standing directly under the given per cent., and opposite to the months, by the present value desired; the product will be the face of the note, or the amount for which it is to be drawn.*

2. To find the compound interest of any sum of money from 1 year to 20, when the rate is 3, 3½, 4, 4½, 5, or 6 per cent.

RULE. *Multiply the amount of* $1 *for the given time and rate, as found in Table II., the product will be the amount. Subtracting the principal from the amount will give the interest.*

3. To find the amount of an annuity at compound interest, at 3, 4, 4½, 5, 5½, or 6 per cent.

RULE. *Multiply the amount of* $1 *for the given rate and time, as found in Table III., by the annuity; the product will be the required amount.*

TABLE I.

Showing the amount of a bankable note, so that when discounted at 6 *or* 7 *per cent. for any number of months, from* 1 *to* 12, *the present worth shall be* $1.

Months.	6 per cent.	7 per cent.	Months.	6 per cent.	7 per cent.
1	1.00553	1.00646	7	1.03681	1.04321
2	1.01061	1.01240	8	1.04210	1.04959
3	1.01574	1.01842	9	1.04767	1.05606
4	1.02093	1.02450	10	1.05319	1.06261
5	1.02617	1.03066	11	1.05877	1.06923
6	1.03146	1.03690	12	1.06440	1.07594

TABLE II.

Showing the amount of $1, at compound interest, from 1 year to 20.

Year.	3 per cent.	3½ per cent.	4 per cent	4½ per cent.	5 per cent.	6 per cent.
1	1.030000	1.035000	1.040000	1.045000	1.050000	1.060000
2	1.060900	1.071225	1.081600	1.092025	1.102500	1.123600
3	1.092727	1.108718	1.124864	1.141166	1.157625	1.191016
4	1.125509	1.147523	1.169859	1.192519	1.215506	1.262477
5	1.159274	1.187686	1.216653	1.246182	1.276282	1.338226
6	1.194052	1.229255	1.265319	1.302260	1.340096	1.418519
7	1.229874	1.272279	1.315932	1.360862	1.407100	1.503630
8	1.266770	1.316809	1.368569	1.422101	1.477455	1.593848
9	1.304773	1.362897	1.423312	1.486095	1.551328	1.689479
10	1.343916	1.410599	1.480244	1.552969	1.628895	1.790848
11	1.384234	1.459970	1.539454	1.622853	1.710339	1.898299
12	1.425761	1.511069	1.601032	1.695881	1.795856	2.012196
13	1.468534	1.563956	1.665073	1.772196	1.885649	2.132928
14	1.512590	1.618694	1.731676	1.851945	1.979932	2.260904
15	1.557967	1.675349	1.800943	1.935282	2.078928	2.396558
16	1.604706	1.733986	1.872981	2.022370	2.182875	2.540352
17	1.652848	1.794675	1.947900	2.113377	2.292018	2.692773
18	1.702433	1.857489	2.025816	2.208473	2.406619	2.854339
19	1.753506	1.922501	2.106849	2.307860	2.526950	3.025599
20	1.806111	1.989789	2.191123	2.411714	2.653298	3.207135

TABLE III.

Showing the amount of an annuity of $1 from 1 year to 20.

Year.	3 per cent.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	2.030000	2.040000	2.045000	2.050000	2.055000	2.060000
3	3.090900	3.121600	3.137025	3.152500	3.168025	3.183600
4	4.183627	4.246464	4.278191	4.310125	4.342266	4.374616
5	5.309136	5.416322	5.470710	5.525631	5.581091	5.637093
6	6.468410	6.632975	6.716892	6.801913	6.888051	6.975319
7	7.662462	7.898294	8.019152	8.142008	8.266894	8.393838
8	8.892336	9.214226	9.380014	9.549109	9.721573	9.897468
9	10.159106	10.582795	10.802114	11.026564	11.256259	11.491316
10	11.463879	12.006107	12.288210	12.577893	12.875354	13.180795
11	12.807796	13.486351	13.841179	14 206787	14.583498	14.971643
12	14.192029	15.025805	15.464032	15.917127	16.385590	16.869941
13	15.617790	16.626838	17.159913	17.712983	18.286798	18.882138
14	17.086324	18.291911	18.932109	19.598632	20.292572	21.015066
15	18.598914	20.023588	20.784054	21.578564	22.408663	23.275971
16	20.156881	21.824531	22.719337	23.657492	24.641139	25.672528
17	21.761588	23.697512	24.741707	25.840366	26.996402	28.212880
18	23.414436	25.645413	26.855084	28.132385	29.481205	30.905653
19	25.116868	27.671229	29.063562	30.539004	32.102671	33.759992
20	26.870374	29.778078	31.371423	33.065954	34.868318	36.785592

SUGGESTIONS TO THE TEACHER.

1. The questions in the smaller sized type are designed for mental and oral exercises.* Although these exercises are more numerous and extensive than are to be found in other similar works, it is hoped that the teacher will not limit his pupils to these exercises, but that he will extend them as far as in his judgment may be practicable.

2. Exercises marked thus, II., are rather more difficult than others, and may be omitted by beginners, until the book, or portions of it, are reviewed.

3. The importance of frequent reviews cannot be overestimated by the teacher. By such reviews, the pupil will acquire that familiarity with first principles, and that facility in performing arithmetical operations, which are necessary to render his progress both rapid and pleasant, and which can be acquired in no other way. Both teacher and pupil should aim to be *thorough*. With this end in view, the teacher will not confine himself to the exercises prepared by the author, but will extend them till the end is attained; for no author can anticipate the precise number of exercises each pupil will need upon any one principle before he will be prepared to advance to another.

4. The pupil should be required to prove his work to be correct, as far as practicable. For example: All his operations in Division should be proved by Multiplication; those in Reduction Ascending, by Reduction Descending, and the reverse, when the pupil has progressed far enough to be able to do it. Operations in Proportion should be proved by Analysis, &c. The pupil's progress by this method will be apparently slow at first, but the facility and correctness which he will acquire in this way, will render his future progress far more rapid and satisfactory than it would be without such training.

5. The teacher should ever bear in mind that all the topics treated of in arithmetic are not of equal importance to every pupil, and that he should adapt his instructions, as far as practicable, to the peculiar wants of the pupil. The scholar whose opportunities for learning arithmetic are very limited should be exercised very thoroughly in the elementary rules, and in their application to as great a variety as possible of common business transactions. He should be encouraged "to make up questions" for himself, and solve them, and every method should be used to render the knowledge he may acquire most useful to him when his short term of pupilage shall have expired. He who is intended for the counting-room should be carefully drilled in Practice, Percentage, Equations, Accounts Current, &c., Art. 100 to 125. While the future mechanic should be as thoroughly drilled in the

* The only exceptions are on pages 55 and 151.

Square and Cube Root, and their application to a great variety of practical questions, in Mensuration and the Mechanic Powers. And he who has sufficient time to devote to study should be made familiar with all the topics treated of, and thus be better fitted to engage in any occupation than he can be whose attention has been confined to only a part of these topics.

The "practical application" of the Square and Cube Root, contrary to the practice of authors generally, is, in this book, postponed till the study of Geometrical Definitions and Mensuration has been commenced. The reason for this is, that such "practical questions" are little else than arithmetical puzzles to the pupil who has no knowledge of the principles of Mensuration; — puzzles which the teacher is generally obliged to solve for the pupil, not only the first time he goes over them, but at every subsequent review of them; at least, such has been the author's experience. Whereas, if taken up in connection with the subject of Mensuration, where, indeed, they belong, no such difficulty exists.

The *beginner* in arithmetic should be taught to call the sign of addition *and*, and the sign of subtraction *less*. Thus $8+6=14$; and $15-5=10$, should be read 8 *and* 6 *are* 14, and 15 *less* 5 *is* 10.

The following answer to the question, "How much and what kind of assistance shall I give my pupils while they are pursuing their studies?" is submitted for the consideration of the teacher.

"If the proposition of the text-book is not understood by a pupil, he should be required to point out definitely to the teacher what it is which he does not understand, and then, *not before*, the teacher may give him the help he needs. If a pupil complains of not understanding the meaning of the text-book, the teacher should, generally, require him to read the passage aloud, telling him to stop when he comes to a word or expression which he does not understand. In four cases out of five, the difficulty will vanish without a word of explanation from the teacher. It is important also to require the pupil, in most cases, so to frame his question that it may be answered by *yes*, or *no*. So that, instead of saying, 'Please tell me what this means;' or, 'How shall I perform this question?' he shall say, 'Does this mean so and so?' or, 'Is this question to be solved by such a rule?' or, 'Is it similar to such a question?' The answer to such questions may be *yes*, or *no;* but more generally it should be, 'Why do you think so?' By such means the pupil will be trained to a careful study of principles, and will learn not to depend upon his teacher to remove every little difficulty."